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Some Observations on Mathematical Programming

by Alex Orden
(From the Opening Session Address at the Boston Symposium, Ed.)

The Constitution of the MP Society calls upon the Chairman of the Council to give a report on the state of affairs of the Society at the end of his term of offfice. It seems a bit odd for that to be included in the Constitution, but so be it. The Constitution also specifies that the Chairman's term of office runs until one year after each of the triennial international symposia, at which time the Chairman-elect becomes Chairman. So at this point a year from now, Michel Balinski will become Chairman, and I will then perhaps, heeding the Constitution, prepare a final report. The time of a symposium seems then to be a time for an interim report, or more accurately in this case, for some general observations.

It is appropriate here to examine broad matters which surround our scientific domain. Let us look at matters for which a much wider time frame than that of scientific developments of the last two or three years is appropriate. I will sweep from 1950 to 2072, from 35 years ago to nearly 100 years into the future.

1950 is a nice round mid-century year back around the time when Mathematical Programming was getting started. No computer program for the simplex method had yet been written; in fact I think no suitable computer had yet been successfully built and put into operation. In the timetable which the MP community devised in about 1965, "Symposium Zero" took place in 1948 as part of a meeting of Econometricians at the University of Chicago at which George Dantzig first formally presented the foundations of linear programming. Tjalling Koopmans subsequently produced the famous conference proceedings on "Activity Analysis." In 1950 Symposium 1, the first devoted entirely to linear programming, had not yet occurred. That took place in 1951, and as you know, Symposium 12 in 1985.

(With June 1950 in mind, I can't resist reminiscing that about 75 yards from this podium, there is a gymnasium which was used at that time for commencement ceremonies. That was where MIT granted me a diploma which confirmed that I had taken enough courses in mathematics and written a suitable dissertation so that I could go on to other things. I expected to engage in mathematical applications of the about-to-explode field of electronic digital computers. And so a couple of months later, totally unaware that the big-bang called the simplex method had been invented, I found myself working with George Dantzig in the Pentagon on the mathematics of linear programming and on initial computer programs for solving LP problems.)

As for the other reference year in this discussion – 2072 – that will be the 100th anniversary of the founding of the Mathematical Programming Society. Everyone here knows, I suppose, that the MP Society is a world-wide organization for fostering research and other activities in its field. In offering some thoughts on the place of the Society in the field along broad lines which we don't usually discuss, I will look first at the professions which have participated in research in mathematical programming during the past 35 years, and then briefly at the role of MP in the world at large, looking speculatively at the far future.

First, a digression about international societies: In the political, economic, and social arenas world-wide organizations

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Martin Beale (1928-1985)

Martin died at his home in Cornwall on December 23rd, 1985, having been seriously ill for several months. Participants at the Mathematical Programming Symposium in Boston last summer witnessed his courage and selflessness, and there seemed to be no end to his enthusiasm for the successful development of our subject. We can remember him sitting at the front of a lecture room, both figuratively and literally, in order to take a close interest in events and to contribute his views to discussions. Occasionally he found himself at centre-stage; the Mathematical Programming Society (MPS) as well as mathematical programming research and applications have gained greatly from his leadership.

He was educated at Winchester and at Trinity College, Cambridge, graduating in Mathematics in 1949 and gaining a diploma in Mathematical Statistics in 1950. He then joined the Mathematics Group of the Admiralty Research Laboratory in Teddington, England, where he worked for 11 years with Stephen Vajda, except for a leave of absence in 1958 at the Statistical Techniques Research Group of Princeton University. In 1961 he moved to become Manager of Mathematical Statistics at CEIR (now Scicon) in London, where he was promoted to Technical Director in 1968. Further, he accepted a visiting Professorship in the Mathematics Department, Imperial College, University of London, in 1967. Thus he was both an academic and an industrial applied mathematician for the last 18 years.

Since the MPS grew from the sequence of mathematical programming symposia, his part in the organisation of the London symposium in 1964 can be

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strive with little visible progress to find paths to harmony in a world of terrifying discord. The United Nations is the clearest example. In the "scientific world" things are much better, but the discord and disorganization of mankind penetrates. Those who undertook in 1972 to establish the MP Society soon learned that they were forming an entity which would have no legal status. We live in a world in which the laws, all laws, are the laws of nations. There are no international laws, only some treaties and other agreements for which there are no enforcement mechanisms. We learned back in 1972 that in setting itself up to be truly international, the MP Society would be born in a legal vacuum, stateless, like a stateless person.

But in a legal vacuum or not, international scientific organizations do exist, and in contrast to political organizations they commonly operate in a fairly good state of harmony. At times one senses a wistful feeling in the international wilderness about how nice the world would be if only international political, economic, and social organizations were as harmonious as scientific organizations. What a utopian thought! Since international scientific organizations operate harmoniously in a legal vacuum, it seems we should aspire to world conditions so benign that laws are unnecessary.

While the MP Society, like many international scientific societies, operates largely in harmony, it does contain factions. I suggest that the professions which participate in MP research be the identifiers of the factions.

