

# Mathematical Programming and the Web: *To surf or not to surf*

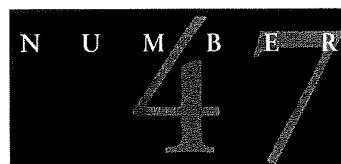
## HEAR NO EVIL, SEE NO EVIL

*Advancements in computer technology have influenced every aspect of our lives: the way we work, the way we communicate, the way we think, and the way we spend our leisure time. Those of us who tried to keep our involvement with computers to the bare minimum could not stop the Internet invasion of our offices and homes. This computer revolution went the extra step when the World Wide Web arrived on the net: One can access most of the Internet's wealth of information by simply clicking on a highlighted text. But, where does mathematical programming fit into all of this? We, as mathematical programming practitioners, can benefit from the Internet and its resources. Rather than watching the boat sail away, we can jump on it and enjoy a nice ride. Oops, let me re-phrase what I said: Rather than watching the board surf away, let's jump on it and start surfing the net.*

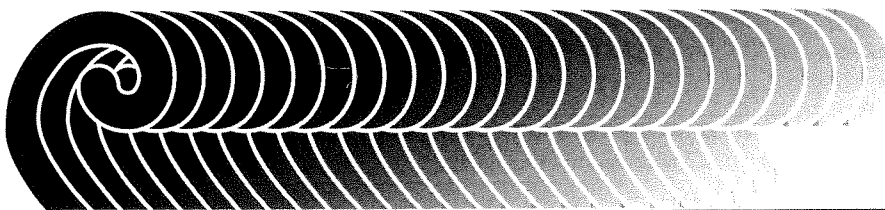
Well, I am a mathematician, can you start with the definitions?

The Internet is the largest computer network in the world. Regional and local computer networks are connected with each other to form this giant computer network. The U.S. Department of Defense started the effort, as usual, by building ARPAnet about 25 years ago. At the same time, many companies, such as IBM, started building their own local networks. The National Science Foundation then built its own network that linked the universities in the U.S. with each other. In 1987, Merit Network Inc., which ran the University of Michigan's network, took over the role of upgrading the NSFnet. I guess you know the rest of the story: Better connections were built, many other networks joined, and we ended up with the Internet.

The World Wide Web is the latest and most sophisticated service on the Internet. Given that the Internet is a big computer network, the amount of information that can be stored on its computers is massive. Many of the Internet users make their ideas, thoughts, research, and computer codes available for others by storing them electronically in designated locations on the net. This information can be retrieved using special computer programs which are called browsers in Web terminology. The two most common Web browsers are Mosaic and Netscape. Both are available at no cost (at this time) on all computer platforms. The World Wide Web and its browsers seem to be the easiest and the most flexible and fun tools on the Internet at present. They are expected to become the predominant method for surfing the net in the next few years. PAGE TWO ►



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## the Web

## C O N T I N U E D

## O P T I M A

## So, what am I going to gain from this Web thing?

This is a tough question to answer precisely. When electronic mail started, many people thought of it as a wasteful tool. Nowadays, most of us use electronic mail to correspond with colleagues and friends, and no one can ignore its usefulness and importance. The same story holds true for the Web. We are delaying our exposure to it by making different excuses, such as "I do not have time for this" or "I am a theoretician and do not have to deal with it." However, the real reason for avoiding this tool as well as many other computer tools is our lack of knowledge and understanding of computers and our failure to put a few hours into learning the new technology.

As someone involved in mathematical programming, this is the least that you can expect to get from the Internet and its Web:

- **RESEARCH.** The Web permits the creation of bulletins that are accessible by any person connected to the Internet. These bulletins can be updated easily and frequently. The moment an update has taken place, the readers of a bulletin are able to access the new version immediately.

Hmm, maybe I am not making myself quite clear here; an example may help to explain my point. Let us say that you are interested in interior point methods for solving linear programs. There is a bulletin, or a page, on the Web that contains all recent papers and reports regarding the subject (<http://www.mcs.anl.gov/home/otc/InteriorPoint/>). You can access this page by giving its electronic address to your Web browser. Remember, these are articles that are dropped there by their authors without going through the lengthy refereeing process. You can save any of these articles on your computer disk or print it out. In return, you can leave an electronic copy of your documents on the Web. Interested people can read and print these documents and give their feedback. There isn't any need to send hard copies by mail any more, to wait for somebody to respond, and to beg for help from secretaries.

- **TEACHING.** This is an area from which I can talk from personal experience (<http://www.engin.umich.edu/dept/ioe/ioe-474/>). I recently taught a simulation class and thought about placing all the class material on the Web. It seemed a little strange in the beginning: class notes on the Web, homework assignments and their solutions on the Web, illustrative figures on the Web, and even grades on the Web. After a couple of weeks, the students realized the convenience of having the Web. They were able to access everything related to the class from any computer connected to the network. Nobody wanted an extra copy of a handout or did not do the

homework because he/she was not in the class. The material was out there in an organized convenient fashion.

- **PUBLICATIONS.** Many professional societies are moving towards electronic publishing <http://ejc.math.gatech.edu:8080/Journal/ejc-wce.html>

An author submits his/her article in an electronic form through the Web. The article is sent electronically to the referees, the corrections and comments are sent back to the author, and finally the article appears on line. It seems like a dream, but Hey, welcome to Cyberspace!

- **OTHER ISSUES.** Believe me, it is a completely different world out there. The amount of information available is unbelievable. When I get into my office in the morning, I read the newspaper, check the stock prices, and look at the weather forecast on the Web. One can even order a pizza on the Web in some places. Millions of bytes are devoted to any subject you can think of; they are waiting for you to click on their links and activate them.

## When can I start surfing the Net? (without getting wet)

As a computer user, you can see that computer interfaces are friendlier than ever. The Web browsers are no exception: They are very simple to use.

- Start your browser (Mosaic or Netscape) by double clicking on it. If you cannot find it on your machine, ask your system administrator to guide you to its location on your system. In a few seconds, a new window appears on your screen.
- Go to the file menu and select **Open Location...** or **Open URL....** In the new box, type in the following

<http://www.cs.rice.edu/~packy/mps/>

then hit return. After a few seconds, the contents of the window changes. Guess where you are!

