OPTIMA
MATHEMATICAL PROGRAMMING SOCIETY NEWSLETTER
OCTOBER 1980
NUMBER 2

Walter Murray

Algorithms: The Influence of Finite Precision Arithmetic

Almost all substantive problems in applied mathematics require the use of a computer for their solution. An article in the April 1980 edition of the SIAM Newsletter likened the introduction of the computer into applied mathematics with the discovery of relativity and quantum mechanics in theoretical physics. Modern computing technology has had the revolutionary effect of changing the emphasis of much applied mathematics from transforming a problem into its most compact form to expressing a problem in its most computable form.

In theoretical physics, the process of change lasted some forty years -- about the length of time to retirement of a Ph.D. graduating prior to Einstein's work. On such a time scale, the computer revolution, although advanced, is still not complete. In this article, my intention is to illustrate where, in my opinion, improvements are still necessary in recognizing the role of the computer in the study of mathematical programming algorithms.

Since computers will obviously be used to implement any algorithm, it would seem that those who devise and study algorithms could not fail to consider the computational aspects. However, such neglect is sufficiently common to cause some concern. Curiously, even algorithms presented in a pseudo-programming style often fail to take full account of the basic limitations of computers.

The explanation for this situation is not entirely clear. Possibly the effects of implementing an algorithm on a computer are considered negligible, or the work required to develop an implementation is regarded as routine. In my view, both these attitudes are erroneous, and often lead to unsound algorithms. In fact, many believe that implementation of an algorithm is anything but routine; for example, Watts and Shampine [1975] state that "...how a method is implemented may be more important than the method itself". The purpose of this article is not to argue the merits of devising algorithms as opposed to implementing them. Rather, I believe that the two processes should be conceived as a whole. In some cases, advantage can be taken of finite-precision arithmetic to the extent that it is possible to define algorithms that have no mathematical counterpart – for example, inverse iteration in the determination of eigenvectors.

Because almost all algorithms for nonlinear optimization are iterative, it is often supposed that any errors introduced by computation are somehow self-correcting. However, the conjugate gradient algorithm clearly illustrates the fallacy of this viewpoint. The conjugate gradient method was first proposed for solving positive definite linear systems in 1951, at a time when it was believed that computational errors in direct methods such as Gaussian elimination would necessarily be overwhelming. By contrast, the conjugate method proceeds by computing a sequence of "improving" estimates of the solution, and is thus iterative in nature. With exact arithmetic, the algorithm will terminate with the exact solution in a finite number of iterations. However, in practice it is not difficult to construct examples for which the conjugate gradient method requires many times the number of iterations that would be taken by the exact procedure. Moreover, the difficulty does not arise purely from the iterative nature of the algorithm. Certain quasi-Newton methods are mathematically equivalent to the conjugate gradient algorithm, yet will converge to the solution in the expected number of iterations.

The computational steps of algorithms as described in research papers are rarely defined completely or even unambiguously. This omission in itself is not necessarily cause for concern, since the authors may intend to study properties of the algorithm that are unaffected by the missing parts. Many authors undoubtedly believe that the trappings associated with clearly defined

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Chairman's Report-

The Constitution of the Mathematical Programming Society requires the retiring Chairman to write an account of the activities of the Society during his term. While this requirement has not always been observed as carefully as others, it is a good one, and our Newsletter is the right place for it.

My term of office ended on August 31. We are all pleased to welcome the new Chairman, Jean Abadie, who will serve through August, 1983, and Manfred Padberg, who will serve as Treasurer for the same term, replacing Freerk Lootsma. (The members-at-large of the Council, Mordecai Avriel, Michael Held, Ailsa Land, and Alex Orden, hold office 1979-82, and the retiring Chairman serves as Vice-Chairman until 1982. Al Williams continues as Chairman of the Executive Committee.)

By its own definition, the Mathematical Programming Society is an international organization dedicated to the promotion and the maintenance of high professional standards in the subject of mathematical programming. The last several years' activities of the Society show a number of accomplishments to that end. New projects have been undertaken and some traditional tasks have been reorganized as a result of what has been learned during the Society's first ten years.

This *Newsletter* is one of the new things. Michael Held, Chairman of the Society's Publications Committee told the story of its founding in the first issue. We know it will be of real service to the mathematical programming community.

Last year the Society inaugurated the first Prize to be given in the broad field of mathematical programming: the Fulkerson prize, for outstanding papers in the area of discrete mathematics. It is sponsored jointly by the Mathematical Programming Society and the American Mathematical Society. The first awards

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were made in Montreal in 1979 at the Tenth International Symposium on Mathematical Programming.

This Spring the Society concluded arrangements with the Society for Industrial and Applied Mathematics for the George B. Dantzig Prize in mathematical programming 'for original work, which by its breadth and scope, constitutes an outstanding contribution to the field.' We expect the first award of the Prize to take place at the Eleventh Symposium in mathematical programming in Bonn in 1982. The Prize is fully described in the first issue of this *Newsletter*.

Every year there are important meetings around the world dedicated to mathematical programming. The Society began offering sponsorship to any properly constituted conference related to mathematical programming or one of its subfields. The Society requires only that the meeting have a scientific or educational purpose, rather than that of profitmaking, that it be open to all members of the Society so far as available space or resources allow, and that there be opportunity to solicit membership in the Society at the meeting. We can provide mailing lists or labels of the Society membership, space in its publications for announcements, facilities for publicity at other sponsored meetings, and the loan of some money. Any one interested in such sponsorship should contact the Chairman of the Society or the Chairman of the Executive Committee. Suggestions for other modes of support of mathematical programming are also welcome.