There are in the first place many specialists in MP who, when among their colleagues, comfortably indicate that professionally they are "mathematical programmers," thereby giving a clearer indication of the nature of their work than saying that they are mathematicians, engineers, economists, or the like. In other words, a considerable number of those here would be glad to be known as "mathematical programmers" if the world at large knew what that meant. But there the phrase is unknown, or worse yet, is understood erroneously. In that renowned vehicle-of-record, the New York Times, in which we have been glad (or I could say proud) to see items on new developments in MP appear occasionally on the very front page, I doubt that there is even one editor or reporter who knows that there are people who identify themselves professionally as "mathematical programmers." And now coming to the worse–yet part, I doubt that anyone connected with the NY Times knows that the word programming in the term, mathematical programming, refers to the planning of activities, not to the programming of computing machines.

Now, while there are, I believe, many at this symposium and elsewhere for whom what I have just said applies, there are in the MP society and in the field overall, other important constituencies. The largest, described in ways with which you may not fully agree, are:

1. Mathematicians: Those who view themselves as mathematicians whose strongest interests happen to lie in theorems and algorithms for extremal problems, or more specifically, constrained optimization. The mathematicians in the MP field are interested in developing appropriate theorems and mathematical properties of algorithms while the "math programmers" are not content unless the results of their work on the mathematics, algorithms, and computation contribute to solution of real-world models.

2. Operations Researchers: Those who consider their professional field to be technical support of planning and decision making by means of all formalizable methods ranging across mathematical modeling, statistics, computer-based heuristic simulation and, nowadays, prospectively the computerization of complex logic which goes under the rubric, "expert systems."

3. Computer Scientists: Those whose view of their profession is that in conjunction with the mammoth phenomenon known as "the computer" there has come to be an underlying scientific field which, for lack of a better-sounding term, is called Computer Science, and within that discipline happen to specialize in optimization and combinatorics algorithms and in related matters involving computational complexity, software for model generation, prospective advances in parallel computing, and possibly the linkage of artificial intelligence to mathematical programming.

There are still other professional constituencies in MP, particularly engineers and economists, but the influence of those disciplines on MP research is currently smaller than those which I have previously listed. Back in 1950, economists were probably the largest group, but the participation of economists in recent symposia in MP has been quite small.

Over a recent time span, say the last 10 years, what has been happening, I believe, is that the proportion of two of the constituencies, those who think of themselves professionally as math programmers and those who view themselves as mathematicians (with strong interest in MP) have not changed. The proportion of operations researchers seems to have declined somewhat, and that of computer scientists has clearly grown. Certainly if you look over the program of this symposium, you will see a significant number of papers which would be equally appropriate at a meeting of computer scientists.

Clearly the presence of several types of researchers invigorates this symposium. Much will be gained from the flow of ideas between the math programmers, the mathematicians, the operations researchers, the computer scientists, and others. It is pertinent at the same time to ask whether the constituencies are factions which, as in international political or economic or social organizations, have conflicting interests. In some ways I believe they do.

Although I've laid some groundwork, I'm not inclined on this occasion to probe those issues. In the operations research societies there are heated controversies between theoreticians and practitioners, but that is not the heart of the matter in the MP Society. For us it is important in the first place to assess which kinds of research in MP have the best prospects of high and lasting significance, and secondly, in my opinion, to identify and practice the scientific paradigms which are best for our kind of science.

To do that calls for a "vision" of the field. Allow me to go beyond vision to visionary. I raise two questions:

1. In a far reaching (perhaps utopian) sense, what are the broad aims of MP, expressed in terms which can be easily understood by the world at large?

2. How important is the MP field to that world at large, not only now, but quite far off in the future?

In order to address these questions, I ask first that you distinguish between modeling for optimization in engineering and MP-type modeling in matters of human organization and business affairs. Engineering models are based on principles of physical science and engineering, and what sometimes comes up is a need for optimization algorithms to solve given problems. But in organizational and business matters, and so to speak in human affairs in general, math programming developments involve not only

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Martin Beale *from page 1*

regarded as his first major contribution of the activities of the Society. In 1971 the original Founding Committee of the MPS (J. Abadie, M.L. Balinski, A. Orden, A.W. Tucker, P. Wolfe and G. Zoutendijk) was extended to an Organising Committee that included Martin. The members of the newly-formed society then chose him as their first "Chairman-Elect," so in 1974 he succeeded George Dantzig as Chairman of the MPS. During his chairmanship the first issue of Mathematical Programming Studies appeared and the Working Committee on Algorithms was established, but his two years of leadership seemed to pass in a flash, for Al Tucker replaced him at the Budapest symposium in 1976. However, he was elected again to Council in 1982. Further, he served as a Senior Editor of Mathematical Programming and of Mathematical Programming Studies after these journals were founded. These appointments give some indication of his contributions to the Society, and we are indebted to him.

His range of professional interests and his participation in many fields are remarkable. In addition to his activities in the MPS, he was a member of the International Statistical Institute, a Fellow of the British Computer Society, a Vice President of the Institute of Mathematics and its Applications, and both an Honorary Secretary and a Vice President of the Royal Statistical Society. He was awarded the Silver Medal of the Operational Research Society and was elected a Fellow of the Royal Society of London in 1979 "for his applications of mathematical and statistical techniques to industrial problems, and for his contributions to the theory of mathematical programming." Further, he became a member of Council of the Royal Society in 1984.

