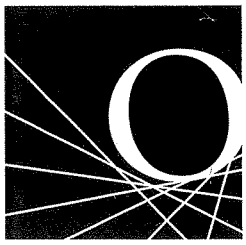


# OPTIMA

MATHEMATICAL PROGRAMMING SOCIETY NEWSLETTER

No. 42 March 1994

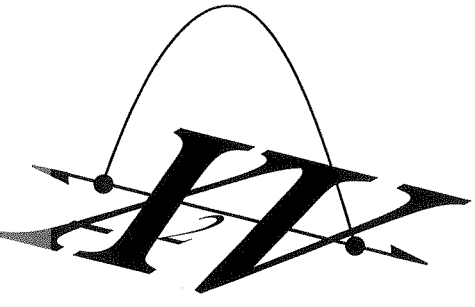

INTERNATIONAL *Symposium*

**IMPORTANT DATES AND DEADLINES**

| | |
|---|---|
| April 29, 1994 | Early registration deadline |
| May 1, 1994 | Last date to send paper copy of abstract |
| June 1, 1994 | Last date to send abstract by e-mail |
| July 15, 1994 | Last date for conference-booked hotel or Residence Hall reservation |
| July 18, 1994 | Deadline to request cancellation refund |
| July 22, 1994 | Fee for cancellation of Residence Hall lodging begins |

Copies of the Second Announcement have been mailed to all MPS members. The symposium coordinators' address is:

University of Michigan
Conferences and Seminars, Room 112
541 Thompson St.
Ann Arbor, MI 48109-1360 USA
Tel. (313) 764-5305
Fax: (313) 764-2990
e-mail: xvismp@um.cc.umich.edu



# Nominations for 1994 Elections

The Constitution of the Mathematical Programming Society sets the term of office for all officers of the Society at three years. Elections for all offices (Chairman, Treasurer and four at-large members of the Council) are held four months prior to each triennial International Symposium. Therefore, the next election will be held in April 1994. the new members-at-large of the Council will take office at the time of the symposium, while the Chairman-elect and Treasurer-elect will take office one year later.

Candidates must be members of the Society and may be proposed either by Council or by any six members of the Society. No proper nomination may be refused, provided the candidate agrees to stand. The procedure to be followed is:

1 *Nomination to any office is to be submitted to Jan Karel Lenstra, Chairman, by April 1, 1994. Such nomination is to be supported in writing by the nominator and at least five other members of the Society.*

2 *In keeping with tradition, the next chairman preferably should be a North American resident. The membership is asked to consider no residents from other continents to be Chairman.*

3 *When the ballots are counted, the four at-large candidates for Council receiving the highest number of votes will be elected, except that not more than two members having permanent residence in the same country may be elected.*


Jan Karel Lenstra, Chairman
Department of Mathematics and Computing Science
Eindhoven University of Technology
P.O. Box 513, 5600 MB Eindhoven,
The Netherlands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# Program INTERNATIONAL *Symposium* Summary

**The 15th International Symposium on Mathematical Programming**, the triennial scientific meeting of the Mathematical Programming Society, will be held Aug. 15-19, 1994, on the central campus of the University of Michigan at Ann Arbor, MI, USA.

Conference activities begin on Sunday afternoon Aug. 14 with a golf game, planned for those interested, at the university golf course. Tee times begin at 1 p.m.

The registration desk for the symposium will be on the second floor concourse of the Michigan League, and will be open from noon to 8 p.m. on Sunday. The desk will be staffed Monday through Friday from 7:30 a.m. to 5 p.m. The symposium's inaugural ceremony will begin Sunday at 8 p.m., with welcoming addresses and a musical invocation by Pulitzer Prize-winning composer William E. Bolcom.

The opening session on Monday, Aug. 15 will be held from 8:30-10:45 a.m. in the Rackham Building. William Cook of Bellcore will address the state of the art in large-scale combinatorial optimization, and Roger Wets will speak on the contributions of George Dantzig, who will receive an honorary award. The Dantzig, Fulkerson and Beale-Orchard-Hays prizes will be awarded and the finalists for the A.W. Tucker Prize will be announced.