Under this program the Society sponsored these meetings in 1980: 'Workshop on polynomial-time algorithms for linear programming', New York City, February; 'Workshop on Large-scale Linear Programming', Laxenburg, Austria, June; 'Workshop in Numerical Methods for System Engineering Problems', Lexington, Kentucky, June; and 'Nonlinear Programming Symposium 4', Madison, Wisconsin, July. In 1981 it will sponsor the four meetings so listed in the *MPS Calendar* in this issue.

The Membership Committee was formed in 1979 with R.W. Cottle as Chairman (now John Mulvey). Having a substantial membership benefits the Society not only in enabling it to speak with

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news from COAL

The Committee on Algorithms has been quite active the last few months. The latest issue of the COAL Newsletter was mailed in August to all members of the Mathematical Programming Society and all "friends of COAL". If, for some reason, you do not receive this newsletter within the next month, please contact me so that this error can be corrected. Also, proceedings of the special session jointly sponsored by COAL and the Computer Science Technical Section of ORSA was published in the April, 1980 issue of SIGMAP. The proceedings entitled "Recent and Future Advances in Mathematical Programming Software" include papers written by **all** major mathematical programming system developers. Copies of this issue are available to "friends" of COAL, free of charge, while supplies last. Any requests for this issue should be sent to Richard P. O'Neill, Department of Energy, Division of Oil and Gas Analysis, Washington, D.C. 20461.

In the near future, all members of the MPS and "friends" of COAL will be receiving a survey requesting information about MP Codes. This survey will be used to compile a list (complete with characteristics) of the mathematical programming software available throughout the world. The Committee intends to sort and collate all information received and to update this information regularly. We believe this information is useful to a wide cross-section of the scientific community and intend to maintain this information service in the future. The results of such an effort can only be successful if we have the cooperation of the entire mathematical programming community. So please help us by completing and returning this survey form.

At the ORSA/TIMS meeting in Colorado Springs, COAL will sponsor a session chaired by Patsy B. Saunders in which the following talks will be presented:

"Testing mathematical software which solves nonlinear systems of equations," K.L. Hiebert

"Exploiting new data structures for generalized network optimization," Fred Glover and Randy Glover

"A study of redundancy in Linear Programming," Mark Karwan, Vahid Lotfi, Jan Telgen and Stanley Zionts

"Computational experience with the ellipsoidal algorithm for Linear Programming," John Barrer and Jan Telgen

This session will take place in the Academy Room of the Broadmoor Hotel at 9:30 - 10:45 on Monday Nov. 10.

On January 5 - 6, 1981, a conference sponsored by COAL on Testing and Validating MP Algorithms and Software will be held in Boulder, Colorado. Dr. Philip Wolfe and Prof. Darwin Klingman will present keynote addresses. Eleven sessions have been scheduled. This conference will bring together both developers and users of optimization software as well as researchers from a variety of other disciplines who have performed computational analysis in order to further develop methodologies for conducting software evaluation. Those interested in attending this meeting should contact Prof. John M. Mulvey, Princeton University, Princeton, New Jersey 08540.

Finally, Doug Shier has been invited to chair a session on computational testing of mathematical programming software at the "Optimation Days, 1981" meeting to be held in Montreal on May 13 -14, 1981. Anyone interested in presenting recent computational results should contact Doug Shier, National Bureau of Standards, Center for Applied Mathematics, Washington, D.C. 20234.

In closing, I encourage each of you to inform me of research being done by you or your colleagues in the area of development and testing of optimization software that might be highlighted in the COAL Newsletter. The deadline for the next issue is November 15, 1980.

—Karla Hoffman
Editor, COAL Newsletter

DEADLINES FOR OPTIMA
Winter Issue — January 1
Spring Issue — March 1

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procedures are superfluous to the theme of their work. Whilst simplicity is a noble goal, it should not be achieved at the price of omitting critically important detail.

To illustrate this point, consider the study of the asymptotic rate of convergence of Newton's method when the matrix of second derivatives is approximated by forward differences of the gradient. It is important to note first that the eventual rate of covergence of any iterative algorithm on a computer is at best linear, regardless of the properties of its exact mathematical counterpart. In fact, a limit is inevitably reached beyond which the solution cannot be further refined (otherwise, solutions of arbitrarily high accuracy could be obtained). A superlinear rate of convergence is typically exhibited in a sequence of computed iterates for only a few iterations before collapsing.

In the case of the finite-difference Newton method, it can be shown mathematically that the convergence rate is linear, with the factor of improvement proportional to the size of the finite-difference interval. This has led to the suggestion that a superlinear rate of convergence can be achieved by reducing the finite-difference interval to zero as the solution is approached.

Despite its theoretical justification, use of this strategy in an implemented algorithm is dangerous, and fortunately unnecessary. If the finite-difference interval is chosen sensibly, the predicted linear rate of convergence occurs only after improvement in the solution has reached the level of rounding error. Moreover, prior to this stage, the finite-difference algorithm exhibits a *quadratic* rate of convergence almost identical to that of Newton's method with analytic second derivatives. It is thus important to establish the effects of rounding errors on a rate of convergence that is derived under the assumption of exact arithmetic.

Although much attention is devoted to proving convergence of algorithms, such results are not always significant in practice. For example, global convergence can be proved for the steepest descent algorithm for unconstrained optimization under very mild assumptions. However, it is well known that this is a very inefficient algorithm. Almost all proofs of convergence are based on showing that some reduction in the function value occurs at each iteration. What can happen in practice with the steepest descent and other methods is that the step taken or the subsequent decrease in the function value becomes negligible with respect to the machine precision, even at points remote from the solution. Consequently, the change cannot be recognized and the computed sequence terminates.