The intersection of his research interests with mine include quadratic programming and the conjugate gradient method for nonlinear optimization. In both of these fields his work is of central importance. Further, I believe that his contributions to discrete optimization have had a greater impact and that his research in statistics was just as strong. Further, he applied mathematical programming and statistical techniques very successfully for the modeling and solution of many serious real problems. This extraordinary range of skills was surely far more than a consequence of the needs of customers of Scicon. Indeed, he was the most assiduous listener to papers at conferences that I have known, his thirst for professional knowledge was unquenchable, and he communicated his expertise very readily. Discussions with Martin helped my own work directly, and I am sure that the indirect benefits are far greater. Here I have in mind not only his contributions to knowledge and their part in further research, but also his influence at the interface between the development of algorithms and real-world computing. It is of vital importance to our subject that much of our work is actually useful, but most computer users find published algorithms indigestible. Therefore, Martin's achievements in bridging this gap are of outstanding value to the global standing of mathematical programming.

The mathematical programming community abounds with affectionate and lively memories of Martin. We can recall his self-effacement, his gentleness, his delightful humor, his dislike of unnecessary mystification in mathematics, and his intellectual honesty. Let me leave you with your personal memories of Martin, as George Dantzig and John Tomlin are compiling a biographical portrait of him. We are thoroughly grateful for his warm friendship and for his illustrious career in mathematical programming.

M.J.D. Powell
March 20, 1986

Some Observations *continued*

algorithms but mathematical structures which people may sooner or later choose as the best way to organize how to think about various problems. There is a profound difference between the latter and the computation of solutions to given engineering problems. We should not carry lightly views and paradigms from engineering and physical science into math programming for human affairs. With this in mind my (visionary) answer to question 1 above is:

All human beings, in most aspects of their lives and particularly in the organization and economic aspects, seek constantly, sort of minute by minute, to attain objectives. Math programming is the science of expressing those human efforts in mathematical form and finding solutions to the mathematical formulations.

It is conceivable that the development and introduction of ways to think in math programming terms about common organizational and business problems are at present in their relative infancy, with a current role in human affairs which is analogous, say, to that of air transportation in about 1920. Such an analogy has, of course, no predictive value, but the view that a time may come when the role of mathematical optimization in society is a great deal larger than it is today is, for researchers in our field, at least a fond hope. The question of whether MP will come to have an extensive role in the world at large may be phrased: will such a time ever come, and if so, how far off is it?

Any attempt to answer this question is wildly speculative. I will go only so far as another, perhaps better, analogy:

The MP Society is an organizational symbol of math programming as a field of knowledge. In 1972 when the Society was established, we made a deal with the International Statistical Institute's headquarters in The Hague to provide us with a secretariat, that is with a place for keeping our membership records, for issuing and tallying election ballots, and the like. It was comforting that the ISI was a distinguished international organization of the kind that we aspired to become. ISI was founded in 1885 and is celebrating its 100th anniversary. The growth of knowledge of statistical concepts and methods and the spread of the scientific use of statistics in innumerable human activities since 1885 has been phenomenal. Will the growth of math programming from 1972 to 2072 be comparable? Will MP or related models and solutions for many everyday problems in human planning be as ordinary on the 100th anniversary of the founding of the MP Society as use of at least the basic techniques of statistics is today, 100 years after the founding of the ISI? Perhaps so. If so, our work is paving the way.

Global Optimization Computational Study

A computational study of algorithms for **global optimization** will be carried out in accordance with guidelines discussed during a recent SDS-IIASA workshop in this area.

As a first step, appropriate test prob- lems are being assembled featuring arbitrary objective functions and con- straints. (Test problems for the special case of concave functions subject to linear constraints are available from J.B. Rosen, 136 Lind Hall, Minneapolis, MN 55455, U.S.A.)

Those interested in contributing to the computational study are invited to contact C.G.E. Boender and A.H.G. Rinnooy Kan, Econometric Institute, Erasmus University Rotterdam, P.O. Box 1738, 3000 DR Rotterdam, The Netherlands.

–A.H.G. Rinnooy Kan

CO'87

April 6-8, 1987
Southampton, U.K.

A conference on the theory and application of Combinatorial Optimization in Operational Research, Management Science, Computer Science and Statistics will be held at the University of Southampton, U.K., from April 6 to April 8, 1987.

The topics include integer program- ming, complexity theory, analysis of algorithms, polyhedral combinatorics, applications to coding theory and cryptography, parallel and sequential computing, telecommunications.

Abstracts of contributed papers should be sent before January 5, 1987 to

The Secretary, CO'87
Department of Mathematics
University of Southampton
Southampton SO2 5NH, U.K.

Position Announcement

George Mason University
Systems Engineering Department

Applications are sought to fill a tenure-track opening at the assistant, associate, or full professor level. Candidates should have a Ph.D. in operations research, systems engineering, applied mathematics, etc., with strong interest in computational issues in mathematical programming. Applicants must demonstrate outstanding achievement or potential for research and a commitment to graduate and undergraduate teaching.

Application letter, vita, and three reference letters should be sent to: Carl Harris, Chairman, Department of Systems Engineering, 4400 University Drive, George Mason University, Fairfax, VA 22030.

