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Are you all salesmen, here?

The Book is Not Closed on the TSP

E.L. Lawler
University of California, Berkeley
J.K. Lenstra
Centre for Mathematics & Computer Science, Amsterdam
A.H.G. Rinnooy Kan
Erasmus University, Rotterdam
D.B. Shmoys
Massachusetts Institute of Technology, Cambridge

IN view of the extensive literature devoted to *The Traveling Salesman Problem* [1] over the last thirty years, with roots as early as 1856, one would expect that all of the most interesting questions concerning the TSP had been settled long ago. Nothing could be further from the truth. In Sherlockian terms, the recently published textbook [1] is no more than *A Study in Scarlet*, and even when *The Final Problem* has been solved, one can await *The Return of the Traveling Salesman.*

The book is certainly not closed on the TSP. We offer the following collection of problems as a guide to further work on combinatorial optimization's most notorious problem. The collection parallels our reference [1] in structure.

For the uninitiated, the TSP is the problem of finding the shortest tour visiting each of n cities exactly once, when the intercity distances are given. We will denote the distance from city i to city j by c_{ij}. The cost of following the route corresponding to the cyclic permutation τ will be denoted by $c(\tau) = \sum_{i=1}^{n} c_{i,\tau(i)}$.

One of the mysteries surrounding the TSP is its origin, at least in its most recent incarnation. Current thought traces the path back to Hassler Whitney, but unless Whitney's (unpublished) notes provide any definite confirmation, it appears to be a difficult task to prove this conjecture.

For the *bottleneck* TSP, the aim is to find a tour through all cities that minimizes the longest intercity distance between two consecutive visits. It is known how to model the bottleneck TSP as a single ordinary TSP with the same number of cities using distances whose length (i.e. number of binary digits) is polynomially bounded. However, the converse is not known: How can one model the ordinary TSP as a bottleneck TSP using even polynomially many cities and distances of polynomial length? As a greater challenge, can either of these transformations be done such that the distances themselves are polynomially bounded?

(It turns out that the first of these two questions is challenging enough. A remarkable recent result of Krentel (to appear in the *Proceedings of the 18th Symposium on the Theory of Computing*) implies that the existence of any polynomial method to model the ordinary TSP as a bottleneck TSP would imply that $P = NP$. This negative result, for all intents and purposes, closes this problem.)

One popular form of the TSP is the *Euclidean* TSP, where each city i is specified by a location (x_i, y_i) in Euclidean space with rational coordinates, so that $c_{ij} = \sqrt{\{(x_i - x_j)^2 + (y_i - y_j)^2\}}$. It is unknown whether the TSP in this form is in *NP*. The fundamental problem here is to find a polynomial procedure to decide whether $c(\tau) \leq k$ for an arbitrary τ and rational k that are input to the procedure.

Circulant matrices (c_{ij}) can be characterized in the following way: if the

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Is P=NP?

By now most of the combinatorial optimization community is aware that E.R. Swart of the University of Guelph has produced a paper asserting a proof that the famous complexity classes P and NP are equal. Given a graph on n vertices, Swart presents a system of linear constraints over nonnegative variables which he argues has a solution if and only if the given graph is Hamiltonian. Since this system has a polynomial (in n) number of constraints and variables, the paper implies the classically NP-Complete question of whether a graph is Hamiltonian can be polynomially resolved by direct application of available (Khachian and Karmarkar) linear programming algorithms.

If correct, Swart's results could be conservatively described as startling and revolutionary. An array of fundamental issues in discrete mathematics would be resolved in a single advance. It is only professional to be suspicious, but (at this writing) we know of no one who has either verified Swart's approach or proved it fatally flawed.

As part of the recent AFOSR sponsored Advanced Research Institute on Discrete Applied Mathematics (ARIDAM) at the Rutgers Center for Operations Research (RUTCOR) seven of us: (D. Crystal, H. Greenberg, A. Kolen, W. Morris, A. Rajan, R. Rardin, M. Trick) did have an opportunity to discuss the paper directly with Swart and study it carefully amongst ourselves. Our summary of the paper, entitled "On the P=NP Paper by E.R. Swart," is available by writing to Ronald L. Rardin, School of Industrial Engineering, Purdue Univ., West Lafayette, IN 47907. Copies of Swart's paper may be obtained by writing to him at the Department of Mathematics and Statistics, Univ. of Guelph, Guelph, Ontario, Canada.

–R. Rardin

Are you all salesmen, here? *from page one*

first row of the matrix has entries a_0, $a_1, \ldots, a_{n-1}$, then all of the entries c_{ij} such that $j - i = k \bmod n$ are equal to a_k. For intercity distances given by circulant matrices, the nearest neighbor rule finds a shortest path visiting each city exactly once and thereby solves what is sometimes called the *Wandering Salesman Problem.* It is unknown, however, whether there is a polynomial procedure to find a shortest tour.

ANOTHER natural constraint on the intercity distances is the *triangle inequality*; that is, $c_{ik} \leq c_{ij} + c_{jk}$ for all i,j,k. An annoyingly simple (but open) question is whether there exists a polynomial approximation algorithm which is guaranteed to find a tour τ such that $c(\tau) \leq kc(\tau^*)$, where τ^* is an optimal tour and k is a constant.