The mere existence of a convergence proof of standard theoretical form does not necessarily assure that the algorithm will succeed. Convergence proofs are often extended by generalizing the class of functions or situations for which they apply. It would be useful for research to be carried out on extending proofs to take account of the computational environment in which the algorithm will be applied. In essence, the gap between theory and practice needs to be narrowed by developing a more sophisticated theory.

Difficulty with the implemented versions of algorithms takes three distinct forms: the algorithms may be inherently unstable, instability may be induced, or the conditioning of the problem may be exacerbated. The last two difficulties can be avoided, although it is often far from obvious how to do so. The reader may wonder why anyone would choose to implement an algorithm in other than the most stable form. Unfortunately, unstable forms of an algorithm are usually more common than stable forms; moreover, instability is often related to closed-form expressions.

An algorithm is said to be inherently unstable if the steps of the procedure are prone to overwhelming computational error, irrespective of how they are implemented. The error does not occur due to ill-conditioning of the problem or the solution, but rather because of the manner in which the iterates are defined. Inherent instability takes two forms: the errors may be inevitable, or they may occur by chance.

An example of inevitable error is provided by some cutting-plane methods. These algorithms solve a nonlinearly constrained optimization problem by transferring it into a sequence of linear programming sub-problems. As the method proceeds, the solution of the linear program contains at least two linearizations of the same nonlinear constraint, unless the number of binding nonlinear constraints is equal to the number of variables. Although the LP constraints retain linear independence mathematically, in practice there is eventually overwhelming computational error in computing the LP solution.

The occurrence of chance error is illustrated by an algorithm that requires the determination of the Cholesky factorization of an arbitrary symmetric matrix. The Cholesky factorization may not even exist unless the matrix is positive definite. Moreover, even if the factorization exists for an indefinite matrix, there may be catastrophic error in its computation. The error is not, however, inevitable because it may happen that the relevant matrices are always positive definite, or that no significant errors occur in forming the factorization. Although such an algorithm is inher-

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The Newsletter

Since this is just the second issue of OPTIMA, it might be well to repeat some of the information about the newsletter. Our purpose is to make information about progress in mathematical programming available to the wide-spread MPS membership. Toward that end we will publish a calendar, technical report titles, meeting announcements, news from COAL, calls for papers, etc. as well as brief items about the activities of individuals and their whereabouts. Brief position announcements and short course announcements will be included for anominal fee ($50).

In each issue of OPTIMA we hope to feature an article of broad interest. In the first issue we had Phil Wolfe's **The Ellipsoid Algorithm** and in this issue we have Walter Murray's article. Contributions are invited. The feature articles may be historical, offer perspectives on our field, suggest directions for research, and, generally, be on any topic likely to be of interest to the membership. Such articles will be edited, but not refereed. If these articles, or others, provoke Letters to the Editor, we can start such a column.

I'm very pleased to announce that Professor **Achim Bachem**, of the University of Bonn, will be OPTIMA's Associate Editor. He will particularly be involved with reporting news from Europe. Items for Professor Bachem should be sent to:

Institute für Ökonometrie und
Operations Research
University of Bonn
Nassestrasse 2
D-5300, Bonn 1, BRD

Professor Bachem's contributions will do much to enhance OPTIMA, and they will undoubtedly prove very important as we approach the Bonn meeting in 1982.

—Don Hearn

OPTIMA
Newsletter of the Mathematical Programming Society
Donald W. Hearn, Editor
Achim Bachem, Associate Editor
Published by the Mathematical Programming Society and Information Services of the College of Engineering, University of Florida. Composition by Lessie McKoy, and Mechanical Production by Dick Dale.

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ently unstable, it may enjoy some success on selected problems.

Induced instability may occur because of the incomplete computational definition of an algorithm. For example, in some methods the search direction is defined as the solution of a set of linear equations. This calculation may be performed in various ways, some of which are numerically unstable even for a well-conditioned system. If a poor technique is used to solve the linear system, the instability has been induced into the algorithm because an alternative, numerically stable method could have been used. Even when the author of an algorithm specifies an unstable procedure for solving a sub-problem, induced instability can be avoided by substituting a numerically sound method.

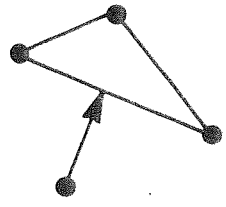
Exacerbation of the condition of a problem can occur in two forms. One is illustrated by the well-known example of forming the "normal" equations when solving linear least-squares problems, which occur in their own right and as sub-problems within methods for nonlinear least-squares. The error in the solution of the normal equations depends on the square of the condition number of the Jacobian matrix. If the mathematically equivalent solution is obtained instead via the QR factorization, then the error in the solution depends on the condition number of the Jacobian, provided that the residual is zero. Even if the residuals are not zero, the accuracy of the solution obtained using the QR factorization is close to that which would be obtained if the solution were computed in infinite precision after representing the original problem in finite precision. That is, the effect of the error in the solution due to errors introduced by computation is of the same order as the inevitable error introduced by representing the problem on the machine.