The Seventh Mathematical Programming Symposium

Nagoya, Japan
November 6-7, 1986

This annual symposium will be held November 6-7, 1986, at Nagoya International Center, Nagoya, Japan. It will consist of the following three sessions:

1. Mathematical Programming, General. Chairman: S. Fujishige.
2. Scheduling and Production Control. Chairman: T. Ibaragi.
3. Applications. Chairman: S. Enomoto.

The first two sessions will consist of three or four talks of an expository nature and those presenting original development. There will be no call for contributed papers; only invited papers will be presented.

Participation from abroad will be welcomed. The conference language is Japanese, but non-Japanese participants may use English.

For further information contact Organizing Chairman, Professor Masao Iri, Faculty of Engineering, University of Tokyo, Bunkyo-ku, Tokyo 113, Japan, or Program Chairman, Professor K. Sawaki, Faculty of Business Administration, Nanzan University, Nagoya 466, Japan.

Call for Papers

International Conference on Vector Optimization

This conference will be held August 4-7, 1986, at the Technical University of Darmstadt, F.R. Germany. Participation is open. Authors are invited to submit abstracts by May 1, 1986. A proceedings will be published. For further information write Vector Optimization, Tech. University of Darmstadt, FB4-AG10, 6100 Darmstadt, F.R.G.

A Model-Management Framework for Mathematical Programming
By Kenneth H. Palmer
John Wiley, New York, 1984
ISBN 0-471-80472-X

This book covers one aspect of the application of linear programming to real problems in real industries, namely the care and feeding of large formulations. It gives a description of the software system PLATOFORM (Planning Tool written in DATAFORM), a support system for mathematical programming applications. PLATOFORM was developed using the Enhanced Mathematical Programming System (EMPS) and its associated language, DATAFORM. The products are currently marketed as MPS III by Ketron, Inc. All six of its authors have been closely involved with the basic design and subsequent growth of this system which, since 1972, has been an indispensable tool used throughout Exxon, one of the largest international corporations. Thus, most of the examples used to illustrate the text are based on an oil-refining application. The main feature of PLATOFORM is the possibility of adapting to any application. PLATOFORM has proven useful in the field of nonlinear models as well.

This book contains 12 chapters. The first two deal with historical development within Exxon. Chapter 3 provides sufficient details of the DATAFORM language. The detailed structure of PLATOFORM is described in chapters 4 through 11: Data and File Structure, System Structure, Input Syntax, Data Management, File Management, Matrix Generation, Optimization, and Report Writing. By the end of Chapter 11, the reader will have been exposed to internal construction of quite a complex system. The purpose of the final chapter is to look at this system from the user's point of view. The authors have taken a simplified oil-refining problem to show how PLATOFORM would be used to set up, solve and report on such a model.

The book is easy to read and well-illustrated. Each chapter concludes with a summary and has many links to other sections. The pros and cons of conception and solution are considered in full detail from a practical point of view. Thus, it is not a big surprise that literature references are not given!

The material presented will be of use and interest to:

- Practitioners of LP who need to know the problems that arise in its application and ways to solve them.
- Developers of LP-software so that their products match the requirements of actual business problems.
- Those in academia who teach and study these topics.

–K.-P Schuster

Stochastic Models in Operations Research Vol. I, II
By D.P. Heyman and M.J. Sobel
McGraw-Hill, Hamburg, 1982/84
ISBN 0-07-028631-0
ISBN 0-07-028632-9

These books are divided into three parts. In Part A the basic theory of stochastic processes is presented with its illustrations in Operations Research and other fields. In Part B operating characteristics of a wide variety of models, primarily of congestion and storage type, are investigated. These two parts form Volume I. Part C, in Volume II, handles optimal control problems of stochastic processes.

Part A, a short and revealing chapter, introduces the reader to the areas of application of stochastic processes. Then a few pages are devoted to definitions which are followed by seven chapters entitled: Birth-and Death Processes, Renewal Theory, Renewal-Reward and Regenerative Processes, Markov Chains, Continuous-Time Markov Chains, Markov Processes, and Stationary Processes and Ergodic Theory.

In Part B there are three chapters. The first has the title System Properties and deals with problems such as reservoir operation, inventory control, and effects of order of service priorities on waiting times concerning queuing models. The second chapter deals with networks of queues. In the third chapter, Bounds and Approximations, primarily queuing and diffusion processes are investigated. Volume I ends with background material on Probability Theory.

In Part C, elementary models of stochastic optimization are mentioned. Then the expected utility criterion is discussed in detail. The next chapter, Myopic Optimal Policies, handles control of finite and infinite horizon decision processes with one and more-than-one decision variable. The following three chapters are devoted to Markov decision processes. The authors always start with nice elementary problems and turn to the more general theory when the practical importance of the processes involved is well understood. Many methods, including linear programming, are mentioned to find optimal policy. The next two chapters analyze classes of models, primarily of production and inventory type, with specially structured optimal policies. Finally, there is a chapter on sequential games which generalizes Markov decision processes. This volume closes with some background material on Probability Theory and Mathematical Analysis.

These books are very well written and can be recommended for teaching in graduate schools. While primarily textbooks, they can be used by practitioners too for the solution of classes of control problems of stochastic processes. The only critical remark of the reviewer is that the very powerful and very practical branch of science, stochastic programming, is entirely left out. Thus the books, with all their merits, handle only part of the stochastic models in Operations Research.