A classic result of Beardwood, Halton and Hammersley states that, for n points distributed independently and uniformly over the unit square, the Euclidean length of the shortest tour through these points is, with probability 1, asymptotically equal to $\beta\sqrt{n}$ for some constant β. However, the proof is existential and does not provide the value of β. Although rough bounds are known for β, the precise value remains elusive and finding it is almost surely a formidable challenge.

An often used approach to find good approximate tours involves neighborhood search, where the current tour is tested for improvement by exchanging a handfull of arcs. Perhaps the most successful procedure of this type is due to Lin and Kernighan. In order to find such a *locally optimal* solution, it appears to be necessary to repeatedly apply the test for improvement, generate the next solution, and then proceed iteratively. Does there exist a fast algorithm which directly generates a local optimum?

THE principal aim of the *polyhedral approach* to the TSP is the identification of facets of the convex hull of incidence vectors of tours. These tours are Hamiltonian cycles in a graph with the cities as vertices and the intercity distances as arc weights. Complexity theory suggests that, in the case of a complete graph, a complete characterization of all the facets is a hopeless task. For special classes of graphs, however, such characterizations have been obtained. A challenging type of research question is to determine the class of all graphs for which a given system of 'nice' facets provides a complete characterization.

A class of rather sophisticated nice facets is the one defined by *clique tree* inequalities. Their algorithmic application is obstructed by the fact that the separation problem has not yet been solved: Given a vector, check whether it satisfies all clique tree inequalities; if not, find one which is violated. A more modest but still unfulfilled aim would be to find a good heuristic for this problem.

ANY optimization algorithm for the TSP is likely to require some form of enumeration of the solution set. Most enumerative methods employ the *branch and bound* principle. The main ingredient of such a method is a relaxation of the TSP, whose solution yields a lower bound on the length of the shortest tour. In the case of the *asymmetric* TSP with independently distributed intercity distances, all successful branch and bound algorithms use the relaxation to the linear assignment problem. In the *symmetric* case, where $c_{ij} = c_{ji}$ for all i,j, the strongest bounds are obtained from the 1-tree relaxation with Lagrangean objective or from the continuous 2-matching relaxation with the addition of facet defining inequalities. So far, no one has succeeded in developing a robust branch and bound algorithm, which performs well in both cases.

A special case of the TSP that was considered before the TSP itself is the problem of finding a *Hamiltonian cycle* in a graph. A graph is called *t-tough* if deleting any set of s vertices leaves the graph with at most s/t connected components. Does there exist some t such that all t-tough graphs are Hamiltonian? In particular, are all 2-tough graphs Hamiltonian?

THE TSP finds application in the context of *vehicle routing,* which is a broad and diverse area. We offer a theoretical and a practical challenge. First, consider the *Chinese multipostman problem.* Given an edge-weighted connected graph, determine a set of m tours, each starting and finishing at a specified vertex, collectively traversing all edges, and no one exceeding a given limit in length. The problem is *NP*-hard in the strong sense when m is arbitrary. What is its complexity status when m is fixed? Secondly, vehicle routing often involves the problem of *period allocation.* Each customer requires a given number of visits in a given period. On which day should each customer be visited such that the collection of solutions to the daily routing problems is overall optimal? This is a neglected problem, and finding good heuristics for its solution is of eminent practical importance.

The problems given above represent just a sample of the work to be done on the TSP. One problem that will probably never be solved is that along with its eye-grabbing name, the traveling salesman problem will never rid itself of associated off-color jokes and inevitable misunderstanding. No waitress would ever inquire of a group of editors who are casting a first glance at their book, *The Minimum Spanning Tree Problem*, 'Are you all trees, here?'

Reference

1. E.L. Lawler, J.K. Lenstra, A.H.G. Rinnooy Kan, D.B. Shmoys (eds.) (1985). *The Traveling Salesman Problem: A Guided Tour of Combinatorial Optimization*, Wiley, Chichester.

Conference Notes

Call For Papers
IFORS '87
11th Triennial Conference on Operations Research
Buenos Aires, Argentina
August 10-14, 1987

The International Federation of Operational Research Societies (IFORS) will be 28 years old in 1987. As an association of 34 national OR societies and 6 kindred societies, its purpose is the development of operations research as a unified science and its advancement in all nations of the world. One of IFORS' main activities is the organization of an international conference every three years. The last conference was in Washington, D.C. You are now invited to the next one to be held in Buenos Aires.

Papers may be contributed by any member of an OR society affiliated with IFORS. Authors are requested to submit an abstract of not more than 100 words, with 5 key words, not later than October 13, 1986, directly to the Chairman of the Program Committee, M.E. Thomas, ISyE, Georgia Tech, Atlanta, GA 30332. The deadline will be strictly enforced. The Program Committee reserves the right to accept or reject these contributions on the basis of the abstract or paper. All the abstracts of the papers to be presented will be published in advance of the conference if they contain no more than 100 words.

Martin Beale Memorial Symposium

A symposium will be held at The Royal Society, London, July 6-8, 1987, in memory of Professor E.M.L. Beale. By covering the range of his professional interests, the symposium will provide useful links between research and applications in Mathematical Programming, Operational Research and Statistics. The programme will include plenary talks for a general audience, submitted papers in parallel sessions, and a conference dinner. For further information please contact Mrs. B.A. Peberdy, SCICON Limited, Wavendon Tower, Milton Keynes MK17 8LX, U.K.