A similar kind of error is not, strictly speaking, an exacerbation of the conditioning of the problem, but rather the introduction of error based on the conditioning that could be entirely avoided. This kind of error is illustrated by an early technique for extending a quasi-Newton method for unconstrained optimization to problems with linear equality constraints. The idea is to choose a rank-deficient matrix as the initial approximation to the inverse Hessian. Although the null space of the constraint normals does not change, at each iteration the inverse Hessian approximation must be updated in order to build up curvature information. Thus, the representation of a constant piece of information is modified because it has been combined with the representation of another piece of information that does vary. The consequence is that unnecessary error is introduced into the computed search direction, and there can be a substantial loss of feasibility in the iterates. This type of error is not the same as induced instability because the error would be small if every part of the problem were well conditioned. However, this case differs from the normal equation example because the error that causes a loss of feasibility can be made independent of the conditioning of other parts of the problem – for example, by storing separate representations of the information concerning curvature of the function and orthogonality to the constraint normals.

This article has presented only a brief overview of one aspect of how the computer influences the study of algorithms. A complete treatment of this subject would require consideration of many other complicated areas, including data structures and the design of software.

Reference

H.A. Watts and L.F. Shampine [1975], *The Art of Writing a Runge-Kutta code: RK45*, paper presented at SIAM National meeting, June 1975.



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a louder voice on behalf of its members, but in lowering the cost of services to the membership. The work of the membership Committee has increased the number of members from 408 to almost 500, the figure at which we will be able to purchase our journal at a reduced price (see below).

A new agreement has been negotiated with North-Holland Publishing Company, publisher of *Mathematical Programming* and the *Studies*. It provides for reductions of the member's price for the journal when membership of the Society exceeds 500, and a novel feature: substantially increased royalties to the Society for new institutional subscriptions which have come about as a result of the Society's efforts. Thus, if your institution does not subscribe to *Mathematical Programming*, you think it should, and are successful in getting it to do so, please inform Michael Held (IBM Systems Research Institute, 205 East 42 Street, New York, NY 10017) so that the Society can claim credit for it and express its gratitude to you.

Michel Balinski, founding Editor-in-Chief, announced his plans in mid-1979 to resign his post as soon as a suitable replacement could be found. We were delighted to find Richard W. Cottle willing to serve in this key position, which he has done since January. He has implemented a reorganization of the Editorial Board aimed at reducing the editorial workload and speeding the handling of manuscripts: Collaborating with the Editor-in-Chief, Co-editors (presently L.C.W. Dixon, B. Korte, and M.J. Todd) can accept submissions and conduct all dealings with authors.

Of course, the major regular event was the Tenth International Symposium on Mathematical Programming in Montreal in August last year, where 450 papers on all aspects of mathematical programming were presented to 700 registered attendees. A new, successful feature of the program were the minicourses in combinatorial algorithms, in pivotal exchange methods, and multiple criteria decision making. Organization of the Symposium was assisted to a modest degree by the newly formed Symposium Advisory Committee (Alex Orden, Chairman; now Jean-Louis Goffin). The Committee is charged with encouraging proposals for sites for the International Symposia, advising the Council on the choice of site, assisting their hosts where possible, and setting guidelines for the Society's role in these official meetings. The Society will continue its practice of giving the host committee charge of the whole affair, but will arrange, at least, for a reduced registration fee for members of the Society.

After considering several proposals, the Council of the Society chose Bonn, Federal Republic of Germany, as the site of the Eleventh International Symposium August 23-28, 1982. It is not too soon to begin the study of possible sites for the Twelfth Symposium, which should be held at a like time in 1985. The Symposium Advisory Committee welcomes all suggestions about possible sites for future meetings. Please address the Chairman (Professor Jean-Louis Goffin, Faculty of Management, McGill University, 1001 Sherbrooke Street West, Montreal, Quebec H3A 1GS, Canada). To page 5

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The fourth issue (August, 1980) of the *Newsletter* of the Society's Committee on Algorithms has just appeared. As editor Karla Hoffman says, 'The Committee on Algorithms has been rather active the past six months...' Indeed, this valuable group has been very active since its formation. Please see its Newsletter, as well as the Committee's column in this and the previous issue of OPTIMA, for more information.

It seems that the year will end with the Society healthy financially as well as morally. Projections of the state of the treasury have encouraged the Council to set the dues for 1981 at the same level (in dollars) as for 1980, and in other currencies the dues should be about the same, or lower. Of course this represents a reduction of dues in real terms, We hope it will encourage members cheerfully to pay when they receive their notices in the near future, and, if you are not a member, encourage you to make application for 1981 -- if you have read this far, then you should join us.

Philip Wolfe

NEW MATHEMATICAL PROGRAMMING SYMPOSIA IN JAPAN

Two research groups on mathematical programming in Japan (one in the Tokyo area chaired by Professor K. Tone of Saitama University and the other in the Kansai area chaired by Professor T. Ibaraki of Kyoto University) have agreed to start the annual Symposia on Mathematical Programming to promote research in, and applications of, mathematical programming in Japan. The symposia will be held in the autumn of every year, in the Tokyo area and in the Kansai area alternately. Tutorial lectures on recent advances in mathematical programming will be delivered and contributed papers on participants' original works will be presented. Participation from abroad is also welcome.

The first symposium will be held November 6-7, 1980, at the Institute of Statistical Mathematics in Tokyo. Lectures and tutorials will be given in the following sessions: **Linear and Nonlinear Programming** chaired by H. Konno, **Combinational Optimization** chaired by T. Ibaraki, **Applications** chaired by S. Suzuki.

Those interested are requested to contact the organizing committee (Chairman: Professor Masao Iri, University of Tokyo, Bunkyo-ku, Tokyo, Japan 113) or the Programme Committee (Chairman: Professor Kaoru Tone, Graduate Institute for Policy Science, Saitama University, Shimo-Okubo, Urawa-shi, Saitama-ken, Japan 338).