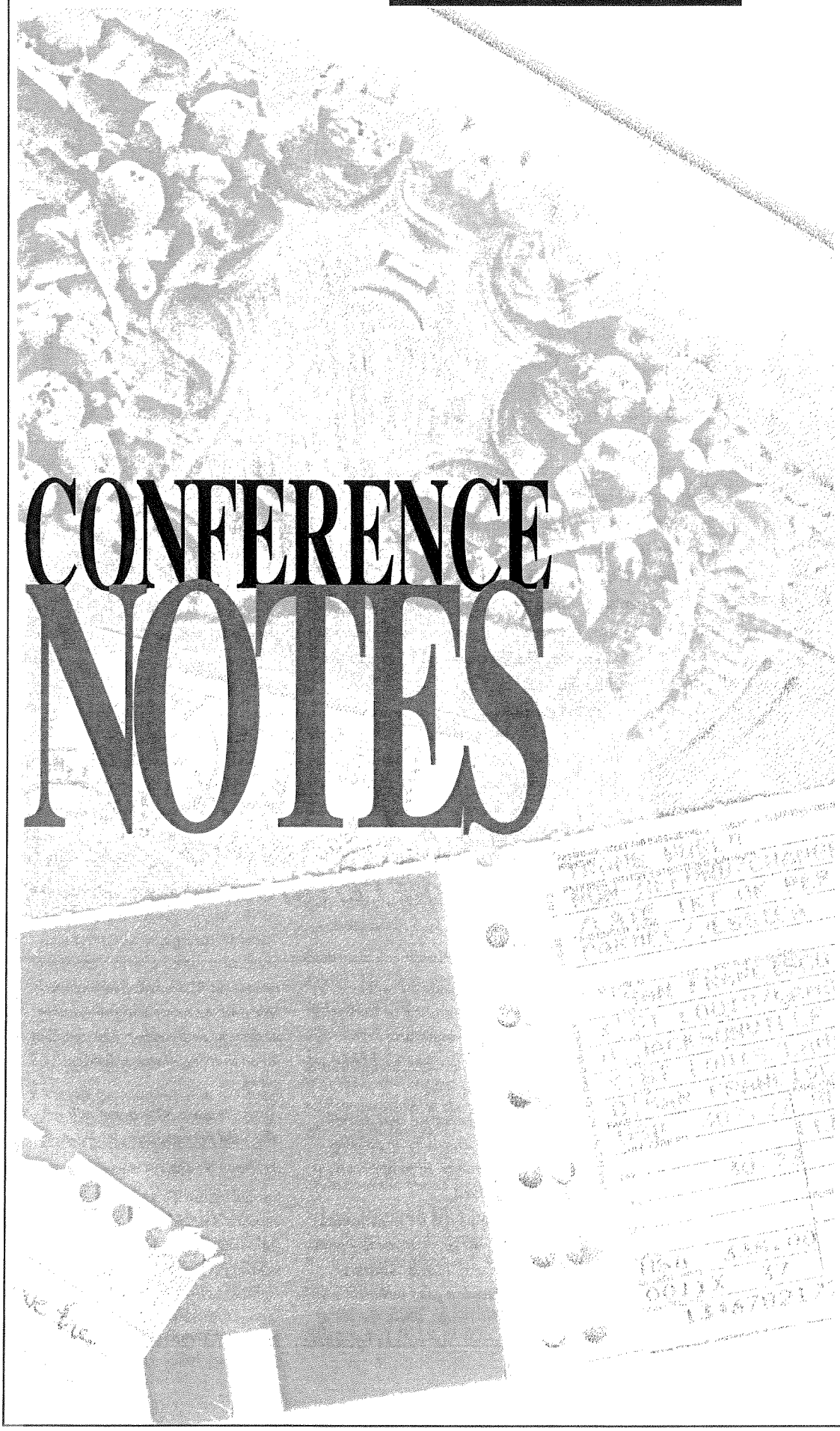
Seminar sessions will be held Monday through Friday. The opening reception for all registered participants and their accompanying guests will be held Monday in the Michigan Union from 6:15-8 p.m. Refreshments will be served.