–M.J.D. Powell

Alan Hoffman Honored by Technion

Dr. Alan Hoffman from IBM Watson Research Center at Yorktown Heights was awarded an Honorary Doctorate of Science by the Technion - Israel Institute of Technology. The ceremony took place at Technion City in Haifa, Israel, on June 16, 1986. The citation for the award reads, "In recognition of his fundamental contributions to Operations Research and its interface with Combinatorics, Graph Theory and Linear Algebra; in particular, for the introduction of unimodularity, thereby laying the foundation for Integer Programming."

–Uriel Rothblum

Journals and Studies

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S.E. Berenguer and R.L. Smith, *"The Expected Number of Extreme Points of a Random Linear Program."*

C. Blair, *"Random Inequality Constraint Systems with few Variables."*

N. Megiddo, *"Improved Asymptotic Analysis of the Average Number of Steps Performed by the Self-Dual Simplex Algorithm."*

M.J. Todd, *"Polynomial Expected Behavior of a Pivoting Algorithm for Linear Complementarity and Linear Programming Problems."*

C.A. Tovey, *"Low Order Polynomial Bounds on the Expected Performance of Local Improvement Algorithms."*

N. Megiddo, *"On the Expected Number of Linear Complementarity Cones Intersected by Random and Semi-random Rays."*

Y-H. Wan, *"On the Average Speed of Lemke's Algorithm for Quadratic Programming."*

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M.J.D. Powell and Y. Yuan, *"A Recursive Quadratic Programming Algorithm that Uses Differentiable Exact Penalty Functions."*

S. Sen and H.D. Sherali, *"A Class of Convergent Primal-dual Subgradient Algorithms for Decomposable Convex Programs."*

S. Fujishige, *"A Capacity-Rounding Algorithm for the Minimum-Cost Circulation Problem: A Dual Framework of the Tardos Algorithm."*

A. Ruszczynski, *"A Regularized Decomposition Method for Minimizing a Sum of Polyhedral Functions."*

R. Lazimy, *"Solving Multiple Criteria Problems by Interface Decomposition."*

M.L. Balinski, Th. M. Liebling, and A.-E. Nobs, *"On the Average Length of Lexicographic Paths."*

N. Megiddo, *"A Note on Degeneracy in Linear Programming."*

S. Zlobec, *"Characterizing an Optimal Input in Perturbed Convex Programming: Corrigendum."*

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J. Herskovits, *"A Two-Stage Feasible Directions Algorithm for Nonlinear Constrained Optimization."*

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J-S. Pang, *"Inexact Newton Methods for the Nonlinear Complementarity Problem."*

A. Dax, *"A Note on Optimality Conditions for the Euclidean Multifacility Location Problem."*

O.L. Mangasarian and T.-H Shiau, *"Error Bounds for Monotone Linear Complementarity Problems."*

J.-P. Dussault, J.A. Ferland, B. Lemaire, *"Convex Quadratic Programming with one Constraint and Bounded Variables."*

J. Kyparisis, *"Uniqueness and Differentiability of Solutions of Parametric Nonlinear Complementarity Problems."*

H.A. Eiselt, *"Continuous Maximum Knapsack Problems with GLB Constraints."*

Technical Reports and Working Papers

Stichting Mathematisch Centrum
Centrum voor Wiskunde en Informatica
Department of Operations Research and System Theory
Postbus 4079 1009 AB Amsterdam
Kruislaan 413 1098 SJ Amsterdam

G.A.P. Kindervater and J.K. Lenstra, *"An Introduction to Parallelism in Combinatorial Optimization,"* OS-R8501.

J.M. Schumacher, *"A Geometric Approach to the Singular Filtering Problem,"* 0S-R8502.

B.J. Lageweg, J.K. Lenstra, A.H.G. Rinnooy Kan e.a., *"Stochastic Integer Programming by Dynamic Programming,"* OS-R8503.

J.B. Orlin, *"Genuinely Polynomial Simplex and Non-Simplex Algorithms for the Minimum Cost Flow Problem,"* OS-R8504.

J.B. Orlin, *"The Complexity of Dynamic/Periodic Languages and Optimization Problems,"* OS-R8505.

J.M. Schumacher, *"Residue Formulas for Meromorphic Matrices,"* OS-R8506.

J.H. van Schuppen, *"Stochastic Realization Problems Motivated by Econometric Modelling,"* OS-R8507.

R.K. Boel and J.H. van Schuppen, *"Overload Control for SPC Telephone Exchanges Refined Models and Stochastic Control,"* OS-R8508.

J.W. Polderman, *"A Note on the Structure of Two Subsets of the Parameter Space in Adaptive Control Problems,"* OS-R8509.

O.J. Boxma and B. Meister, *"Waiting-time Approximations for Cyclic-Service Systems with Switch-Over Times,"* OS-R8510.

O.J. Boxma, *"A Queueing Model of Finite and Infinite Source Interaction,"* OS-R8511.

G.A.P. Kindervater and H.W.J.M. Trienekens, *"Experiments with Parallel Algorithms for Combinatorial Problems,"* OS-R8512.