–A. Prekopa

Progress in Combinatorial Optimization
by W.R. Pulleyblank
Academic Press, London, 1984
ISBN 0-12-566780-9

This volume contains 21 papers presented at a Conference on Combinatorics held at the University of Waterloo in the summer of 1982. Included are five papers on matroid theory, two on perfect graphs, two on total dual integrality and two on scheduling. The remaining subjects are: facets of polyhedra, finite interval orders, submodular flows, submodular functions, the ellipsoid method, combinatorial problems on matrices, greedoids, problems in dynamic periodic graphs, heuristics for graph partitioning and integer programming.

All the material has been reviewed, and it may be considered as an up-to-date reference for graduate students and researchers on Combinatorial Optimization.

–F. Barahona

Algorithms & Software (Special Bibliography)

Fachinformationszentrum Energie, Physik, Mathematik GmbH

Karlsruhe, 1985

This bibliography is a loose-leaf edition containing information about both software and mathematical solution algorithms published in scientific literature. More than 4000 documents dating to 1984 and concerning mathematical problems as well as problems of computer science, physics, operations research, etc., are collected in five files. A sixth file containing an introduction and three indices (an author's index, a cross-reference list and a KWIC-index based upon keywords) is added.

The entries are classified according to the "Mathematics Subject Classification Scheme." The information given to each entry is divided into three blocks. First, bibliographical data are listed, such as the name of the algorithm, its author and its source. The second block yields information about the documentation, especially concerning the programming language, the complexity of the algorithm, the implementation, availability, etc. Finally, the purpose of the algorithm is described.

It is worth mentioning that no control of quality or proof of correctness of the documents has been carried out. However, corrections, counterexamples or a proof to the contrary, if known, are mentioned.

–M. Alfter and K. Mattar

Matrices and Simplex Algorithms

by A.R.G. Heesterman

Reidel, Dordrecht, 1983

ISBN 90-277-1514-9

The book is devoted to mathematical programming algorithms and is to serve as a textbook. It consists of five chapters, all quite long.

The first chapter deals with the fundamental tools of linear algebra which are used in linear programming. This undergraduate material describes the uses of matrices, systems of equations and determinants. In the second chapter, dealing with linear programming, the author gives the basic theory of the Simplex Method augmented by a number of valuable hints and proposals for the numerical treatment of problems. The third chapter introduces mathematical programming problems under more general circumstances. Here we find many interesting results on optimality conditions and feasible sets. In chapter four quadratic programming and efficient algorithms for solving quadratic problems are discussed. The author starts with linear restrictions and then explains the general quadratic case. The last chapter introduces the concept of integer programming. Here the branching method and the use of cutting planes are described. A main feature of the book is the great number of examples, graphical illustrations and the extensive and valuable offer of code listings for the algorithms under discussion. These code listings are written in a readable manner with many comments such that they are instructive for the reader. Most of the theoretical results are explained with numerical examples which are discussed in detail.

But here I see a certain danger of the presentation. It is not easy to see the theoretical and general background clearly because one is concentrating on the example under discussion. An additional difficulty results from a number of non-standard definitions, such as interior point or convex function, etc. Also the derivation of the results and algorithms is long due to the less-formalized language.

As I see it, the main value and purpose of the book (agreeing with the author's intention) is as a textbook for readers with knowledge of mathematical programming rather than as an instruction book for undergraduate students. The textbook will be very helpful to the reader acquainted with the basic concepts of mathematical programming and interested in numerical details as well as in efficient codes. It will also benefit students as a resource for numerical examples and code details.

–K.H. Borgwardt

Linear Programming

by Katta G. Murty

John Wiley, New York, 1983

ISBN 0-471-09725-X

This book is the first of a "trilogy" consisting of textbooks on linear programming, combinatorial programming and linear complementarity. This series is meant to extend the contents of the book, *Linear and Combinatorial Programming* (1976), by the same author. It is the aim of this first book to cover all theoretical, practical and computational aspects of linear programming.

The first chapter discusses the methods for obtaining a linear programming (LP) model for a given practical problem (including e.g. piecewise linear approximation and multiobjective LPs). Chapter 2 presents the original version of the simplex method of G.B. Dantzig. Chapter 3 deals with the mathematical background. All the material of linear algebra needed to understand the mathematical theory of LP is presented. In the next chapter duality, especially the economical aspects, is discussed in detail. This chapter contains the various theorems of stability in the LP models and conditions for unique solutions. Computational and numerical aspects of linear programming (which are often omitted in textbooks) are extensively discussed in Chapters 5-7. Guidelines for designing computationally efficient implementations of the simplex method are given; column-generation techniques, dual simplex methods and the various factorization methods are explained.

Chapters 8 and 9 deal with the postoptimal analysis of a linear programming problem. In Chapter 8 we can find the parametric linear programs in detail while Chapter 9 presents the sensitivity analysis. Degeneracy problems are considered in Chapter 10. Here Lexico-Simplex algorithms, both in the primal and dual case, are presented. Chapter 11 considers the bounded variable LPs, and Chapter 12 discusses the methods for large-scale LPs. Transportation problems are the focus of Chapter 13.