– M. Iri

Conference on OPTIMIZATION : THEORY & ALGORITHMS March 16-20, 1981

The Conference will be held at Confolant (Miremont, Puy-de-Dôme) France, about 50 km from Clermont-Ferrand. It has been organized by **J.-B. Hiriart-Urruty**, University of Clermont-Ferrand II, **W. Oettli**, Universität Mannheim, and **J. Stoer**, Universität Würzburg. Forty researchers, mainly from France and West Germany, have been invited to speak on the latest developments in the theory, algorithms and applications of optimization. The official languages of the conference are French and English.

For more information and registration, prospective participants should contact one of the organizers.

–J.-B. Hiriat-Urruty

IFORS '81 - HAMBURG, GERMANY JULY 20-24, 1981 9TH INTERNATIONAL CONFERENCE ON OPERATIONAL RESEARCH

The International Federation of Operational Research Societies (IFORS) will hold its 9th INTERNATIONAL TRIENNAL CONFERENCE in Hamburg, Germany, on July 20-24, 1981.

The aims of IFORS are the development of Operational Research as a unified science and its advancement in all nations of the world. Up to now, 31 national OR societies have become Member Societies of IFORS. In addition, 6 OR societies, such as the Mathematical Programming Society, are associated with IFORS as Kindred Societies.

One series of sessions among several others will be dedicated to the theme of IFORS '81 which reads: **Operational Research in the Interest of International Cooperation.** In addition to the mainstream Sessions which are dedicated to the theme, there will be Technical Sessions, National Contributions Sessions, Contributed Papers Sessions, Workshops, and a special Exibition of computer hardware and software.

Authors who would like to contribute a paper should submit an abstract no later than December 15, 1980, to the Chairman of the Programme Committee: Prof. Dr. Marc Roubens, Faculté polytechnique, Rue de houdain 9, B-7000 Mons, Belgium.

–Heiner Muller-Merbach
IFORS'81: Public Relations
Am Lowenter 11

POSITIONS AVAILABLE

U.S. Department of Energy

The Mid-Term Energy Market Model (MEMM) forecasts energy supplies and prices for the period 1985-1995. In addition to making a congressionally mandated annual forecast, we are using the model for a comprehensive assessment of government interventions in the energy market. The model is also used for special studies of proposed legislation and regulatory programs.

We are looking for good analysts with a background in operations research, economics or computer science who would like to work in a demanding atmosphere. Two types of people would have the highest priority - a fresh graduate with a bachelor's or master's degree, extensive computer programming experience, and some knowledge of economics or programming; or someone who has received a PhD within the last five years, preferably in mathematical programming, has excellent analytic capability, and a willingness to do some computer programming. Some other combination of talents than those stated above might be acceptable. The most important requirements are excellent analytic capability and a willingness to work hard.

For more information, contact:

Julie H. Zalkind
Director
Midterm Analysis Division
Energy Information Administration
12th & Pennsylvania Ave., N.W. - Rm. 4433
Washington, DC 20461
(202) 633-8505

Technical Reports & Working Papers

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Ithaca, NY 14853

J.J. Billera and D. Heath, "*A Unique Procedure for Efficient Allocation of Shared Costs,*" TR 430.

W-L. Hsu and G.L. Nemhauser, "*A Polynominal Algorithm for the Minimum Weighted Clique Cover Problem on Claw-free Perfect Graphs,*" TR 434.

W-L. Hsu, "*How to Color Claw-free Perfect Graphs,*" TR 435.

M.J. Todd, "*Approximate Labelling for Simplicial Algorithms and Two Classes of Special Subsets of the Sphere,*" TR 442.

M.J. Todd, "*A Constructive Proof of Sonnenschein's Lemma,*" TR 443.

D. Goldfarb and M.J. Todd, "*Modifications and Implementation of the Shor-Khachian Algorithm for Linear Programming,*" TR 446.

R.G. Bland, "*Linear Programming Duality and Minty's Lemma,*" TR 449.

W.F. Lucas, "*Game Theory,*" TR 450.

M.J. Todd, "*PLALGO: A FORTRAN Implementation of a Piecewise-Linear Homotopy Algorithm for Solving Systems of Nonlinear Equations,*" TR 452.

M.J. Todd, "*An Implementation of the Simplex Method for Linear Programming Problems with Variable Upper Bounds,*" TR 461.

UNIVERSITY OF COLGONE
Department of Mathematics
Weyertal 86-90, D-5000 Koln 41, West Germany

R.E. Burkard, H. Hamacher, "*Minimum Cost Flows in Regular Matroids,*" 79-12.

U. Zimmermann, "*Linear Optimization for Linear and Bottleneck Objectives with One Nonlinear Parameter,*" 79-15.

U. Zimmermann, "*A Primal Dual Method for Algebraic Linear Programming,*" 79-16.

R. Euler, "*On a Classification of Independence Systems,*" 80-7.

H. Hamacher, "*Optimal Cuts,*" 80-8.

R.E. Burkard, J. Krarup, P.M. Pruzan, "*Efficiency and Optimality in Minisum, Minimax 0-1 Programming Problems,*" 80-9.

R.E. Burkard, H. Keiding, J. Krarup, P.M. Pruzan, "*A Relationship Between Optimality and Efficiency in Multicriteria 0-1 Programming Problems,*" 80-10.

R. E. Burkard, U. Zimmermann, "*Combinatorial Optimization in Linearly Ordered Semimodules: A Study,*" 8012.

UNIVERSITY OF WISCONSIN-MADISON
Mathematics Research Center
610 Walnut Street
Madison, Wisconsin 53706

Stephen M. Robinson, "*Generalized Equations and Their Solutions, Part II: Applications to Nonlinear Programming,*" TR 3048.