P.J.C. Spreij, *"Recursive Parameter Estimation for Counting Processes with Linear Intensity,"* OS-R8513.

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College of Management (COM)
School of Mathematics (Math)
School of Information and Computer Science (ICS)

F.A. Al-Khayyal, T.J. Hodgson, G.D. Capps, J.A. Dorsch, D.A. Kriegman and P.D. Pavnica, *"A Lagrangian Dual Approach for Solving a Structured Geometric Program Arising in Sample Survey Design,"* ISYE Report Series No. J-85-1.

F.A. Al-Khayyal, *"Necessary and Sufficient Conditions for the Existence of Complementary Solutions and Characterizations of the Matrix Classes Q and Q_O,"* PDRC Report Series 85-5, ISyE.

F.A. Al-Khayyal, *"Minimizing a Quasiconcave Function Over a Convex Set: A Case Solvable by Lagrangian Duality,"* ISyE.

F.A. Al-Khayyal, C.A. Tovey, *"Extending Simulated Annealing to Continuous Global Optimization,"* ISyE.

M.A. Berger and A. Felzenbaum, *"Sign Patterns of Matrices and Their Inverses,"* Math.

M.A. Berger, A. Felzenbaum and A.S. Fraenkel, *"Lattice Parallelepipeds and Disjoint Covering Systems,"* Math.

M.A. Berger, A. Felzenbaum and A.S. Fraenkel, *"A Non-analytic Proof of the Newman-Znam Result for Disjoint Covering Systems,"* Math.

M.A. Berger, A. Felzenbaum and A.S. Fraenkel, *"Improvements to the Newman-Znam Result for Disjoint Covering Systems,"* Math.

M.A. Berger, A. Felzenbaum and A.S. Fraenkel, *"Covers of Product Sets and the Korec-Znam Result,"* Math.

M.A. Berger, A. Felzenbaum and A.S. Fraenkel, *"Disjoint Covering Systems with Precisely One Multiple Modulus,"* Math.

M.A. Berger, *"A Trotter Product Formula for Random Matrices,"* Math.

C.E. Blair, R.G. Jeroslow, and J.K. Lowe, *"Some Results and Experiments in Programming Techniques for Propositional Logic,"* University of Illinois and COM, Georgia Inst. Tech.

C.Cardelino and P.Y. Chen, *"Algorithm for Solving a Complex Function $F(z)=0$,"* IT-ICS-85/04.

D. Crystal, *"Min Algebraic Duality,"* ISyE.

M. Graham, N. Griffeth, and B. Smith-Thomas, *"Algorithms for Providing Reliability in Unreliable Database Systems,"* IT-ICS-84/22.

W. Green and E.W. Kamen, *"Stabilizability of Linear Systems Over a Commutative Normal Algebra with Applications to Spatially-Distributed and Parameter-Dependent Systems,"* Math, Georgia Institute of Technology and Electrical Engineering, University of Florida.

W. Green and T.D. Morley, *"Operator Means, Fixed Points and the Norm Conveyance of Monotone Approximants,"* Math.

N. Griffeth and J.A. Miller, *"Performance Modeling of Database Recovery Protocols,"* IT-ICS-84/21.

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R. Jeroslow, *"Alternative Formations of Mixed Integer Programs,"* COM.

R. Jeroslow, *"A Simplification for some Disjunctive Formulations,"* COM.

R. Jeroslow, *"Spatial Embeddings for Linear and Logic Structures,"* COM.

R. Jeroslow, "On Monotone Chaining Procedures of the CF Type," COM.

R. Jeroslow, *"An Extension of Mixed-Integer Programming Models and Techniques to Some Database and Artificial Intelligence Settings,"* COM.

J. Miller and N.G. Griffeth, *"Approximate Queuing Network Analysis of Database Recovery Protocols,"* IT-ICS-85/12.

J. Miller and N. Griffeth, *"Markovian Analysis and Optimization of Database Recovery Protocols,"* GIT/ICS/86/0Z.

D.C. Nachman, *"Arbitrage Operations and Market Expansion,"* COM.

D.C. Nachman, *"Options and Market Expansion,"* COM.

D.C. Nachman, *"Efficient Funds for Meager Asset Spaces,"* COM.

M.B. Richey and R.G. Parker, *"On Multiple Steiner Subgraph Problems,"* ISyE Report Series.

M.B. Richey and R.G. Parker, *"Minimum-Maximal Matching on Series-Parallel Graphs: An Algorithm and Its Implications for Other Problems,"* ISyE Report Series.

M.B. Richey and R.G. Parker, *"Solving General Eulerian Subgraph Problems,"* ISyE Report Series.

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G. Vijayan, *"Some Properties and Conjectures of the Geometry of Planar Graphs,"* IT-ICS-85/17.

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Department of Operations Research
Nassestr. 2
D-5300 Bonn 1, West Germany

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Systems Optimization Laboratory
Department of Operations Research
Stanford University
Stanford, California 94305

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Northwestern University
Department of Industrial Engineering and Management Sciences
Evanston, Illinois 60201

W.J. Hopp, *"Sensitivity Analysis in Discrete Dynamic Programming,"* TR 85-07, November 1985.

P.C. Jones, J.L. Zydiak and W.J. Hopp, *"Prices and Depreciation for Machine Replacement I: Deterministic Case,"* TR 85-08, December 1985.