Chapter 14 discusses worst-case and average-case behaviour of the simplex algorithm. The last three chapters extend the 1976 book. They contain the ellipsoid method, iterative methods for LPs and vector minima.

The book does not contain new results (of course, this was not intended). It provides an introductory course in linear programming as well as a basic reference for the experienced reader since it covers all important aspects of linear programming. A main advantage of the book is its clear and understandable style. The chapters are constructed to require a minimal mathematical background; numerous examples and exercises are given. At the end of each chapter a brief reference list can be found. Unfortunately, no solutions to at least a subset of the exercises are given. This would have helped control the reader's knowledge.

In summary, Murty's book can be rated as a valuable textbook for the inexperienced as well as for the advanced reader.

–G. Galambos

Selected Topics in Graph Theory 2
Edited by Lowell W. Beineke and Robin J. Wilson
Academic Press, New York, 1983

Four years after the success of Selected Topics in Graph Theory 1, which is considered one of the top six mathematical books for 1979, Beineke and Wilson have published the second volume.

The exceptionally favorable critique of the first volume was well merited, and those who have read it would be as pleased with the second. It is indeed as good as the first. This second book, although dealing with different topics from the first one, has kept the same spirit and style. Each chapter, written by an expert in the field, gives a survey of that field of graph theory. The chapters are titled as follows: Eulerian Graphs, Perfect Graphs, Automorphism Groups of Graphs, Infinite Graphs, Extremal Graph Theory, Random Graphs, Graphs and Partially Ordered Sets and Graphs and Games.

The list presents a large and diversified panorama of graph theory. The chosen topics are still the field of interest of a large number of researchers. The choice of the topics was guided by the need for surveys on particular topics, the timeliness of certain areas, suggestions from colleagues and friends, and the editors' own preferences.

In the introduction we find the same notations used by all the authors as well as certain definitions necessary for the understanding of any part of the book. From one chapter to another we find a great resemblance in style, manner of presentation, and degree of difficulty. This facilitates the transition from one chapter to another, making it one of the more interesting aspects of the book. Each author presents a complete survey of his topic. Moreover, the absence of superfluous technical details lightens the task of the reader. Most proofs are voluntarily omitted or simply replaced by sketches of proofs or outlines of proofs. Thus, the reader is always in touch with the main ideas. Each author knows how to present the motivations and the important results of the topic. Even more recent results are given. Each chapter is 30 to 40 pages and gives a quick and comprehensive overview of the subject. Furthermore, many (50 to 120) references are given, including the most recent. This is important in a discipline which progresses very rapidly. This book will interest specialists as well as advanced beginners who are interested in a particular topic.

However, we regret the absence of the algorithmic aspect of a large majority of the topics. Specifically, the application of the complexity theory in graph theory should not be omitted. This probably could be explained by the emphasis given to the pure aspects of graph theory. Note that the graph isomorphism problem has not been proven NP-complete yet (if it has been proven, please give a reference) and that a line graph is not a perfect graph. It is up to the reader to verify the rectification of these mistakes elsewhere in the book. Note also that it would be useful to mention the topics treated in the first volume.

Nevertheless, this is an excellent collection and is indeed useful for professional graph theorists, for newcomers in the field, and for experts in other fields who may want to learn about specific topics. This book should definitely be in libraries accessible to researchers in combinatorics. Note that Selected Topics in Graph Theory 1 and 2 are already accompanied by a volume which is a bit more applied, Applications of Graph Theory, also edited by Beineke and Wilson. We would like to inform the reader that the third volume of Selected Topics in Graph Theory is in its gestation period.

–M. Burlet

Handbook of Algorithms and Data Structures
by G.H. Gonnet
Addison Wesley, London, 1984

Although a lot of books on data structures and algorithms have been written, this book is outstanding and new in its field. It is a real "handbook." A great number of useful data structures and algorithms are listed and analysed in a compact and convenient form.

The major advantage of this book is its extensive average case analysis of the algorithms considered which is outstanding and state of the art. However, I feel that the description of multidimensional searching algorithms falls short in comparison to the extensive treatment of hashing. This might be because the author himself has been very active in the latter area, but such considerations are inappropriate for a handbook.

The structure of the book and partitioning of the algorithms into searching, sorting, selection and arithmetic algorithms are useful for programmers and computer scientists who wish to find a suitable data structure for a given problem. The author intends to contribute to a change of computer science from an art to a science and describes all his algorithms in PASCAL and C directly. Thus, a programmer has something "to stand on." However, some doubts remain about its ability to make available the wealth of information in one book which this field has generated in the last 20 years, as claimed by the author.

In summary, this is an outstanding book with a highly condensed description and analysis of a great number of algorithms. Especially the extensive average case analysis makes this book very interesting for the programmer as well as for the computer scientist.