One hour, state-of-the-art tutorial and survey lectures, dealing with recent developments covering the broad spectrum of mathematical programming and related areas, have been organized. These will be held in special sessions, two in parallel, 1:30-2:30 p.m. and 4:30-5:30 p.m. on Monday, 9:45-10:45 a.m. and 1:30-2:30 p.m. on Wednesday, and 9:45-10:45 a.m. and 3:15-4:15 p.m. on Tuesday, Thursday and Friday. Following is a list of speakers who have accepted the invitation to lecture in this series.

| | |
|---|---|
| **R. Bixby** | *Recent results in linear programming computation* |
| **W. Cook** | *Large-scale combinatorial optimization* |
| **G. Cornuejols** | *Balanced matrices* |
| **A. Frank** | *Strongly polynomial algorithms* |
| **C. Gonzaga** | *Interior and pathfollowing methods for LP* |
| **A. Griewank** | *Automatic differentiation* |
| **J. Holland** | *Genetic algorithms* |
| **N. Karmarkar** | *Interior methods in combinatorial optimization* |
| **U. Karmarkar** | *Mathematical programming in manufacturing* |
| **R. Karp** | *Approximate solutions to NP-optimization problems* |
| **L. Lovasz** | *Number theory, the algebra of polynomials, M and MP* |
| **J. Mulvey** | *Finance* |
| **W. Murray** | *Large-scale nonlinear optimization* |
| **G. Nemhauser** | *Integer programming solution strategies* |
| **A.S. Nemirovskii** | *Interior methods for convex programming* |
| **J. Nocedal** | *Nonlinear methods* |
| **P. Pardalos** | *Global Optimization* |
| **R.T. Rockafellar** | *Nonsmooth optimization* |
| **S. Smale** | *Newton methods and complexity* |
| **P. Toth** | *Routing and transit* |

*These speakers have been requested to prepare state-of-the-art survey articles or extended abstracts, based on their talks. A paperback booklet of these articles will be handed out to each registered participant on arrival at the conference.*

John Birge, General Chair
Katta G. Murty, Program Chair



# CONFERENCE NOTES

## ANNOUNCEMENT & CALL FOR PAPERS

## *International Conference on Operations Research*

**Berlin, Aug. 30-Sept. 2, 1994**

Operations Research 1994 is the second conference in a series of quadrennial international conferences that take place under the auspices of the OR societies, DGOR, GM ÖOR, ÖGOR, SVOR, of the German-speaking countries and (for the first time) the Special Interest Group on Optimization, SIGOPT, of the DMV.

The conference serves as a forum for scientists and practitioners in all areas of Operations Research. Conference languages are English and German. The scientific program includes invited plenary and semiplenary lectures, as well as contributed papers. Presentation of the latter is limited to 30 minutes, including discussion. The deadline for submission of abstracts is April 15, 1994. Contributions will be documented in a conference volume.

Participants are encouraged to present software solutions to their contributions or software systems. A software exhibition of OR-related systems is planned.

MAILING ADDRESS:

Prof. Dr. R.H. Möhring
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Strabe des 17. Juni 136
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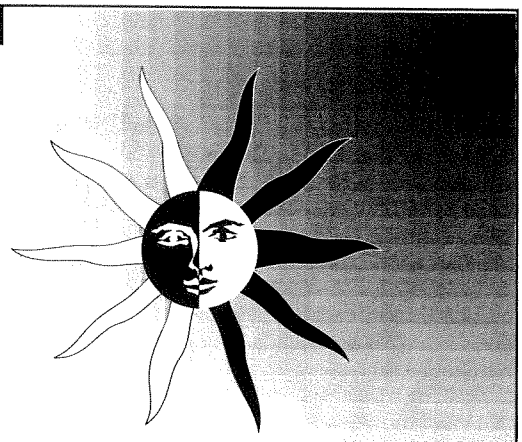
Tel: 0049-30-314-24594/25728
e-mail: or94@math.tu-berlin.de



# CALL FOR PAPERS

## *Fifth Stockholm Optimization Days*

Theoretical, computational and applied papers are welcome for the Fifth Stockholm Optimization Days, a two-day conference on optimization, to be held at KTH (Royal Institute of Technology) in Stockholm, Sweden, June 27-28, 1994. Plans call for sessions on crew and vehicle scheduling, dual optimization methods and nonlinear programming, among other areas.

Abstracts (a maximum of 200 words) should be sent by May 1 (preferably by e-mail) to: *optdays@math.kth.se* or by mail to Optimization Days, Division of Optimization and Systems Theory, KTH, S-100 44 Stockholm, Sweden. The FAX number is +46 8-22 53 20.

Further information can be obtained from the same addresses.

The conference is financially supported by the Goran Gustafsson Foundation and the Swedish National Board for Industrial and Technical Development (NUTEK). Organizers are Ulf Brannlund, Anders Forsgren (head), Per Olov Lindberg and Krister Svanberg from the Division of Optimization and Systems Theory, Department of Mathematics, Royal Institute of Technology (KTH).