F. Fourer, *"A Simplex Algorithm for Piecewise-Linear Programming III: Computational Analysis and Applications,"* TR 86-03, June 1986.

W.-L. Hsu, *"Recognition and Optimization Algorithms on Planar Perfect Graphs,"* TR 86-04, June 1986.

R.E. Bixby and R. Fourer, *"Finding Embedded Network Rows in Linear Programs I: Extraction Heuristics,"* TR 86-05, July 1986.

Book Reviews

Combinatorial Optimization for Undergraduates
By L.R. Foulds
Undergraduate Texts in Mathematics
Springer, Berlin, 1984
ISBN 0-387-90977-X

This text covers a broad range of topics in combinatorial optimization as may be inferred from the following chapter titles: 0. Introduction; 1. Linear Programming and Extensions (including the transportation and assignment problems); 2. Solution Techniques (integer and dynamic programming, complexity, heuristics); 3. Optimization on Graphs and Networks (trees, paths, flows, CPM and PERT); 4. Some Applications (facilities layout, traveling salesman, vehicle routing, capacitated trees, evolutionary trees); 5. Appendix (linear algebra and graph theory).

There are some positive aspects of this effort by Foulds. We agree with him that there is a need for a good undergraduate level text devoted to combinatorial optimization. We also admire his attempt to motivate theory through small examples. Nevertheless, we feel that the book does a disservice to the area that it purports to serve. It suffers from a number of major deficiencies.

The first paragraph of the text sets the stage. It paraphrases a continuous optimization problem from Virgil's *Aeneid* that is solvable by variational calculus - a truly surprising motivation for the study of combinatorial optimization. The remainder of the introductory chapter will most likely confuse any novice in the area of optimization. After a notationally laborious description of the shortest Hamiltonian path problem, Foulds provides some techniques for local improvement. He then gives the following definition: "If, for all such x_i $f(x_0) \leq f(x_i)$, x_0 is said to be a *local minimum of f*." But there is no antecedent for the word "such". Indeed, one would have to know the meaning of the term "local minimum" to find sense in the definition.

Chapter 1 falls short of any expectations raised by its title. The treatment of linear programming lacks care at the level of detail. It is confusing to read "$\mathbf{b} > 0$" where "$\mathbf{b} \geq 0$" is intended (page 25). It is troublesome to work with lower as well as upper case notation for vectors (but, if it has to be done, then using both for one and the same vector is a good exercise (page 36)). It is frustrating to try to apply phase 2 of the simplex method without being told first how artificial variables should be removed from the basis (page 29). And it is impossible to solve assignment problems to optimality by a quasi-Hungarian method that relies on a simple greedy heuristic for finding minimum bipartite vertex covers (pages 73, 74). Proclaiming optimality of this rule is more than a detail and worse than lack of care.

We are also displeased with the section on complexity in Chapter 2. At the technical level, the definition of *P* is wrong, that of *NP* is lacking, and the statement, "If p_1 is *NP*-complete and $p_1 \propto p_2$ then p_2 is also *NP*-complete," is false. Our main objection, however, is that by making one-and-a-half pages of inexact statements about complexity and by not integrating the concept into the rest of the text, Foulds has mystified rather than clarified the subject.

The next section, on heuristics, is not satisfactory either. The main message here is that one has to be realistic and clever when solving problems in business and industry. While we do not disagree with realism or creativity, we do disagree with the looseness of the approach. No attempt is made to provide a theoretical framework of heuristics, and no word is spent on worst case or probabilistic analysis. All this is saddening. A formal treatment of computational complexity and approximation algorithms can be so easy, so illuminating, and so exciting.

Chapter 5 contains an eclectic collection of toy applications that are solved by straightforward heuristics, mostly of a greedy nature. The presentation falls short on two counts: the techniques introduced in the previous chapters are largely disregarded, and the reader is not even informed about the existence of more sophisticated solution methods.

We strongly recommend against the adoption of this text for any course in combinatorial optimization.

–J.K. Lenstra, J.B. Orlin

Linear and Nonlinear Programming
By R. Hartley
Horwood, Chichester, 1985
ISBN 0-85312-644-5

This nice little book gives an informal introduction to linear programming and its main related topics. The presentation is done with a minimum of mathematical prerequisites without any formal statement and proof of theorem; therefore, as such, the book may be attractive to lecturers having to face students majoring in business administration, economics or engineering where mathematical formalism is not always much appreciated. The book is written in a clear and easily readable manner. Each chapter is followed by exercises for which hints and answers can be found at the end of the book. With its 221 pages, the book covers quite a lot of material.

After an introduction where linear programming is presented via a "real world" example and is illustrated graphically, the first four chapters are devoted to the simplex method including the revised version. All this is done without using matrix notation. Chapters 5 to 7 are concerned with duality, sensitivity analysis and bounded variables. Related standard topics such as the transportation problem, the multiobjective and the integer programming cases are treated in chapters 8 to 11. The last chapter is on quadratic programming. To have written more about nonlinear programming, even in an informal manner, probably would have made the book too long.