- The underlined text is a link to another document that contains more information about this text. Click on the text and see what you get.
- There are many buttons and menu items, such as **Back** and **Forward**, that can make your ride smoother. If you get lost on the Web, do not worry, just click on the **Home** button. When you find something you like, you can add it to your list of interests by selecting **Add a Bookmark** or **Add to Hotlist**. Later, it can easily be retrieved using **View Bookmarks** or **Hotlist....**

Here we go, you are an expert surfer after this small lesson. Remember, surfing the Internet is Markovian: It does not matter how long you have been using the net. We are all experts.

Here are some pointers to Web pages that are related to Mathematical Programming:

- Open

<http://e-math.ams.org/>

to access the electronic system of the American Mathematical Society

- Open

<http://www.informs.org/>

to access the Institute of Operations Research and the Management Sciences home page

- Open

<http://www.siam.org/>

to access the page of the Society for Industrial and Applied Mathematics

- Open

<ftp://math.liu.se/pub/MPS/index.html>

to access the Linköping Mathematical Programming Library

- Open

<http://www.mcs.anl.gov/home/otc/Guide/OptWeb/>

to access the NEOS Guide Optimization Tree at Argonne National Laboratory

- Open

<http://mat.gsia.cmu.edu/>

to access Michael Trick's Operations Research Page

- Open

<http://moa.math.nat.tu-bs.de/opt-net/opt-net.html>

to access the Opt-Net Home Page

**Wow, this is getting interesting.**

**How can I spin my own Web?**

Building your own Web page is not hard at all. The Web is based on a technology called hypertext. Usually, you need to create a file, called **index.html**, that contains the information which you want to place on the Internet. Believe it or not, all that you need to know is out there on the Web itself. A small hint, whenever you see something that you like, select the menu item **View Source...** from your browser. A new window shows the commands used in creating the original page. I found this feature to be the best Hypertext tutor around. If you want to learn more, access the home page of the World Wide Web Initiative

<http://www.w3.org/>

It contains extensive documentation regarding Hypertext and other Web related protocols.

Finally, you are warned that the Internet and the Web can be addictive. Enjoy at your own risk!

-Samer Takriti

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# Report FROM THE CHAIR

JAN KAREL LENSTRA

The society was formed in the early 1970's, primarily to support a series of international symposia that started in 1949. The symposium that was held in Ann Arbor in August 1994 under the chairmanship of John Birge and Katta Murty was a memorable event. The 1997 symposium will take place in Lausanne; Tom Liebling and Dominique de Werra are the organizers. For the 2000 symposium, an advisory committee chaired by Bob Fourer welcomes proposals.

In addition to the general symposia, the society also sponsors meetings on more specific topics. The most prominent of these are the triennial conferences on stochastic programming and the IPCO conferences, on integer programming and combinatorial optimization, held in every non-symposium year. A stochastic programming meeting was held last June in Nahariya, Israel; the next one is scheduled for 1998, possibly in Vancouver. The fourth IPCO meeting took place in Copenhagen at the end of May; the fifth one will be held in Vancouver in June 1996.

Our journal, *Mathematical Programming*, is the main publication outlet in the area of optimization. In August 1994, the editors of Series A and Series B, Bob Bixby and Bill Pulleyblank, resigned after many years of distinguished service. They broadened the scope of the journal, which now contains, in addition to theory, more material on computation and implementation. Bob also managed to decrease the backlog of Series A quite substantially. His successor is Don Goldfarb. Bill Pulleyblank, who started Series B and made it a successful series of special issues, was succeeded by John Birge. In the coming years, the journal will have four volumes of three issues per year, including at most three issues of Series B.

Then, as you will have noticed, OPTIMA began its second youth. The editorial staff, still led by the founding editor, Don Hearn, was expanded. The newsletter itself expanded from three annual issues of twelve pages each to four issues of sixteen pages.

Our prize program requires few comments. The Fulkerson Prize, the Dantzig Prize, the Beale-Orchard Hays Prize, and the Tucker Prize are widely recognized awards. To facilitate the work of future prize committees, the council is considering clarifying the prize rules on a few points.

A membership committee, chaired by George Nemhauser, has been asked to advise the council on issues regarding the recruitment of new members, the relation between the society and its regional and technical sections, and special membership arrangements. The society has an arrangement with its Hungarian members, which, after 24 years, may require reconsideration. There is one regional section, the Nordic Section, which achieves an admirable activity level, and there could be more. There also is one technical section, the Committee on Stochastic Programming, which, among other things, organizes the main conferences in the area. The society's first technical section, the Committee on Algorithms, declared its mission achieved and voted to disband in August 1994. The membership committee will submit its report shortly, and you will be able to read more about it in one of the future issues of OPTIMA.

The most recent activity is the establishment of an MPS page on the WorldWide Web. As most of you probably know better than I do, this provides virtually unlimited possibilities. Paying annual dues, signing up for symposia, and consulting the membership list are some of the first, easier options. At a later stage, the table of contents of the journal, the text of OPTIMA, an updated version of Phil Wolfe's history of the society, and our constitution and bylaws can be made accessible. Finally, the journal is likely to be available on-line, which will lead us into the era of electronic publishing. But again, this refers to the future rather than the past.

The society has fewer than 1,000 members, who are evidently more interested in mathematical programming (or optimization, which is a much better term) than in administrative work. The organizational overhead is light. For such a small and quiet group, it is remarkable that it has achieved a truly international character, with an active and high-level program of meetings, publications, prizes, and regional and technical sections.

While your primary business should stay in optimization, I want to encourage you to take part in these activities. The society is in the position to support initiatives and to provide leverage. It can help to start up meetings and sections of a regional or technical nature. And OPTIMA needs your contributions. Our newsletter is a volunteer effort, which can exist only on the basis of feature articles, news items and book reviews written by individual members.