Invited speakers include:

| | |
|---|---|
| **T. Coleman** | Cornell University, Ithaca, NY, USA |
| **J. Desroisiers** | HEC, Montreal, Canada |
| **D.M. Gay** | AT&T, Murray Hill, CA, USA |
| **P.E. Gill** | University of California, San Diego, CA, USA |
| **J.-L. Goffin** | McGill University, Montreal, Canada |
| **D. Hearn** | University of Florida, Gainesville, FL, USA |
| **K.C. Kiwiel** | Systems Research Institute, Warsaw, Poland |
| **R. Mifflin** | Washington State University, Pullman, WA, USA |
| **W. Murray** | Stanford University, Palo Alto, CA, USA |
| **M.L. Overton** | Courant Institute, NY, USA |
| **M.J.D. Powell** | Cambridge University, UK |
| **A. Ruszczynski** | IIASA, Laxenburg, Austria |
| **A. Sartenaer** | FUNDP, Namur, Belgium |
| **R.B. Schnabel** | University of Colorado, Boulder, CO, USA |
| **H.D. Sherali** | Virginia Tech, Blacksburg, VA, USA |
| **Ph. Toint** | FUNDP, Namur, Belgium |

## *16th Symposium on Mathematical Programming with Data Perturbations*

May 26-27, 1994

This symposium is designed to bring together practitioners who use mathematical programming optimization models and deal with questions of sensitivity analysis, with researchers who are developing techniques applicable to these problems.

Contributed papers in mathematical programming are solicited in the following areas:

- Sensitivity and stability analysis results and their applications;
- Solution methods for problems involving implicitly defined problem functions;
- Solution methods for problems involving deterministic or stochastic parameter changes; and
- Solution approximation techniques and error analysis.

*Clinical presentations that describe problems in sensitivity or stability analysis encountered in applications also are invited.*

*Abstracts of papers for presentation should be sent in triplicate to Professor Anthony V. Fiacco. Abstracts should provide a good technical summary of key results, avoid the use of mathematical symbols and references, not exceed 500 words and include a title and the name and full mailing address of each author. The deadline for submitting abstracts is March 15, 1994.*

*Approximately 30 minutes will be allocated for presenting each paper.*

**Anthony V. Fiacco, organizer**

Sponsored by the Department of Operations Research and the Institute for Management Science and Engineering, School of Engineering and Applied Science, The George Washington University, Washington, D.C. 20052. Tel. (202) 994-7511

# *Call for Nominations*

# 1993 Lanchester PRIZE

EVERY year since 1954 the Council of the Operations Research Society of America has offered the Lanchester Prize for the best published contributions in operations research in the English language. For 1993, the prize is $5,000 and a commemorative medallion.

Books and papers for the 1993 prize will be screened by a committee appointed by the Council of the Society. To be eligible for consideration, the book or paper must be nominated to the Committee. Anyone can make nominations.

To be eligible for the Lanchester Prize, a book, paper, or group of books or papers must meet the following requirements:

- It must have been on an operations research subject;
- It must have been published in 1993; or two years prior to 1993, or, in the case of a group, at least one member of a group must have been published during that time period;
- It must have been written in the English language; and
- It must have appeared in the open literature. Books or papers may be case histories, reports of research representing new results, or primarily exposition. For a nominated set (group of either articles or books) published over more than one year, it is expected that each element in the set is part of one continuous effort, such as a multiyear project or a continuously written, multivolume book.

The Committee will use the following criteria in making judgments:

- The extent to which the contribution advances the state of the art of operations research;
- The originality of the ideas or methods;
- The new areas of application it opens up;
- The degree to which existing theory or method is unified or simplified; and
- The clarity and excellence of the exposition.

Nominations may be in any form, but must include the titles of paper(s) or book(s), author(s), place and date of publication, and six copies of the material. Supporting statements bearing on the worth of the publication, in terms of the five criteria, will be helpful, but are not required. Each nomination will be carefully reviewed by the Committee.

Nominations must be received by March 30, 1994, to allow adequate time for review.

The decision by the Committee and the ORSA Council will be announced, and any prizes approved will be awarded, at the National Meeting of the Society, Oct. 23-26, 1994, in Detroit, MI.