–Jacques Gauvin

Tree Automata
By F. Gécseg and M. Steinby
Akadémiai Kiadó, Budapest, 1984

Tree automata were invented in the mid-sixties as a natural generalization of finite automata, accepting finite (valued) trees instead of strings. Although the resulting theory of recognizable (or "regular") sets of trees soon offered appealing results and interesting problems and also provided useful insights into questions of classical formal language theory, until now it was not covered by a textbook collecting the relevant notions and most important theorems.

The book by Gécseg and Steinby fills this gap. There are four chapters, each of them supplemented by historical notes and references to further literature. The first chapter presents some universal algebra as the terminological background upon which the exposition of tree automata theory is based. In the second chapter one finds an introduction to the theory of recognizable forests (sets of trees), including a comparison of root-to-frontier and frontier-to-root tree automata, Kleene's theorem in the context of trees, algebraic characterizations of recognizability, and several theorems concerning minimal tree automata. The following chapter deals with applications of these results to context-free (string-) languages. In the fourth chapter, tree transducers are investigated. First, Engelfriet's classification of tree transformations and the fundamental decomposition results for these relations are presented. Then follows a discussion of tree transducers with regular look-ahead, surface forests, hierarchies of tree transformations, and the equivalence problem for tree transducers. The book ends with an extensive bibliography consisting of about 280 items which covers virtually all relevant literature up to 1982.

It should be noted that some major developments of the subject which are important in current research are only mentioned in the notes but not treated in the main text. In particular, this applies to automata on infinite trees, the connection with logic (monadic second-order logic and logics of programs), and issues related to the concept of context-free tree grammar, like pushdown tree automata or macro tree transducers.

Nevertheless, this carefully written monograph contains all that could be called the "basic theory" of tree automata; it will be a very useful reference for anyone who is interested in this rapidly developing area.

–W. Thomas

Introduction to Stochastic Dynamic Programming
By Sheldon M. Ross
Academic Press, London, 1983

This book provides an introduction to the theory of stochastic dynamic programming or Markov decision processes. As can be seen from the table of contents, both classical and modern fields of research are covered as follows: I. Finite-Stage-Models; II. Discounted Dynamic Programming; III. Minimizing Costs – Negative Dynamic Programming; IV. Maximizing Rewards – Positive Dynamic Programming; V. Average Reward Criterion; VI. Stochastic Scheduling; and VII. Bandit Processes. The author presents these subjects in a concise and elegant way, evading technicalities and appealing to the imagination of the reader.

The author does not build up a huge overall theory but presents numerous important examples illustrating the methods and underlying structures, counterexamples for expected results, and useful exercises for the reader. The emphasis lies on qualitative and structural results concerning the optimal policy and the value function. However, computational approaches are also dealt with, and the author states that few mathematical prerequisites are needed. This is true as long as one is willing to accept some intuitively obvious relations. Without mathematical proofs, it takes some time

continued on following pages

to really understand the foundations. The optimality equation is proved in a rough way and only for special cases. Furthermore, it seems possible to prove it for all cases without the heavy measure-theoretic apparatus using the fact that the action space is finite or the probability measures have a countable support.

The book is strongly recommended to economists and operations researchers as well as to mathematicians ready to invest some additional work.

–M. Schäl

Introduction to Sensitivity and Stability Analysis in Nonlinear Programming
By A.V. Fiacco
Academic Press, New York, 1983
ISBN 0-12-254450-1

This book is concerned with sensitivity and stability issues related to the following general parametric nonlinear programming problem P(ε): minimize {f(x,ε): gi (x,ε)≥ 0 for i=1, ..., m, hj (x,ε) = 0 for j=1, ..., p} where xεEn and where ε is a parametric vector in Ek. Given an ε in Ek, a local solution x(ε) along with a corresponding set of Lagrange multipliers u(ε) and w(ε) associated with the inequality and the equality constraints in P(ε), respectively, satisfying the first-order optimality conditions, are referred to as the Karush-Kuhn-Tucker (KKT) triple, and the function f*(ε) defined as f[x(ε), ε] is referred to as an optimal value function. The sensitivity information generated is concerned with the first-order variations in the KKT triple and with the first-order and second-order variations in f*(ε) with respect to the problem parameter vector ε. This is hence a local perturbation analysis. The stability analysis, on the other hand, is a finite perturbation analysis and is concerned with generating parametric bound information on the optimal value function f*(ε) or on the solution point x(ε). The book conducts this type of analysis using both an algorithm-independent as well as an algorithm-dependent approach and provides specializations for particular generic cases of P(ε) such as problems involving right-hand-side perturbations only as well as for particular practical applications.

Following a brief motivational introduction in Chapter 1, Chapter 2 presents a collection of basic sensitivity and stability results for P(ε) and its various realizations which have been studied in the literature. These results deal with: (a) the continuity of f*(ε) and of the point-to-set map which maps ε to the set of alternative optimal solutions; (b) differentiability properties of f*(ε) based on a rate of change of this function with respect to ε at a solution point and the existence and computation of directional derivatives of this optimal value function; (c) the behavior of the KKT triple with respect to continuity and differentiability properties, and (d) bounds on optimal value functions and on local solutions to the parametric problem P(ε).

Chapter 3 begins to extend the existing results for generating the desired sensitivity information. Using standard second-order sufficiency optimality conditions for a strict local minimum along with appropriate constraint qualifications, the existence and behavior of first-order variations of the KKT triple with respect to the problem parameters are explored, and explicit representations are developed for these partial derivatives. Based on this, formulae are derived for first and second-order changes in the optimal value function f*(ε) with respect to the problem parameters ε.