Some words about our discipline. We often hear that much of the research in optimization that is being done in academia is baroque and of no relevance to the outside world. Our work does have its frivolous aspects. They give a certain charm to it. And research that is driven by direct practical needs only and not by academic curiosity is less likely to be innovative. But optimization as a whole is not baroque at all. It is just reaching its maturity. It is ideally positioned in between mathematics, computing, and practice. Modeling insights and computational methods developed during the last half century have now come together with new computer architectures and programming techniques to enable us to solve truly large and truly hard real-world problems. Optimization is very much alive, and it is alive in its full breadth.

I want to thank all of my friends and colleagues with whom I have had the privilege to work. In particular, I express my gratitude to Michel Balinski and George Nemhauser, past chairmen of the society, to Les Trotter, our treasurer during two terms, and to Rolf Möhring and Steve Robinson, who chaired the Executive and Publications Committees. I hope that our successors will enjoy themselves as much as I did.

—Jan Karel Lenstra  
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# Optimal Design of Engineering Structures

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**S**tructural design is an engineering discipline aimed at creating constructions (bridges, cantilevers, inner skeleton of airplane wing, etc.) capable of carrying external loads under different loading scenarios. For example, a bridge should withstand forces corresponding to rush hour morning traffic, rush hour evening traffic and perhaps an earthquake. The criteria for "good" design are either certain characteristics of rigidity, such as stiffness and stability of the construction, or cost related measures, such as total amount of material used, structure lifetime, or financial cost of the construction. In this article we focus on discrete structures, so-called *trusses*. A truss consists of a finite number of thin elastic *bars*, connected to each other at *nodes* (joints). Typical examples are transmission towers and cantilever arms, but the most widely known truss is doubtlessly the Eiffel Tower. When designing a truss, an engineer bears in mind a set of *active nodes* where the external forces are applied in the two-dimensional (2D) plane or in three-dimensional (3D) space, a set of *loading scenarios*, where each scenario reflects a particular distribution of the external forces, and a set of *fixed nodes* (supports), such as the ground, a wall, or a ceiling at which the construction can be supported. The final design is given in terms of (1) the geometric location of the joints where bars are linked, (2) the topology of the interconnections between the nodes, and (3) the sizes (cross sections, areas, volumes) of the bars. *Layout optimization* is concerned with the simultaneous selection of all three design components: *geometry*, *topology* and *sizing*, and it constitutes one of the newest and most rapidly expanding fields of structural design; see the excellent review article [10]. Although mathematically and computationally the most challenging design task, layout optimization is economically the most rewarding one, as an efficient layout of a truss uses the given amount of material in an optimal way to create the most *rigid* structure. Such a structure can be significantly more stable than the layout obtained by ad hoc methods.

To see how rigidity is determined, let us look at what happens when a truss made of elastic material is put under a given load. External forces cause a certain deformation of the truss, which means that the free, i.e., the unsupported, nodes move slightly (called *node displacements*) until the tension caused by the elongations of the bars compensates for the external forces. As a result of the deformation, the truss stores certain potential energy called *compliance*. The compliance is thus a good global characteristic of the rigidity of the truss with respect to a given load, i.e., the smaller compliance, the more rigid is the structure, and serves as a reasonable objective function. In the *single-load* case, the compliance is minimized with respect to a unique loading scenario, and in the *multi-load* case, the compliance corresponding to the worst possible scenario, out of a given set of such nonsimultaneous loading scenarios, is minimized.

The approach to layout optimization as discussed in this article is based on creating a fine mesh of potential node locations, allowing all possible connections between all pairs of nodes. Thus, the geometry problem is circumvented by solving a large-scale *truss topology design* (TTD) *problem* which, fortunately, has surprisingly good mathematical and computational characteristics; see the review paper [2].

**Mathematical model.** The simplest formulation of a TTD problem is derived as follows. Given a node set consisting of  $N$  elements in the 2D plane or 3D space, one can naturally associate an  $n$ -dimensional space  $\mathbf{R}^n$  of virtual node displacements with it. Here,  $n$  is about  $2N$  for planar trusses and  $3N$  for spatial ones ("about" - since supported nodes are not free to move in all directions). Displacements of the nodes, and likewise the external loads, can be represented by vectors in this space. It is convenient to identify the truss with an  $m = \frac{1}{2}N(N-1)$  dimensional vector  $t$  of *bar volumes*, where the entries of this vector are indexed by the distinct pairs of nodes. The entry corresponding to the unordered pair  $(j,k)$  of nodes is equal to the volume of the bar linking nodes  $j$  and  $k$ . A zero entry means that the corresponding pair of nodes is not linked. The tension/compression caused by displacement  $x$  of the truss  $t$  is  $A(t)x$  for some  $n \times n$  matrix  $A(t)$ . Consequently, the displacement caused by external load  $f \in \mathbf{R}^n$  is determined by the equilibrium equation

$$A(t)x=f,$$

and the compliance of the truss under the load is

$$c=f^T x.$$

In the *linear elastic* model of the material the matrix  $A(t)$  is linear in  $t$ , i.e.,

$$A(t)=\sum_{i=1}^m t_i A_i,$$

where  $A_i$  is a positive semidefinite symmetric matrix, in fact, a rank-1 dyadic matrix