***Nominations should be sent to:***

John J. Bartholdi III, Chairman
Lanchester Prize Committee
School of Industrial and Systems Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0205

# *Contents of Mathematical Programming* JOURNALS

## Vol. 62 No. 3



## Vol. 63 No. 1


# *New Trends in Discrete and Computational Geometry*

J. Pach, ed.
Algorithms and Combinatorics 10
Springer-Verlag, Berlin, 1993
ISBN 3-540-55713-X

Computational geometry has come a long way. When it emerged as a discipline in its own right in the early '80s, it looked like a branch of computer science. It dealt with problems that, for example, arise in motion planning and robotics, in image processing and computer graphics, but also in dealing with multidimensional data structures. To start with, there was not that much geometry in it: the main point was an efficient handling of data structures, to get good (theoretical) bounds on algorithms which did not use too much knowledge about the geometry of the objects under consideration.

This has changed. Already, the first and basic books that have appeared about the subject observed that a lot of "classical" discrete geometry is useful for the analysis, but even more so for constructing geometric algorithms. At the time Goodman and Pollack started their journal, "Discrete & Computational Geometry," this may have looked like bringing two foreign (or at least alien) fields together. The fields of discrete geometry and computational geometry have merged, for the betterment of both. The volume under review, which borrows its title from the journal, is proof of this.

In 12 loosely linked chapters, the volume gives a "tour d'horizon" of what the field looks like today and it makes for a lot of interesting reading. The topics range from geometrical problems and topological tools for analyzing them, to methods for the construction and analysis of deterministic and randomized algorithms. Below the surface, however, the strong link between the combinatorial and algorithmic points of view can be felt throughout. Instead of detailed discussions — many of the chapters certainly could be worth an extended, separate review — here is a list of the authors and chapter headings.

I. L. Guibas & M. Sharir: Combinatorics and Algorithms of Arrangements

II. R. Seidel: Backwards Analysis of Randomized Geometric Algorithms

III. J. Matoušek: Epsilon Nets and Computational Geometry

IV. L. Khachiyan: Complexity of Polytope Volume Computation

V. J.E. Goodman & R. Pollack: Allowable Sequences and Order Types in Discrete and Computational Geometry

VI. N.M. Korneenko & H. Martini: Hyperplane Approximation and Related Topics

VII. J.E. Goodman, R. Pollack & R. Wenger: Geometric Transversal Theory

VIII. K. Bezdek: Hadwiger-Levi's Covering Problem Revisited

IX. I. Bárány: Geometric and Combinatorial Applications of Borsuk's Theorem

X. G. Fejes Tóth & W. Kuperberg: Recent Results in the Theory of Packing and Covering

XI. W. Moser & J. Pach: Recent Developments in Combinatorial Geometry

XII. P. Komjáth: Set Theoretic Constructions in Euclidean Spaces

Although they are not exceptional in their modesty, I mostly agree with the publisher's notes on the back of the book:

> "The most important new developments in discrete and computational geometry are summarized in this book. ... The chapters are self-contained surveys written by leading experts in the field. The book will be used by researchers, graduate students, and engineers interested in applications."

Of course, most of the material in this book is **not** directly useful for the "engineer interested in applications." However, it does make a lot of inspiring reading, and it is related more closely than one might think at first glance to the problems and algorithms that actually "work" in some of the software for optimization, graphics and robotics.

*In 12 loosely linked chapters, the volume gives a "tour d'horizon" of what the field looks like today and it makes for a lot of interesting reading.*

**References**

[1] H. Edelsbrunner: *Algorithms in Computational Geometry,* Springer-Verlag, Berlin Heidelberg, 1987.

[2] J.E. Goodman & R. Pollack, eds.: Discrete & Computational Geometry, Springer-Verlag, New York, Vol. 1 (1986).

[3] K. Mehlhorn: *Data structures and algorithms 3: Multi-dimensional searching and computational geometry,* Springer-Verlag, Berlin Heidelberg, 1984.

[4] F.P. Preparata & I.M. Shamos: *Computational Geometry: An Introduction,* Springer-Verlag, New York, 1985.