The actual numerical aspects related to the efficient determination of these partial derivatives are addressed in Chapter 4. The utility of obtaining these derivatives lies in being able to provide a first-order estimation or representation of the KKT triple, and a second-order representation of the optimal value function. This in turn is useful in characterizing the convexity of f*(ε) and for characterizing the stability of a solution to the problem subject to perturbation.

Chapter 5 continues the analysis of Chapter 4, examining a special case of a right-hand-side perturbation. It is noted that by treating ε as a variable in P(ε), and by adding a constraint ε=α, the foregoing analysis may be viewed as being also related to a perturbation of the right-hand-side α. Nonetheless, an explicit treatment of this is provided for the problem $P_2(\varepsilon)$: minimize {f(x): g(x) $\geq\varepsilon^1$, h(x) = ε^2}, where ε = $(\varepsilon^1,\varepsilon^2)$. A historical digression first points out the well-known relationships between the variation of f*(ε) with respect to ε and the Lagrange multipliers u(ε) and w(ε). Thereafter, as before, explicit formulae are developed for first-order derivatives of the KKT triple and for the first and second-order derivatives of the optimal value function under stated second-order assumptions. The utility of these expressions in generating stability information, as well as its role in cyclical decomposition schemes for solving nonlinear programming problems is also discussed. In particular, the use of the second-order results in designing second-order procedures for generating search directions in resource-directed decomposition procedures for suitable nonlinear programming problems is pointed out as an important application of these results.

In contrast with the algorithm-independent approach of Chapters 3-5, Chapters 6 and 7 demonstrate how standard nonlinear programming algorithms have an inherent capability of generating sensitivity information during the normal course of their solution procedure, provided they terminate with a KKT triple satisfying the usual first-order optimality conditions. Chapter 6 makes this point using twice-differentiable penalty functions and, in particular, employs a logarithmic barrier term with respect to the inequality constraints, and a quadratic penalty term with respect to the equality constraints for the purpose of giving a specific illustration. Again, first-order sensitivity results for the KKT triple are obtained. Further, directly from the estimates of the gradient and hessian of the penalty function in the limit as the penalty parameter vanishes, first and second-order results for the optimal value function are also obtained. Specializations to right-hand-side perturbations are also pointed out.

Chapter 7 continues to illustrate this fact using other popular algorithms including Newton-based procedures, projected-gradient and reduced-gradient algorithms, and the augmented Lagrangian methods. The basic idea here is that these and other algorithms typically determine a solution to a nonlinear program via a sequence of problems, each of which is some perturbation of the original problem, and hence the solution procedure naturally contains sensitivity information. Besides, having successfully obtained a KKT triple satisfying first-order conditions, one directly has the information for the first order derivatives of $f^*(\varepsilon)$ and the KKT triple, from which second-order derivative information for $f^*(\varepsilon)$ may be derived. This is done via an estimate of the inverse of the Jacobian of the first-order necessary optimality conditions, which is typically available when employing a Newton-type procedure for solving the sequence of subproblems in determining a KKT triple.

Chapter 8 employs the penalty function of Chapter 6 in a computer program SENSUMT, which also performs the associated sensitivity analysis in order to study a particular application relating to a multi-item continuous-review inventory system. Brief mention is also made of the study of other applications including a geometric programming model of a stream water pollution abatement system and a nonlinear structural design problem.

Chapter 9 deals with the stability analysis, developing piecewise linear, continuous, global upper and lower bounding functions which envelope $f^*(\varepsilon)$, as a function of this parameter ε, when $f^*(\varepsilon)$ is known to be convex or concave. Connections between the construction mechanism of these optimal value bounds and duality theory are also explored. A similar scheme can be used for nonconvex parametric programs whenever suitable convex or concave underestimating or overestimating problems can be constructed. This is extended for constructing parametric bounds for the solution point $x(\varepsilon)$ as well. Again, it is emphasized that most nonlinear programming algorithms have an in-built capability of generating these parametric bounds via the information generated during the normal course of their operation.

Finally, Chapter 10 looks to future research directions. The author anticipates movement toward a further unification of theory which will lead to an extension of the ideas discussed herein to other areas as well as a transfer of computational techniques from other developments to analyze sensitivity and stability issues.

The book itself is a major step toward this unification. It is a first book which deals in a comprehensive fashion with sensitivity and stability issues related to nonlinear programming problems. It is based largely on the author's own original work and an extension of existing analysis which relates to the subject of this book. In this respect, the book represents a rich collection of ideas and concepts which are woven together in a clear and well-written presentation. This book will not only encourage research in this area, certainly some along the many directions pointed out throughout the text, but will also encourage the study and incorporation of sensitivity and stability information generation capability in standard nonlinear programming algorithmic developments and codes, and the use and interpretation of this type of information by practitioners working with various applications.

–H.D. Sherali

Trees and Hills
By R. Greer
North-Holland, Amsterdam, 1984
ISBN 0-444-87578-6

The title of this book is misleading with regard to its contents. Only the subtitle ("Methodology for Maximizing Functions of Systems of Linear Relations") informs the reader about the problem dealt with by the author. In this monograph a set of linear relations defining a subset of R^d is considered. Any relational operator $\{<, \leq, =, \neq, \geq, >\}$ may define a condition. The problem treated is to determine all of those vectors $x \in R^d$ which satisfy or do not satisfy elements of this set of linear relations in such patterns as will extremize certain functions of interest.