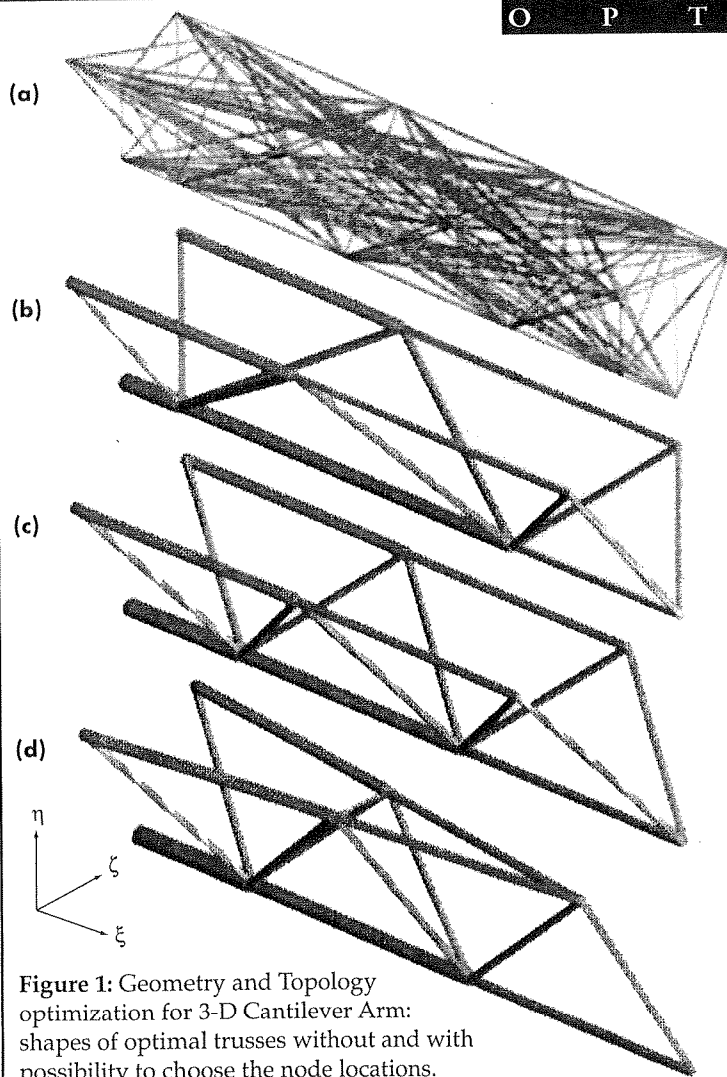
$$A_i=b_i b_i^T.$$

The vector  $b_i \in \mathbf{R}^n$  contains the sines and cosines of the direction of bar  $i$  and it also depends on some material characteristic, the so-called Young modulus. Thus, a typical setting of the TTD problem is

$$(\text{TTD}) \min_{f \in F} \max_x \left\{ f^T x \mid A(t)x=f, t \geq 0, \sum_{i=1}^m t_i = V \right\}$$

where  $F \subset \mathbf{R}^n$  is a set of loading scenarios, and  $V$  is the given total bar volume. The fact that the entries of  $t$  are allowed to take value zero takes care of both the topology and sizing aspects of the design. In the full layout optimization problem, the matrices  $A_i$  depend on the geometric positions of the nodes, and those are part of the design variables.

## O P T I M A



**Figure 1:** Geometry and Topology optimization for 3-D Cantilever Arm: shapes of optimal trusses without and with possibility to choose the node locations.

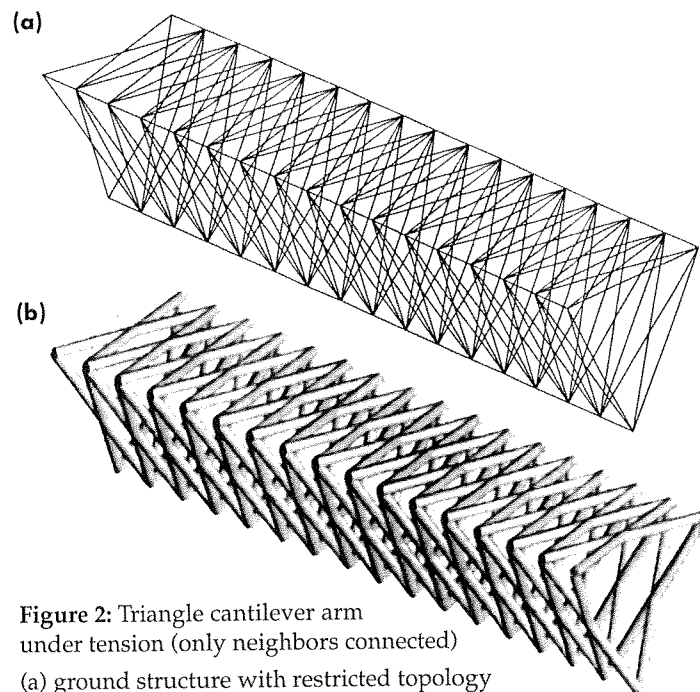
(a) ground structure; 120 potential bars

(b) fixed node set; optimal truss has 17 bars and compliance 8.944

(c) nodes can move in  $x$ -coordinate direction; optimal truss has 17 bars and compliance 6.563

(d) nodes can move in  $x$  and  $y$  coordinate directions; optimal truss has 18 bars and compliance 6.326

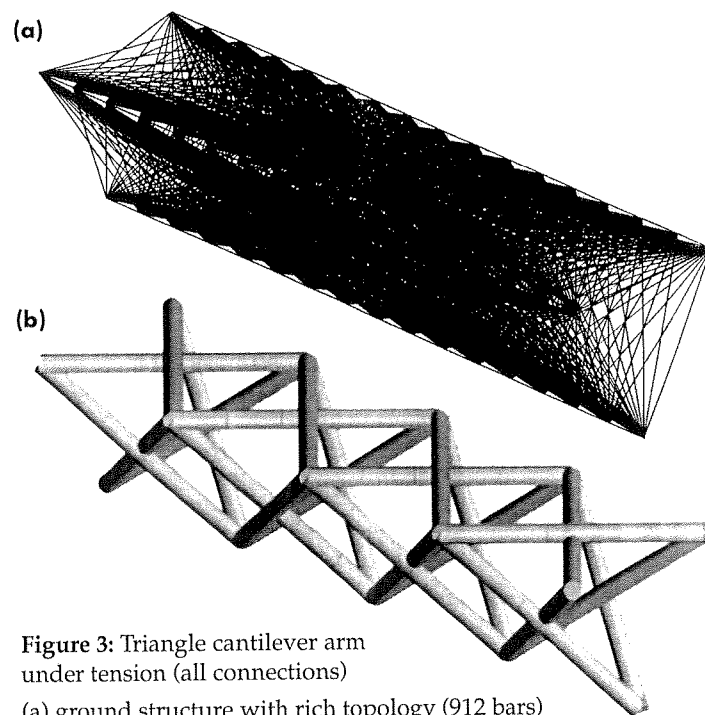
Both the variables corresponding to the bar volumes and the positions of the nodes are very important. In particular, even relatively small variations of the node set may result in significantly different shapes of the optimal truss; see **Figure 1**. Unfortunately, even for fixed cardinality of the node set, the problem is nonconvex in the positions of the nodes and may require techniques of nonsmooth optimization. This approach is developed and used by the group of Professor J. Zowe in Bayreuth (see [1,4]) and is illustrated in **Figures 1 - 3** which are courtesy of Achtziger, Kocvara, and Zowe. Here, in order to get a computationally tractable problem, we are forced to fix the node set and treat  $t$  as the only design vector. This approach is not as limited as it may sound. In fact, it allows us to capture the geometry part of the design as well. By choosing a fine 2D/3D grid as the node set, and allowing all possible links between the nodes (this initial choice is called ground structure), we can approximate the true layout optimization problem with arbitrarily high accuracy. It can be proven that the number of nonzero bar volumes in an optimal solution of TTD does not