—GÜNTER M. ZIEGLER

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## *Computer Algorithms for Solving Linear Algebraic Equations: The State of the Art*

NATO ASI Series
Series F: Computer and Systems Sciences, Vol. 77
Emilio Spedicato, ed.
Springer-Verlag, Berlin, 1991
ISBN 3-540-54187-X

This book contains a selection of the contributions presented by some of the world's leading authorities at the NATO Advanced Study Institute meeting on computer algorithms for solving linear algebraic equations. The meeting was held Sept. 9-21, 1990, at Il Ciocco, Barga, Italy. Of the 14 con-

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tributions in the book, some contain sequential algorithms, some contain both sequential and multiprocessor algorithms, and some are devoted solely to the implementation of algorithms on multiprocessor architectures.

*Of the 14 contributions in the book, some contain sequential algorithms, some contain both sequential and multiprocessor algorithms, and some are devoted solely to the implementation of algorithms on multiprocessor architectures.*

In the first paper, by C.G. Broyden, direct methods for the solution of the system of linear algebraic equations $Ax=b$, where $A \in R^{n\times n}$ is non-singular and $b \in R^n$ is given, are described in a rather novel way. After considering Gauss elimination, pivoting and $LU$ decomposition, the methods of Choleski for symmetric positive definite $A$, of Aasen for symmetric indefinite $A$, of Levinson for Toeplitz $A$, and of Björck and Pereyra for Vandermonde $A$ are described. The last part of Broyden's paper is devoted to conjugate direction methods, especially appropriate for large sparse positive definite $A$.

In the second paper, by V. Pan, the complexity of algorithms for the solution of systems of linear algebraic equations $Ax=b$ is considered in the cases for which $A$ is dense and unstructured, sparse, and dense but structured. Attention also is given to iterative algorithms such as Jacobi, Gauss-Seidel, SOR, SSOR, etc. Multigrid methods, which are important in the solution of partial differential equations, also are considered.

The third paper, by A. Björck, presents a survey of the singular value decomposition and its use for analyzing and solving linear least squares problems, and two recent algorithms for numerically rank deficient problems, based on QR factorization, are presented. New backward stable methods, based on the modified Gram-Schmidt factorization, are given for linear least squares problems and underdetermined linear systems. Iterative methods for the factored normal equations, $A^T(b-Ax) = 0$, and preconditioners are surveyed, and parallel block preconditioners for least squares problems in which the matrices have block structure are developed.

In the fourth paper, by E. Spedicato, the main properties of the ABS class of algorithms for the solution of a general system of linear algebraic equations, $Ax=b$, with $A \in R^{m\times n}$, $b \in R^m$ and $m \leq n$, are surveyed. As explained in a recently published monograph titled, "ABS Projection Algorithms," by J. Abaffy and E. Spedicato (AS89), the majority of existing algorithms for the solution of systems of linear and nonlinear algebraic equations are embedded in the ABS class where they correspond to particular choices of the free parameters. In his paper, Spedicato states that if $m>n$, then there are several ABS approaches for solving $Ax=b$ in the least-squares sense without explicitly forming the normal equations. The paper also states that extensive numerical experiments have shown that several of the methods that have been investigated are compatible with LINPACK and with NAG codes in terms of accuracy on illconditioned rank deficient problems.

In the paper by J. Abaffy, the application of ABS algorithms to $Ax=b$, where $A \in R^{m\times n}$ is sparse, is considered. The implicit $LL^T$ algorithm is applied to matrices having nested dissection form. The implicit $LU$ and $QR$ algorithms and the Huang method are applied to various types of band matrices. Numerical evidence is given to illustrate the assertion that, in many cases, the ABS algorithms are advantageous, in terms of storage, compared to methods using standard factorizations.

The sixth paper, by W. Hackbusch, contains a comprehensive survey of multigrid methods, with examples and pseudocode algorithms.

In the seventh paper, by H. Yserentant, the hierarchical basis method [Y86] of Yserentant and the preconditioner of Xu [X89] and Bramble, Pasciak and Xu [BPX90] are described.

The eighth paper, by O. Axelsson, presents a relaxed incomplete $LU$ factorization method without pivoting for the solution of a linear system $Ax=b$, in which $A$ is a block $H$-matrix. A pseudocode version of the factorization is given.

The ninth paper, by G. Meurant, contains a review of some domain decomposition techniques for solving symmetric sparse linear systems. These techniques are used to construct efficient parallel preconditioners for the conjugate gradient method.