UNIVERSITY OF CALIFORNIA AT BERKELEY
Electronics Research Laboratory
College of Engineering
Berkeley, CA 94720

E. Polak and S. Tishyadhigama, "*New Convergence Theorems for a Class of Feasible Directions Algorithms,*" UCB/ERL M78/25, 1980.

E.Polak and A. Sangiovanni-Vincentelli, "*Theoretical and Computational Aspects of the Optimal Design Centering, Tolerancing and Tuning Problems,*" UCB/ERL M/79/6, 1979.

R. Trahan and E. Polak, "*A Derivative-Free Algorithm for a Class of Infinitely Constrained Problems,*" UCB/ERL M78/75, 1977.

C. Gonzaga, E. Polak and R. Trahan, "*An Improved Algorithm for Optimization Problems With Functional Inequality Constraints,*" UCB/ERL M78/56, 1977.

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
System and Decision Sciences Area
2361 Laxenburg
Austria

A. Propoi, "*Iterative Methods for Structured Linear Programming,*" WP 79-013.

M. Kallio, "*On the Simplex Method Using an Artificial Basis,*" WP 79-025.

K. Tone, "*Multistage Benders' Decomposition Applied to Multiperiod, Multicommodity Production, Distribution and Inventory System,*" WP 79-045.

E. Nurminski, "*Conceptual Newton Method for Solving Multivalued Inclusions: Scalar Case,*" WP 79-050.

A. Wierzbicki, "*The Use of Reference Objectives in Multiobjective Optimization--Theoretical Implications and Practical Experience,*" WP 79-066.

M. Kallio, W. Orchard-Hays, A. Propoi, "*Linkage of Optimization Models,*" WP 79-083.

M. Kallio, W. Orchard-Hays, "*Experiments with the Reduced Gradient Method for Linear Programming,*" WP 79-084.

J. Birge, "*Some Conditions for Optimal Deterministic Solutions to Stochastic Dynamic Linear Programs,*" WP 79-101.

E. Nurminski, "*Some Theoretical Considerations on Linkage Problems,*" WP 79-117.

A. Wierzbicki, "*A Methodological Guide to Multiobjective Optimization,*" WP 79-122.

A. Wierzbicki, "*The Use of Reference Objective Levels in Group Assessment of Solution of Multiobjective Optimization,*" WP 79-124.

M. Peschel, J. Ester, Nguyen Thuc Loan, "*Reference, Preference, Convexity and Efficiency--Basic Notions in Multiobjective Decision Making,*" WP 80-022.

M. Kallio, A. Lewandowski, W. Orchard-Hays, "*An Implementation of the Reference Point Approach for Multiobjective Optimization,*" WP 80-036.

C. Lemarechal, "*Nonsmooth Optimization: Use of the Code DYNEPS,*" WP 80-036.

E. Nurminski, "*Numerical Experiments with Decomposition of LP on a Small Computer,*" WP 80-037.

A. Wierzbicki, "*A Mathematical Basis for Stafisficing Decision Making,*" WP 80-090.

C. Lemarechal, E. Nurminski, "*Differentiability of a Support Function of an E-Subgradient,*" WP 80-101.

Y. Ermolev, "*Some Problems of Linkage Systems,*" WP 80-102.

TECHNICAL REPORTS & WORKING PAPERS. . . .

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Systems Optimization Laboratory
Department of Operations Research

George B. Dantzig, *"Expected Number of Steps of the Simplex Method for a Linear Program with a Convexity Constraint,"* SOL 80-3.

Walter Murray, Michael L. Overton, *"A Projected Lagrangian Algorithm for Nonlinear l_1 Optimization,"* SOL 80-4.

Zachary F. Lansdowne, *"Network Formulation of the Midterm Energy Market Model,"* SOL 80-5.

Philip E. Gill, Walter Murray, Michael A. Saunders, Margaret H. Wright, *"Computing Finite-Difference Approximations to Derivatives for Numerical Optimization,"* SOL 80-6.

Philip E. Gill, Walter Murray, Michael A. Saunders, Margaret H. Wright, *"Aspects of Mathematical Modeling Related to Optimization,"* SOL 80-7.

Philip E. Gill, Walter Murray, Michael A. Saunders, Margaret H. Wright, *"Methods for Large-Scale Nonlinear Optimization,"* SOL 80-8.

Robert M. Freund, *"A Constructive Proof of the Borsuk-Ulam Antipodal Point Theorem,"* SOL 80-9.

Robert M. Freund, *"Variable-Dimension Complex with Applications,"* SOL 80-11.

Robert M. Freund, Michael J. Todd, *"A Constructive Proof of Tucker's Combinational Lemma,"* SOL 80-12.

Bruce A. Murtagh, Michael A. Saunders, *"Minos/Augmented User's Manual,"* SOL 80-14.

R.W. Cottle, J.S. Pang, *"On the Convergence of a Block Successive Over-Relaxation Method for a Class of Linear Complementarity Problems,"* SOL 80-17.

George B. Dantzig, *"Time-Staged Methods in Linear Programming; Comments and Early History,"* SOL 80-18.

Philip G. Abrahamson, *"A Nested-Decomposition Approach for Solving Stair-Case-Structured Linear Programs,"* SOL 80-20.

UNIVERSITY OF BONN
Department of Operations Research
Nassestr. 2, D-5300 Bonn, West Germany

Annual Report 1979, WP 80150-OR

M. Grötschel, L. Lovász, A. Schrijver, *"The Ellipsoid Method and its Consequences in Combinatorial Optimization,"* WP 80151-OR.