**Figure 2:** Triangle cantilever arm under tension (only neighbors connected)

(a) ground structure with restricted topology (only neighboring nodes are connected-192 bars)

(b) optimal truss (96 bars), compliance = 16628.9



**Figure 3:** Triangle cantilever arm under tension (all connections)

(a) ground structure with rich topology (912 bars)

(b) optimal truss (24 bars), compliance = 3685.4 [22 percent of the compliance for the optimal truss in Figure 2(b)]

exceed a certain value, which depends solely on the number of active nodes and the cardinality of the set  $F$  of loading scenarios, and not on the cardinality of the ground structure. Hence, an optimal solution to TTD selects appropriate nodes and links "automatically" from the structure, and solves, in principle, the geometry, topology and sizing problems simultaneously.

THE bad news is that in order to capture all components of the design, we inevitably have to deal with large-scale TTD problems. Indeed, even in the planar case, to approximate the actual, continuous node universe within accuracy  $h$ , we should deal with a node set of cardinality  $N=O(1/h^2)$ . Moreover, in order not to impose *a priori* restrictions on the topology, we should allow basically all possible links between the nodes so that the design dimension of TTD will be  $m=O(N^2)=O(h^{-4})$ . See Figures 2 and 3 for the effect of using restricted versus rich topology. In the spatial case, the situation is even worse:  $m=O(h^{-6})$ . To get an impression of the sizes of realistic instances of TTD problems, note that designing a simple planar console with a quite moderate ground structure of  $15 \times 15 = 225$  nodes results in TTD problem with  $m=15,556$  bars. This is by order of magnitudes greater than the number (around 500) of bars in the Eiffel Tower! Well, we should pay somehow for the fact that we are not as ingenious as Eiffel...

**Solving the truss topology design problem.** The TTD problem is the main subject of the research carried out during the last five years in our Lab—the Optimization Laboratory of the Faculty of Industrial Engineering and Management at Technion, Israel. What attracted our attention was its challenging large-scale character combined with its convexity properties and rich mathematical structure. Moreover, it is indeed rewarding to deal with a large-scale problem and yet to have the possibility to *see*, in the direct meaning of the word, the solution! We started by processing the problem mathematically, which by itself was an exciting adventure. The goal was to find an equivalent reformulation having smaller design dimension, see [1]. What enabled us to achieve this goal was extensive use of *duality*. It turned out that there are two “computationally tractable” settings of TTD:

- The one where the set of loading scenarios is finite:  $\{F=f_1, \dots, f_k\}$ . In this case, the dual of TTD is equivalent to the following minmax quadratic-fractional problem:

$$(QF) \quad \min_{x_1, \dots, x_k \in \mathbf{R}^n, \lambda \in \mathbf{R}^k} \left\{ 2 \sum_{j=1}^k f_j^T x_j + V \max_{i=1, \dots, m} \sum_{j=1}^k \frac{x_j^T A_i x_j}{\lambda_j} \mid \lambda \geq 0, \sum_{j=1}^k \lambda_j = 1 \right\};$$

In the *single-load* case ( $k=1$ ) the  $\lambda$ -variables disappear, and the problem becomes simply a minmax problem with  $m$  convex quadratic forms  $\frac{1}{2} x^T A_i x + f_i^T x$  of  $n$  variables  $x$ , see [3].

- The one where  $F$  is a  $k$ -dimensional ellipsoid in  $\mathbf{R}^n$  given as the image of the unit ball in  $\mathbf{R}^k$  under the linear mapping  $u \mapsto Qu$ , where  $Q$  is an  $n \times k$  matrix. In this case, using Fenchel-Rockafellar duality, the problem is converted into the following *semidefinite program*

$$(SD) \quad \min_{\Lambda, X} \left\{ 2 \text{Trace}(QX^T) + V \max_{i=1, \dots, m} \text{Trace}(X \Lambda^{-1} X^T A_i) \mid \Lambda \geq 0, \text{Trace}(\Lambda) = 1 \right\},$$

where  $\Lambda$  is  $k \times k$  symmetric matrix,  $X$  is  $n \times k$  matrix, and the constraint  $\Lambda \geq 0$  reads “ $\Lambda$  is positive semidefinite.” Problem SD is indeed a semidefinite program: it can be rewritten equivalently as

$$\min \left\{ \text{Trace}(QX^T) + \frac{V}{2} p \mid \begin{pmatrix} \Lambda & X^T b_i \\ b_i^T X & \rho \end{pmatrix} \geq 0, i = 1, \dots, m, \text{Trace}(\Lambda) = 1 \right\},$$

where  $b_i$  are the vectors involved in the dyadic representation  $A_i = b_i b_i^T$  of the matrices  $A_i$ .

Note that both QF and SD are convex programming problems, in contrast to the original formulation TTD, which is not convex jointly in its variables  $(x, t)$ .

A second major advantage of the dual reformulations is a dramatic reduction of the design dimension,  $kn + k - 1$  for QF and  $kn + \frac{k(k+1)}{2} - 1$  for SD, instead of  $m=O(n^2)$  for TTD. Note that  $k$  is usually small, and  $n$  is in the order of hundreds. The huge original design dimension, of course, does not disappear completely: now  $m$  becomes the number of minmax components in the dual or, which is basically the same, the number of smooth constraints in the inequality constrained reformulation of the dual. Nevertheless, the “swapping of sizes” that we get when passing from the original problem to the dual one is very promising from the computational viewpoint, since a majority of the available optimization methods are much more sensitive to the design dimension of the problem than to the number of constraints.