In the 10th paper, I. Galligani presents a description of the implementation, on a computer consisting of two vector processors, of the preconditioned simultaneous displacements method for solving large sparse systems of linear equations. The convergence properties of the method are treated under the assumption that the matrix of the system is symmetric positive definite or is an irreducible L-matrix with strong diagonal dominance. Nu-

merical results for a test matrix arising from the discretization of the diffusion problem on a rectangular domain are given.

In the 11th paper, by M. Cosnard, a review of some of the results obtained in the last decade on the design and analysis of parallel factorization algorithms for solving dense linear systems is presented.

In the 12th paper, by Y. Robert, the implementation, on distributed memory architectures, such as systolic arrays and general-purpose hypercubes, of linear algebra algorithms is discussed.

In the 13th paper, by I.S. Duff, several algorithms for the solution of sparse linear systems on parallel architectures is discussed.

In the final paper, by L.C.W. Dixon, the task of forming and solving the sets of linear equations that arise from nonlinear problems in optimization, in ordinary differential equations and in partial differential equations, is considered. It is shown that in each case the formation of the linear system can be handled efficiently by automatic differentiation. Results are given to show that this can be very efficiently performed on a parallel computer. Regarding the solution of the set of equations, it is shown that, in general, the truncated Newton method out-performs more traditional optimization codes and that when the optimization problem arises in a two-dimensional finite element context, the SIMD/DAP processor is very effective. It is shown that it is difficult to obtain effective speed-up for large sparse systems on small MIMD machines of the Sequent or transputer network type. It is shown for large sparse optimization that the use of accurate dot products can greatly reduce the number of iterations when using a conjugate gradient algorithm.

**References**

[AS] J. Abaffy & E. Spedicato: *ABS Projection Algorithms: Mathematical Techniques for Linear and Nonlinear Equations*, Ellis Horwood Ltd., Chichester, 1989.

[BPX90] J.H. Bramble, J.E. Pasciak & J. Xu: *Parallel Multilevel Preconditioners*, Math. Comp. 55 (1990), 1-22.

[X89] J. Xu: *Theory of Multilevel Methods*, Report No. AM48, Department of Mathematics, Pennsylvania State University, 1989.

[Y86] H. Yserentant: *On the Multi-level Splitting of Finite Element Spaces*, Numer. Math. 49 (1986), 379-412.

–BY M.A. WOLFE

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# *Dynamic Economic Models and Optimal Control*

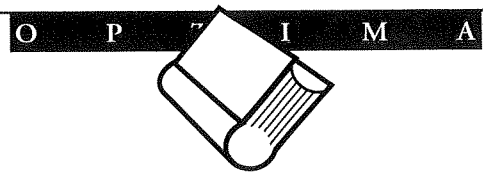
G. Feichtinger, ed.

North Holland, Amsterdam, 1992

ISBN 0-444-884G1-Z

This book is the fourth in a series reporting selected presentations at the Viennese Workshop on Dynamic Economic Models and Optimal Control. This book, as well as the occurrence of the workshop (held in 1981, 1984 and 1987), are remarkable achievements, both by the quality of the papers presented and by the concentration of the conference on the modeling and analysis of dynamic systems in economics and management science.

The current volume, reflecting an evolving interest in the study of nonlinear dynamic systems has several new and special features. For example, papers in chaos and chaotic economic systems provide an important linkage between the classical approach of control and the new ideas arising from bifurcation theory and chaotic oscillations (11 contributions). There also are important collections of papers on the application of differential game and optimal control theory to economics (11 and 20 contributions, respectively). The book is focused on deterministic systems and, conspicuously, no contributions in stochastic control theory and applications are included. This is an advantage, however, as it




focuses on other aspects of uncertainty, arising from gaming situations and nonlinear dynamics.

The book has numerous contributions. Some examples include the paper by Léonard, "On the ubiquity of trade in capital goods: Jumps in the sate variables," which generalizes well-known results by Arrow and Kurz. Kemp et al. present a paper on a dynamic formulation of the foreign aid process, a problem which has been dealt with previously in a static form. Feichtinger et al. discuss the limit cycles of resource-employment in a regulated fishery (based on Hopf bifurcation theory).

Tamar Basar's paper discusses the application of differential game theory in robust controller design for economic systems, an important contribution to $H^\infty$ control. Gradus and Kort discuss "Optimal taxation on profit and pollution within a macroeconomic framework," providing an insightful analysis to a timely problem — taxation and pollution. Jorgensen's paper is on the "Dynamics of extramarital affairs," providing a stimulating presentation of a problem that is a permanent fixture of our social makeup.