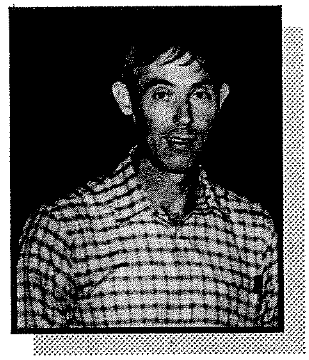
–H. Noltemeier

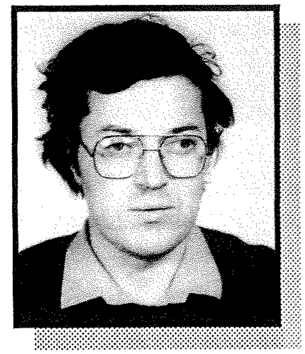
Gallimaufry

The *New York Times* recently reported that **Professor L.V. Kantorovich** died on April 7, 1986. Professor Kantorovich was a Nobel laureate and a senior editor of *Mathematical Programming*... Georgia Institute of Technology announces the appointment of **George L. Nemhauser** to the A. Russell Chandler III Chair. . .IFORS reports that its new officers are **J. Lesbourne** (Paris) as President and **S. Bonder** (USA), **B. Kavanagh** (Australia) and **F. Ridgway** (Ireland) as Vice-Presidents with **J.R. Borsting** continuing as Treasurer. . .**Carl-Louis Sandblom**, formerly of Concordia University, has been appointed as Professor of Industrial Engineering at the Technical University of Nova Scotia. . .The IFORS '87 conference will be held August 10-14 in Buenos Aires.

OPTIMA No. 17 contained a Dantzig prize article which should have been attributed to **George Nemhauser** and **Mike Powell** as well as **M. Balinski** and **R. Wets.**

Deadline for the next *OPTIMA* is August 15, 1986.





Prize winners at Boston:
Michael Saunders (top), winner of Orchard-Hays Prize and **Jozsef Beck**, a Fulkerson Award Winner.

Books for review should be sent to the Book Review Editor, Prof. Dr. Achim Bachem, Mathematiches Institute der Universität zu Köln, Weyertal 86-90, D-5000 Köln, West Germany.

Journal contents are subject to change by the publisher.

Donald W. Hearn, Editor
Achim Bachem, Associate Editor
Published by the Mathematical Programming Society and Publication Services of the College of Engineering, University of Florida.
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FIRST CLASS MAIL

C A L E N D A R

15 April 1986 Maintained by the Mathematical Programming Society (MPS)

This Calendar lists noncommercial meetings specializing in mathematical programming or one of its subfields in the general area of optimization and applications, whether or not the Society is involved. (The meetings are not necessarily 'open'.) Anyone knowing of a meeting that should be listed here is urged to inform Dr. Philip Wolfe, IBM Research 33-2, POB 218, Yorktown Heights, NY 10598, USA; Telephone 914-945-1642, Telex 137456.

Some of these meetings are sponsored by the Society as part of its world-wide support of activity in mathematical programming.Under certain guidelines the Society can offer publicity, mailing lists and labels, and the loan of money to the organizers of a qualified meeting.

Substantial portions of meetings of other societies such as SIAM, TIMS, and the many national OR societies are devoted to mathematical programming, and their schedules should be consulted.

1986

April 30-May 2: "Optimization Days" at École des Hautes Études Commerciales, Montréal, Canada.
(Submission deadline January 31.) Contact: Prof. Alain Haurie, GERAD, École des H.E.C., 5255 avenue Decelles, Montréal, Quebec, Canada H3T 1V6; telephone 514-340-6042.

May 14-16: Third SIAM Conference on Discrete Mathematics, Clemson, South Carolina, USA. Contact: SIAM Conference Coordinator, 117 South 17 Street-14th Floor, Philadelphia, PA 19103. Telephone 215-564-2929.

May 22-23: 'Eighth Symposium on Mathematical Programming with Data Perturbations', The George Washington University, Washington, DC, USA. Contact: Professor Anthony V. Fiacco, Department of Operations Research, School of Engineering and Applied Science, The George Washington University, Washington, DC 20052, USA; Telephone 202-676-7511. Deadline for abstracts, 10 March 1986.

June 16-19: The International Conference on Numerical Optimization and Applications, Xi'an, Shaanxi, China. Contact: Professor You Zhao-Yong, Department of Mathematics, Xi'an Jiaotong University, Xi'an, Shaanxi, China. Telex 70123 XJTU CN.

July 29-August 1: Conference on Continuous Time, Fractional and Multiobjective Programming, Canton, New York, USA. Contact: Professor Chanchal Singh, Department of Mathematics, St. Lawrence University, Canton, NY 13617, USA. Telephone 315-379-5293.

August 12-14: SIAM Conference on Linear Algebra in Signals, Systems, and Control; Boston, Massachusetts, USA. Contact: SIAM Conference Coordinator, Suite 1400, 117 South 17 Street, Philadelphia, PA 19103.
Telephone 215-564- 2929.

September 15-19: International Conference on Stochastic Programming, Prague, Czechoslovakia. Contact: Dr. Tomas Cipra, Dept. of Statistics, Charles University, Sokolowska 83, 18600 Prague 8, Czechoslovakia. Cosponsored by the Committee for Stochastic Programming of the Mathematical Programming Society.

1988

August 29-September 2: Thirteenth International Symposium on Mathematical Programming in Tokyo, Japan. Contact: Professor Masao Iri (Chairman, Organizing Committee), Faculty of Engineering, University of Tokyo, Bunkyo-ku, Tokyo 113. Official triennial meeting of the MPS.