L.Butz, *"Charakterisierungen Zusammenhängender Zweifacher Blockpläne,"* WP 80152-OR.

L. Butz, *"Characterizations of Connectivity in Row-Column Designs,"* WP 80153-OR.

A. Bachem, *"Anwendungen der Komplexitätstheorie im Operations Research: Eim Überblick,"* WP 80154-OR.

A. Bachem, R. Schrader, *"Einfuhrung in des Operations Research,"* WP 80155-OR.

B. Korte, R. Schrader, *"A Note on Convergence Proofs for Shor-Khachian Methods,"* WP 80156-OR.

V. Klee, *"A Note on Convex Cones and Constraints Qualifications in Infinite-Dimensional Vector Spaces,"* WP 80157-OR.

M.A. Frumkin, G.V. Gens, Ju. I. Hmelevakii, E.V. Levner, *"On Computational Complexity of Optimization Combinatorial Problems,"* WP 80157-OR.

"IV Bonn Workshop on Combinatorial Optimization," August 28-30, 1980, Abstracts, WP 80159-OR.

Journals & Studies

Richard W. Cottle, Editor-In-Chief, has announced the following contents of **Mathematical Programming**, Volume 19, Numbers 2 and 3:

Volume 19 No. 2

U.G. Rothblum and H. Schneider, *"Characterizations of Optimal Scalings of Matrices."*

W.Gochet and Y. Smeers, *"A Modified Reduced Gradient Method for a Class of Nondifferentiable Problems."*

P.H. Zipkin, *"Bounds for Aggregating Nodes in Network Problems."*

T.F. Coleman and A.R. Conn, *"Second-Order Conditions for an Exact Penalty Function."*

R.D. Armstrong and D.S. Kung, *"A Dual Method for Discrete Chebychev Curve Fitting."*

O.L. Mangasarian, *"Locally Unique Solutions of Quadratic Programs, Linear and Nonlinear Complementarity Problems."*

K.G. Murty, *"Computational Complexity of Parametric Linear Programming."*

A. Ruszczynski, *"Feasibile Direction Methods for Stochastic Programming Problems."*

M.C. Cheng, *"New Criteria for the Simplex Algorithm."*

R. Horst, *"A Note on the Convergence of an Algorithm for Nonconvex Programming Problems."*

Volume 19 No. 3

A.F. Perold, *"A Degeneracy Exploiting LU Factorization for the Simplex Method."*

M.E. Dyer, *"Calculating Surrogate Constraints."*

R.D. Wollmer, *"The Two-Stage Linear Programming Model Under Uncertainty with 0-1 First Stage Variables."*

S. Schaible and I. Zang, *"On the Convexifiability of Pseudonconvex C^2-Functions."*

J. Gauvin, *"Shadow Prices in Nonconvex Mathematical Programming."*

P.S. Brooks, *"Infinite Retrogression in the Eaves-Saigal Algorithm."*

L.Van der Hyden, *"A Variable Dimension Algorithm for the Linear Complementarity Problem."*

R. W. Cottle, *"Completely-Q Matrices."*

J. Ch. Pomerol, *"About a Minimax Theorem of Mattheis, Strang, and Christiansen."*

Mathematical Programming Study 13, Combinational Optimization II, Edited by V.J. Rayward-Smith will be published in the near future. In addition, the following studies are in preparation:

Network Models and Applications, Edited by D.W. Klingman and J.M. Mulvey.

Mathematical Programming at Oberwolfach, Edited by H. Konig, B. Korte and K. Ritter.

Optimality, Duality and Stability, Edited by M. Guignard-Spielberg.

Constrained Optimization, Edited by A.G. Buckley and J.-L. Goffin.

Applications, Edited by J.-L. Goffin and J.M. Rousseau

Mathematical Programming with Data Perturbations, Edited by A.V. Fiacco.

Matrix Generation, Report Writing and Computer-Assisted Analysis, Edited by H.J. Greenberg.

Numerical Methods for System Engineering, Edited by R.J.B. Wetts.

MPS CALENDAR

1980

November 6-7: "First Japan Mathematical Programming Symposium", Tokyo, Japan. Contact: Professor Masao Iri, University of Tokyo, Bunkyo-ku, Tokyo, Japan 113.

November 6-8: Fall meeting of SIAM, Houston, Texas, U.S.A. Theme: Mathematical applications in energy production, mathematical modeling in medical science, advances in mathematical optimization. Contact: H.B. Hair, Society for Industrial and Applied Mathematics, 1405 Architects Building, 117 South 17 Street, Philadelphia, PA 19103, U.S.A.

December 10-12: "19th IEEE Conference on Decision and Control", Albuquerque, New Mexico, U.S.A. (Submission deadline was 31 March 1980) Contact: Prof. Michael K. Sain, Dept. Electrical Engineering, Notre Dame University, South Bend, IN 46556, U.S.A.

1981

January 5-6: "Mathematical Programming: Testing and Validating Algorithms and Software". U. S. National Bureau of Standards, Boulder, Colorado. Organized by the Committee on Algorithms of the MPS, the Bureau of Standards, and the Department of Energy. Contact: Dr. Richard H. F. Jackson, Center for Applied Mathematics, National Bureau of Standards, Washington, D.C. 20234, U.S.A.; telephone 301-921-3855.

January 26-31: "Mathematische Optimierung", Mathematisches Forschungsinstitut Oberwolfach, Oberwolfach, Federal Republic of Germany. Contact: Institut für Ökonometrie und Operations Research (see 1982, August 23-28).