The minmax problem QF was something we could try to solve numerically, and we started with attempts to solve its single-load version using available software. It turned out, however, that the traditional methods like bundle, augmented Lagrangeans, and SQP, are quite inefficient in that they were unable to handle “small” TTD problems with tens of nodes and hundreds of bars. What actually worked were interior point methods, and we strongly believe that these methods constitute, if not the only, then definitely the most appropriate tool for this kind of application. First of all, these methods are theoretically efficient. With properly chosen log-type penalties for the constraints, we get a *polynomial time* complexity result (see [5,6]) as follows: to solve QF with accuracy  $\epsilon$  in the objective, it suffices to perform  $O(\sqrt{m+k} \ln(1/\epsilon))$  Newton-type iterations having arithmetic cost  $O(mn^2 k^2)$  each. When evaluating this latter quantity, one should take into account the nice structure of the TTD data: due to its origin,  $A_i = b_i b_i^T$  with at most 4 (planar case) or 6 (spatial case) nonzero entries in  $b_i$ . The actual behavior of the polynomial time interior point method was even better than could be predicted by this complexity result; e.g., a single-load QF problem coming from the ground structure with 225 nodes and 15,556 bars, was solved in 138 Newton steps [5].

Although promising, the polynomial time interior point method we used was far from being the most efficient. The number of Newton steps turned out to be sensitive to the number of loads and sometimes the computations lasted more than 500 hours. Therefore, we definitely needed something more efficient. Intensive research yielded two essential modifications of the standard interior point scheme — the *penalty/barrier multiplier* (PBM) method [8] and the *truncated log-barrier* (TLB) method [7]. As one can see from the table below, both these methods solve single- and multi-load QF problems to high accuracy, i.e., 8-12 digits, in 30-50 Newton steps. The number of Newton steps needed is basically independent of the problem size.

Variables ( $n$ )	Constraints ( $m$ )	Newton steps, PBM	Newton steps, TLB
98	150	12	22
126	1234	22	29
242	4492	23	33
342	8958	22	30
450	15556	41	40
656	32679	47	42

## O P T I M A

Both PBM and TLB turned out to be very promising for general-type convex optimization problems, not only for TTD. Now it is time to present several nice pictures of trusses.

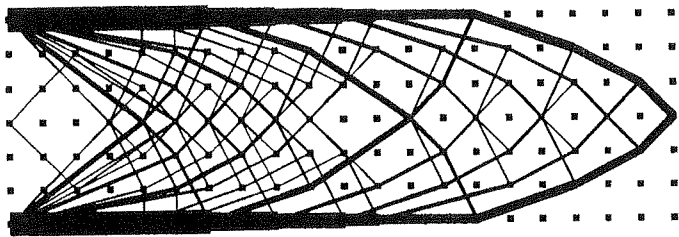


Figure 4: Optimal truss with  $21 \times 7$  nodes. The compliance is 121.059. The problem has 294 variables and 6574 constraints.

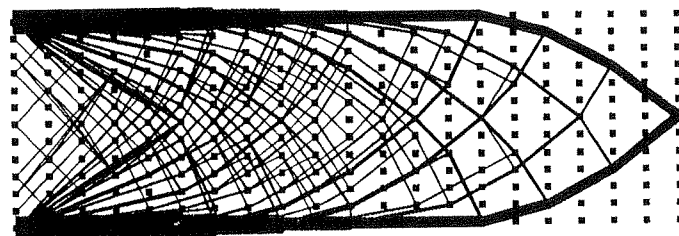


Figure 5: Optimal truss with  $21 \times 13$  nodes. The compliance is 120.264. The problem has 546 variables and 22764 constraints.

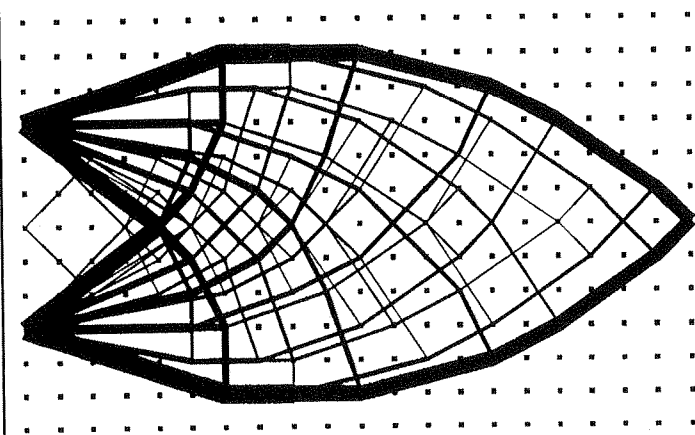


Figure 6: Optimal "big" truss with  $21 \times 13$  nodes. The compliance is 106.145. The problem has 546 variables and 22764 constraints.

Figures 4 - 6 illustrate the importance of a "rich" ground structure. They deal with a single-load QF problem where the console to be designed should transmit a single vertical force acting downward at the middle node of the extreme right column to a vertical line of the very left column, all of whose nodes are supported. We see that enrichment of the ground structure makes the design close to the solution of the corresponding continuous problem (a Mitchell truss).

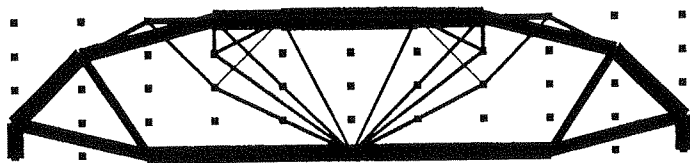


Figure 7: Optimal bridge with  $11 \times 5$  nodes. The compliance is 417.901. The problem has 110 variables and 934 constraints.