Papers by Nishimura on "Factor intensity and Hopf bifurcations," Hommes on "Periodic, Quasi-periodic and Chaotic dynamics in a simple macro model with Hicksian nonlinearities," Lorenz on "Multiple attractors, complex basin boundaries, and transient motion in deterministic systems," Fienkenstadt and Kuhbier on "Principle component analysis and overall sphericity: An application of chaotic time series" provide further motivation to the stability analysis and a growing interest in chaotic economic dynamics.

Overall, this is a useful reference text for researchers and graduate students alike in the field of modeling, and the analysis of dynamic systems and their applications in economics.

–CHARLES TAPIERO

*Overall, this is a useful reference text for researchers and graduate students alike in the field of modeling, and the analysis of dynamic systems and their applications in economics.*

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## *Optimality and Equilibria in Stochastic Games*

by F. Thuijsman

Centre for Mathematics and Computer Science, Amsterdam, 1992

ISBN 90-6196-406-7

This monograph, which differs from the Ph.D. thesis of the author only in some minor points, deals with two-person, non-terminating stochastic games with finite state and action spaces. The introductory Chapter 1 discusses formal definitions of the stochastic game model (zero-sum versus general-sum) with its solution concepts (value and optimal strategies versus equilibria), as well as the major historic results in the field. Generally speaking, both the zero-sum and the general-sum case can be considered for three evaluation criteria of incomes: the $\beta$-discontinued reward criterion, the limiting average reward criterion and the total reward criterion. Fine solutions, in terms of stationary strategies (i.e. strategies for which the action choices of the players only depend on the state and not the history), are known to exist with respect to the $\beta$-discontinued reward criterion, whereas, with respect to the limiting average reward criterion, similar problems as for the total reward criterion occur.

In chapters 2, 3 and 4, the emphasis is on stochastic games, with respect to the limiting average reward criterion (mainly because the existence of limiting average $\varepsilon$-equilibria can be seen as the major open problem in stochastic game theory nowadays). Involving the zero-sum case, Chapter 2 provides an alternative and straightforward proof for the known result that for each player there exists easy initial states, i.e. starting states for which this player has a stationary limiting average optimal strategy. In the same setting, with respect to some other set of initial states (with maximal or minimal limiting average value), a sufficient condition is presented for each player to have stationary limiting average $\varepsilon$-optimal strategies. For the general-sum case, Chapter 2 also provides a similar result in that there always is a non-empty set of initial states for which an "almost-satisfactory" limiting average $\varepsilon$-equilibrium exists, i.e. stationary strategies which are amplified with some threat to prevent profitable

deviations of the opponent. So, if both players stick to their $\varepsilon$-equilibrium strategies, then with probability close to 1, they will use stationary strategies throughout the whole game.

In Chapter 3, the general-sum result of Chapter 2 is extended by formulating sufficient conditions for the existence of an "almost-stationary" limiting average $\varepsilon$-equilibrium (for all starting states). The relevant conditions are formulated in terms of asymptotic properties of sequences of stationary $\beta$-discounted equilibria. It is not clear whether these sufficient conditions hold for any general-sum stochastic game.

In Chapter 4, it is established that these sufficient conditions are automatically fulfilled for three special classes of stochastic games that have been examined in literature: unichain stochastic games (satisfying the property that, for any pair of stationary strategies, there is just one irreducible set of states); stochastic games with state-independent transitions (SIT); and repeated games with absorbing states.

Chapter 5 focuses on the total reward criterion for zero-sum stochastic games. Because the total value may fail to exist in general, the emphasis is on games satisfying the condition of limiting average value 0 for all initial states and both players possessing stationary limiting average optimal strategies. For such a game, the total reward criterion can be seen as a refinement of the limiting average reward criterion, since a total ($\varepsilon$-)optimal strategy is necessarily limiting average optimal. By means of an example, it is illustrated that, even with the above property, history-dependent behavior strategies are indispensable for the player to achieve total $\varepsilon$-optimality. The existence of stationary total optimal strategies for stochastic games with the above property is characterized.

Chapter 6 is devoted to mathematical programs connected to stochastic games. In the framework of all three evaluation criteria, nonlinear programs are presented that completely characterize the existence of stationary equilibria or ($\varepsilon$-)optimal strategies.

–THEO DRIESSER

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