April 6-8: "International Congress on Mathematical Programming", Rio de Janeiro, Brazil. (Abstract deadline 1 December 1980; special forms required) Contact: Professor Milton Kelmanson, Caixa Postal 1507 - CEP 20100, Rio de Janeiro, R.J., Brazil. Sponsored by Sociedade Brasiliera de Pesquisa Operacional and the MPS.

May 13-14: "Optimization Days", Université du Quebéc à Montréal. Contact: Professor Efim Galperin, Département de mathématiques, Université du Québec à Montréal, C.P. 8888 Succ. "A", Montréal, Québec, Canada H3C 3P8; telephone 514-282-3221. Sponsored by the MPS.

July 13-24: "NATO Advanced Research Institute on Nonlinear Optimization", Cambridge, England. Contact: Professor M.J.D. Powell, Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge CB3 9EW, England. Sponsored by the MPS.

July 20-24: "Eighth British Combinatorial Conference", Swansea, England. Contact: A.D. Keedwell, Department of Mathematics, University of Surrey, Guildford, Surrey GU2 5XH, U.K.

July: "Stochastic Programming", Budapest, Hungary. Contact: Bolyai János Mathematical Society, Budapest VI, Anker köv 1-3, I. Em. III, Hungary.

August 24-28: "CO81: Conference on Combinatorial Optimization", Stirling, Scotland. Contact: Professor L. Wilson (CO81), Department of Computing, Stirling University, Scotland, U.K.

1982

August 23-28: Eleventh International Symposium on Mathematical Programming in Bonn, Federal Republic of Germany. Contact: Institut für Ökonometrie und Operations Research Universität Bonn, Nassestraße 2, 5300 Bonn 1, Federal Republic of Germany; Telex 886657 unibo b, Telephone (02221) 739285. Official triennial meeting of the MPS. (Note: The International Congress of Mathematicians will be held August 11-19 in Warsaw, Poland.)

Post-Conference Notes

VANCOUVER

Approximately sixty-five researchers from 14 countries attended the NATO sponsored Advanced Research Institute on "Generalized Concavity in Optimization and Economics," held in Vancouver in early August. The meeting was directed by Mordicai Avriel (Technion), Siegfried Schaible (University of Alberta), and Bill Ziemba (UBC).

Forty-nine papers were presented in seventeen half-day sessions. Session topics included **Characterizations and Properties of Generalized Concave Functions; Generalized Duality and Pseudoduality; Multiobjective Optimization Problems; Algorithms for Generalized Concave Programs; F-Convexity; Optimization Theory; Fractional Programming; Applications of Generalized Concavity in Economics, Portfolio Analysis, and Stochastic programming**; and **Concavifiable Functions**. Academic Press will publish the conference proceeding (edited by the Directors). The volume will be an estimated 500 pages, and should be available by late Spring, 1981.

The technical program was complemented by a well-organized social program consisting of hikes, tours, luncheons, and receptions.

—Don Hearn

BONN

The IV Bonn Workshop on **Combinational Optimization** was organized by the Institute of Operations Research, University of Bonn, August 28-30. Fifty-four participants from 17 countries attended and presented 39 papers. There will be a special issue of **Discrete Mathematics** where some of the material will appear. A list of the participants as well as abstracts of presented papers will appear as report number 80154-OR of the Institute. Copies are available on request.

—A Bachem

COLOGNE

Rainer Burhard and Thesdor Ellinger were local organizers of the V **Symposium of Operations Research** held at the University of Cologne, August 25-27. The meeting was arranged by Gesellschaft für Mathematik, Ökonomie und Operations Research, Germany. There were 380 participants of whom 160 came from Germany and 220 from foreign countries. Plenary lectures were given by **C. Berge** (Paris), **L. Lovasz** (Szeged), **R.T. Rockafellar** (Seattle) and **W. Vogel** (Bonn). In seven parallel contributed paper sessions, 250 papers were presented.

—A. Bachem

IIASA

A very successful **Workshop on Large-Scale Linear Programming** was held at the International Institute for Applied Systems Analysis, Laxenburg, Austria, on June 2-6, 1980. The workshop was co-sponsored by the Systems Optimization Laboratory of Stanford University as well as by MPS. The approximately 40 participants were mainly invited Eastern and Western scientists who actively contribute to the research in the field. A proceedings is being edited by George Dantzig, Michael Dempster and Markku Kallio, and it will be published by IIASA during Fall, 1980, in the form of two volumes.

—M. Kallio and A. Wierzbicki

PISA

An **International Workshop on Advances in Linear Optimization Algorithms and Software** was held in mid-summer. Fifteen papers were presented on linear programming, network flows, and combinatoral optimization. The workshop was sponsored by Centro Scientifico IBM di Pisa, Instituto di Matematica, L. Tonelli, University of Pisa and Instituto di Elabrazione dell' Informazione (CNR) Pisa. Local arrangements were well taken care of by C. Snadi.

—A. Bachem

Call For Papers..

MATRIX GENERATION, REPORT WRITING AND COMPUTER-ASSISTED ANALYSIS

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September 8 - 10, 1981, at Austin, Texas

An Organizing Committee of 30 members from 14 countries is soliciting abstracts for 30 minute presentations.

Abstracts (200 to 300 words) should be sent to the Program Chairman, S.Zlobec, Mathematics Department, McGill University, Montreal 110, Quebec H3A 2K6, Canada, by January 30, 1981.

For further information contact K.O. Kortanek (General Chairman), IE/OR Department, Virginia Polytechnic Institute, Blacksburg, VA, or James Vick (Arrangements Chairman), Mathematics Department, University of Texas, Austin, Texas 78712.