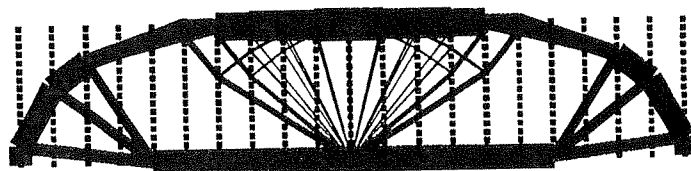


Figure 8: Optimal bridge with  $21 \times 17$  nodes. The compliance is 403.981. The problem has 714 variables and 38896 constraints.

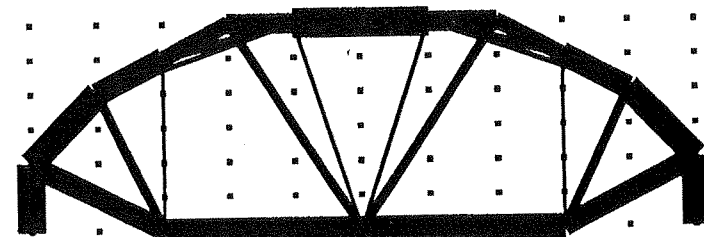


Figure 9: Optimal "high" bridge with  $11 \times 7$  nodes. The compliance is 283.755. The problem has 154 variables and 1828 constraints.

Figures 7 - 9 illustrate the same effect for bridges. The construction is supported in the vertical direction at the very southwest and southeast nodes, i.e., "the river banks". The single loading scenario is formed by three equal vertical forces applied equidistantly on the segment linking the supported nodes, i.e., "the road."

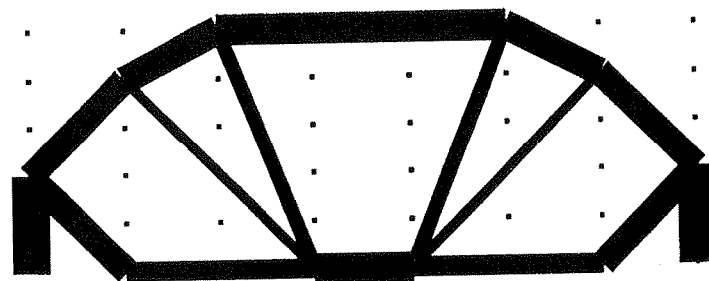


Figure 10: Optimal bridge - single-load formulation. The problem has 96 variables and 730 constraints.

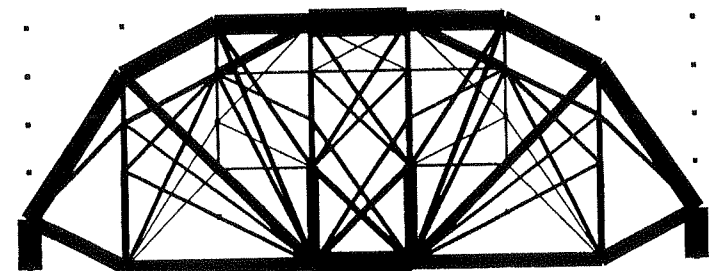


Figure 11: Optimal bridge - minmax multi-load formulation. The problem has 384 variables and 730 constraints.

Figures 10 - 11 demonstrate the difference between single- and multi-load settings. Both designs relate to a  $6 \times 8=48$ -node ground structure where links between all pairs of nodes are allowed. The extreme southwest and southeast nodes are supported in vertical direction. The design in Figure 10 corresponds to the case of a single loading scenario where four equal forces are applied *simultaneously* and equidistantly on the segment linking the supported nodes. The bridge in Figure 11 corresponds to a multi-load design with the

same four loads acting *nonsimultaneously*. In the latter case we typically obtain a design with many more bars, one of which is far more rigid.

**Further research.** It is now time to confess that our research up to now has contributed more to large-scale convex optimization than to practical truss topology design. From the practical viewpoint, the indicated approach leads to designs which should serve as "reference points" rather than to readily implementable constructions. The reason is that QF only partly models the actual design constraints and that there are at least three important restrictions it does not take care of:

- **Bounds on node displacements.** In the practical design, there are restrictions on the movements that the nodes are allowed to take.
- **Buckling.** The linear elasticity model underlying TTD has restricted applicability. For thin trusses, it is appropriate for a rather wide range of external loads which extend the bar. In contrast to this, the forces *compressing* the bar may cause arc-type or sine-type deformations which a good design should avoid.
- **Stability with respect to occasional loads.** Problem QF takes care only of the given loading scenarios. As a result, it may happen that a small "occasional" load may cause inappropriately large deformations of the resulting construction.

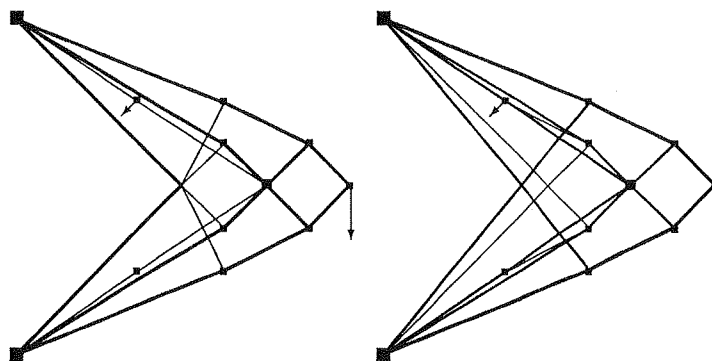
Attempts to incorporate the "anti-buckling" restrictions and restrictions on the node displacements straightforwardly lead to essentially large-scale nonconvex optimization problems, which are hardly tractable. One could prevent the indicated inappropriate phenomena by imposing lower bounds on the bar volumes, which basically preserves the nice convex structure of the problem. This approach, however, has rather restricted value: it can be used only in postoptimality analysis as it makes no sense to impose nontrivial lower bounds on bar volumes before the topology of the construction is identified. In contrast to this, we can take certain care of the stability of the resulting truss. The idea is as follows: let us pass from the finite set of loading scenarios underlying the usual multi-load TTD to an *ellipsoid* of loads, thus thinking of stability of the construction not only with respect to the "loads of interest," but also with respect to all small enough "occasional loads." The most natural way to construct such an ellipsoid is to take the "ellipsoidal envelope" of the initial finite set,  $F_{ini}$ , of loading scenarios and the Euclidean ball,  $B$ , of all possible occasional loads of reasonable magnitude, i.e., to take as  $F$  the ellipsoid of minimal volume containing  $F_{ini} \cup B$ . The immediate question is to which nodes the occasional loads are applied. It could definitely not be the initial node set of the ground structure, since it is natural to expect that the majority of the initial nodes will not appear in the resulting truss. There are, however, nodes which certainly will appear in this truss, i.e., the active nodes to which the forces participating in  $F_{ini}$  are applied, and we could choose  $B$  to be the ball in the subspace of virtual displacements of the active nodes. With this approach, we take from the very beginning certain, although incomplete, care of the stability of the resulting construction. And, of course, we could, and in our opinion also should, apply the outlined approach in the postoptimality analysis, resolving the problem on the node set given by preliminary design with "incomplete" stability constraints, i.e., with a "flat" ellipsoid of loads in the subspace of virtual displacements of active nodes. When resolving the problem, we deal with the "full-dimensional" ellipsoid of loads in the space of virtual displacements of the new, reduced node set.