Technical Reports & Working Papers

Universität zu Köln
Mathematisches Institut
Preprints in Optimization
Weyertal 86-90
D-5000 Köln 41

U. Zimmermann, *"Submodulare Flusse: Verfahren zur Minimierung Linearer Zielfunktionen,"* WP 84.11.

A. Bachem, *"Dualitat und Polaritat in Diskreten Strukturen,"* WP 84.12.

R. Euler and A.R. Mahjouk, *"On a Composition of Independence Systems by Circuit-Identification,"* WP 85.13.

A. Bachem and H. Hamacher, *"Joint US/FRG Seminar: Applications of Combinatorial Methods in Mathematical Programming - Abstracts and Open Problems,"* WP 85.14.

W. Kern, *"Verbandstheoretische Dualitat in Kombinatorischen Geometrien und Orientierten Matroiden,"* WP 85.15.

W.Kern, *"An Efficient Algorithm for Solving a Special Class of LPs,"* WP 85.16.

A. Bachem and A. Wanka, *"On Intersection Properties of (Oriented) Matroids (Extended Abstract),"* WP 85.17.

A. Bachem and W. Kern, *"On Sticky Matroids,"* WP 85.18.

A. Bachem and A. Wanka, *"Separation Theorems for Oriented Matroids,"* WP 85.19.

W. Kern, *"On the Existence of Modular Embeddings of the Lattice of a Matroid (Extended Abstract),"* WP 85.20.

J. Bokowski and B. Sturmfels, *"Coordination of Oriented Matroids,"* WP 85.21.

J. Bokowski and B. Sturmfels, *"Programmsystem zur Realisierung Orientierter Matroide,"* WP 85.22.

W. Kern, *"On Finite Locally Projective Planar Spaces,"* WP 85.23.

The Johns Hopkins University
Department of Electrical Engineering
and Computer Science
Baltimore, Maryland 21218

N. Adlai and A. DePano, *"On k-Envelopes and Shared Edges,"* 85/03.

N. Adlai and A. DePano, *"Efficient Polygonal Enclosures That Cover the Plane,"* 85/04.

J. O'Rourke and S.R. Kosaraju, *"Computing Circular Separability,"* 85/05.

W.J. Rugh, *"An Extended Linearization Approach to Nonlinear System Inversion,"* 85/06.

W.T. Baumann and W.J. Rugh, *"Feedback Control of Nonlinear Systems by Extended Linearization: The Multi-Input Case,"* 85/07.

S. Suri and J. O'Rourke, *"Finding Minimal Nested Polygons,"* 85/08.

M. McKenna, J. O'Rourke, and S. Suri, *"Finding the Largest Rectangle in an Orthogonal Polygon,"* 85/09.

S. Suri, *"A Linear Time Algorithm for Minimum Link Paths Inside a Simple Polygon,"* 85/11.

S. Suri and J. O'Rourke, *"Worst-Case Optimal Algorithms for Constructing Visibility Polygons with Holes,"* 85/12.

J. O'Rourke, *"Reconstruction of Orthogonal Polygons from Vertices,"* 85/13.

N. Adlai and A. DePano, *"Finding Smallest Perimeter Rectangles for a Given Convex Polygon, Or Cropping for the Cheapest Frame,"* 85/14.

J. O'Rourke, *"A Lower Bound on Moving a Ladder,"* 85/20.

JOURNALS & STUDIES

Vol. 34, No. 3

W. Cook, A.M.H. Gerards, A. Schriver, and E. Tardos, *"Sensitivity Theorems in Integer Linear Programming."*

R.E. Stone, *"Linear Complementarity Problems with an Invariant Number of Solutions."*

Y. Fathi and C. Tovey, *"Affirmative Action Algorithms."*

J. Boucher and Y. Smeers, *"The Manne-Chao-Wilson Algorithm for Computing Competitive Equilibria: A Modified Version and its Implementation."*

J.L. Nazareth, *"The Method of Successive Affine Reduction for Nonlinear Minimization."*

R. Hettich, *"An Implementation of a Discretization Method for Semi-Infinite Programming."*

J. Semple and S. Zlobec, *"On the Continuity of a Lagrangian Multiplier Function in Input Optimization."*

Vol. 35, No. 1

M.E. Dyer, A.M. Frieze, and C.J.H. McDiarmid, *"On Linear Programs with Random Costs."*

J.T. Fredricksen, L.T. Watson, and K.G. Murty, *"A Finite Characterization of K-Matrices in Dimensions Less than Four."*

R.H. Byrd and R.B. Schnabel, *"Continuity of the Null Space Basis and Constrained Optimization."*

J.-P. Crouzeix and P.O. Lindberg, *"Additively Decomposed Quasiconvex Functions."*

M. Fukushima, *"A Relaxed Projection Method for Variational Inequalities."*

E.W. Sachs, *"Broyden's Method in Hilbert Space."*

J.M. Borwein and H. Wolkowicz, *"A Simple Constraint Qualification in Infinite Dimensional Programming."*

J.L. Nazareth, *"The Method of Successive Affine Reduction for Nonlinear Minimization."*

J. Guelat and P. Marcotte, *"Some Comments on Wolfe's 'Away Step.'"*