This general class of problems includes, for example, the NP-complete Weighted Closed, Open, or Mixed Hemisphere problem. Also linear programming falls into this category of problems of extremizing functions of systems of linear relations.

The present monograph is mainly concerned with the development of a tree algorithm for solving the general problem. According to the author, it is the only known nonenumerative algorithm for solving the problem above, which furthermore enables the Weighted Open Hemisphere (WOH) problem to be solved in a much shorter time than by other algorithms. The author proves this latter assertion by giving some test examples in Chapter 9.

The monograph extends part of the author's Ph.D. dissertation. It contains a very detailed tutorial on polyhedral convex cones (Chapter 2, pp. 15-82!). The tree algorithm is developed in two stages. First, in Chapter 5, a tree algorithm for solving the WOH problem is presented. Then after discussing in Chapter 4 how problems of extremizing functions of systems of linear relations may be reduced, the general tree algorithm is presented in Chapter 5. In Chapter 6 the computational complexity of the tree algorithm is discussed. Another methodology for extremizing functions of systems of linear relations is compared and contrasted with the tree algorithm in Chapter 7, and applications of the tree algorithm are discussed in Chapter 8. A general conclusion is given in the last chapter.

This book will be primarily interesting for the experts in the area mentioned, though a more compact edition would be sufficient for them. However, due to the very detailed introduction to the area of polyhedral convex cones and the full description of the WOH problem, the book will also be useful to the interested reader who is only slightly familiar with these fields. The uncommon symbols used in parts do not really make it more difficult to handle.

–H.-J. Kruse

C A L E N D A R

20 August 1986

Maintained by the Mathematical Programming Society (MPS)

This Calendar lists noncommercial meetings specializing in mathematical programming or one of its subfields in the general area of optimization and applications, whether or not the Society is involved. (The meetings are not necessarily 'open'.) Anyone knowing of a meeting that should be listed here is urged to inform Dr. Philip Wolfe, IBM Research 33-2, POB 218, Yorktown Heights, NY 10598, USA; Telephone 914-945-1642, Telex 137456.

Some of these meetings are sponsored by the Society as part of its world-wide support of activity in mathematical programming.Under certain guidelines the Society can offer publicity, mailing lists and labels, and the loan of money to the organizers of a qualified meeting.

Substantial portions of meetings of other societies such as SIAM, TIMS, and the many national OR societies are devoted to mathematical programming, and their schedules should be consulted.

1986

September 15-19: International Conference on Stochastic Programming, Prague, Czechoslovakia.
Contact: Dr. Thomas Cipra, Department of Statistics, Charles University, Sokolowska 83, 18600 Prague 8, Czechoslovakia. Cosponsored by the Committee for Stochastic Programming of the Mathematical Programming Society.

1987

April 6-8: "CO87", a Conference on Combinatorial Optimization, Southampton, U.K.
Contact: Dr. C.N. Potts, Faculty of Mathematical Studies, University of Southampton, Southampton SO9 5NH, United Kingdom. (Sponsored by the London Mathematical Society. Deadline for abstracts, 5 January 1987.)

July 6-8: Martin Beale Memorial Symposium, London, U.K.
Contact: Professor M.J.D. Powell, Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge (CB3 9EW, United Kingdom. Telephone (0223) 337889, Telex 81240.

1988

August 29 - September 2: Thirteenth International Symposium on Mathematical Programming in Tokyo, Japan.
Contact: Professor Masao Iri (Chairman, Organizing Committee), Faculty of Engineering, University of Tokyo, Bunkyo-ku, Tokyo 113. Official triennial meeting of the MPS.

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Kenneth O. Kortanek, formerly of Carnegie-Mellon, has been appointed Murray Research Professor of The Management Sciences, College of Business Administration, University of Iowa....**Horst Hamacher** (Florida) is visiting the Technical University of Graz, Austria for the academic year 19886-87...**Bruce Golden** (Maryland) announces the availability of NETSOLVE, an interactive software package for network manipulation and optimization.

The October 26-29 ORSA/TIMS meeting in Miami Beach will feature several 90-minute tutorials on state-of-the-art mathematical programming topics. Included are lectures by **George Nemhauser** (Georgia Tech) on Integer Programming, **Dimitri Bertsekas** (MIT) on Network Algorithms, and **Jan Karel Lenstra** (Mathematisch Centrum, Amsterdam) on Sequencing and Scheduling. There will also be many regular sessions on mathematical programming, including several on the Karmarkar algorithm. Additionally, **Bob Jeroslow** (Georgia Tech) has arranged a set of sessions on the AI/OR Interface.

Deadline for the next OPTIMA is November 15, 1986.

Books for review should be sent to the Book Review Editor, Prof. Dr. Achim Bachem, Mathematiches Institute der Universität zu Köln, Weyertal 86-90, D-5000 Köln, West Germany.

Journal contents are subject to change by the publisher.

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MATHEMATICAL PROGRAMMING SOCIETY

303 Weil Hall
College of Engineering
University of Florida
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