From the practical viewpoint there is nothing very specific with ellipsoids. The only, and strong, reason why we focus on ellipsoids is the already indicated fact that the only computationally tractable versions of TTD seem to be those related to the case when  $F$  is an ellipsoid, or to the case where  $F$  is a polytope given by a list of its vertices, which is the usual multi-load TTD resulting in QF.

Mathematically, a TTD problem with an ellipsoidal set of loads results in a semidefinite program SD which seems to be more difficult than QF. Note, however, that in practical design the set of given loading scenarios comprises a very small number (1-5) of "localized" loads, so that there are very few active nodes. As a result, SD associated with the "pre-optimization stable formulation" of TTD involves low-dimensional matrix inequality constraints and is basically as computationally tractable as the usual multi-load TTD. In the postoptimality analysis we deal with "full-dimensional" ellipsoid of loads, but this ellipsoid is associated with the reduced node set given by the preliminary design, and we again may need to deal with a semidefinite program of tractable size.

The outlined *stable truss topology design via semidefinite programming* seems to be quite promising. In particular, we hope that this setting implicitly takes care of large node displacements and buckling phenomena.

To illustrate the advantages of the "stable" truss topology design, let us look at the following example. The left part of **Figure 12** represents the results of the optimal single-load truss topology design on a  $9 \times 9$  square planar grid (81 nodes, 2040 tentative bars); the extreme left nodes are completely supported, the remaining are free, the external load is shown by the long arrow. The truss looks quite attractive as its compliance with respect to the given load is 382.5. It turns out, however, that the construction is highly unstable since, when the initial load is replaced by a 10-times smaller "occasional" one at the node shown by the short arrow on the picture, the compliance becomes 18392.1—48 times larger. The "occasional" load results in the displacement of the node where the load is applied, which is almost 500 times larger than for the "scenario" load.



**Figure 12:** Single-load optimal design (left) and its postoptimal "stable correction" (right).



The right part of **Figure 12** is the truss given by postoptimal "stabilization" of the solution. To be precise, we selected the bars with relative volumes more than 1% from the aforementioned truss, and chose, as the reduced node set, the nodes incident to the selected bars. Then we resolved the problem, taking the 14 selected nodes as the node set, allowing all 66 tentative links of the nodes, and choosing as  $F$  the "ellipsoidal envelope" of the initial load and the ball consisting of all the 10-times smaller loads in the 20-dimensional space of virtual displacements of our new node set. The minmax compliance over our new 20-dimensional ellipsoid of loads of the resulting construction is 395.6—which is 3.4% greater than the minmax compliance of the first truss with respect to the single scenario load. Moreover, the compliance of the "stable" truss with respect to the original load is only 0.4% larger than the compliance for the first truss.

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# from S THE Nordic Section

The Nordic Section of the Mathematical Programming Society was created at its first meeting in Copenhagen, Denmark, in 1990. Stein Wallace, who played an important role in the forming of the section, was elected its first chairman. The stated goal of the Section was to provide a framework for inter-Nordic collaboration within the field of mathematical programming. We believe that the Section has indeed contributed positively to the development of the field and will hopefully continue to do so for a long time. The present board consists of Kaj Holmberg, Sweden; Kim Allan Andersen, Denmark; and Dag Haugland, Norway.

At the first meeting it was decided to hold biannual meetings of the Section, trying to bring together all researchers in the Nordic countries working on mathematical programming. The second meeting was held in Trondheim, Norway, in 1992, and the third was held in Linköping, Sweden, in 1994 (in February so as not to conflict with The International Symposium on Mathematical Programming in Ann Arbor). The fourth meeting will be held in 1996 in Aarhus, Denmark. These meetings are basically held over a weekend and are kept simple in order to be self-financed with reasonable registration fees (reduced, of course, for members of MPS). There has been some funding for participating students obtained from NorFA ("Nordisk ForskerAkademi").

In addition to the biannual meetings, we have started to produce a Newsletter of the Nordic Section; Number 2 has recently been issued. Our goal is to make the newsletter annual, but delays will possibly occur. The newsletter mainly contains lists of mathematical programming research reports and theses produced in the Nordic countries, in order that information about current research will be available before the actual publication which, as we all know, may take some time. These lists are particularly useful to people who cannot easily access all the different journals in our field.

Another simple but very useful feature is the e-mail list by which anyone can reach all the members of the Nordic Section (address: [mps@iok.unif.no](mailto:mps@iok.unif.no)). We are now also beginning to collect information about Nordic WWW-pages and ftp-sites containing mathematical programming material, all in order to facilitate communication between Nordic researchers and to make information easily available.

-Kaj Holmberg  
([kahol@math.liu.se](mailto:kahol@math.liu.se))

# CONFERENCE NOTES

## FORTHCOMING CONFERENCES

- ▶ **ICCP-95-International Conference on Complementarity Problems: Engineering & Economic Applications, and Computational Methods, Baltimore, Maryland, U.S.A. Nov. 1-4, 1995**
- ▶ **Third Workshop on Global Optimization, Szeged, Hungary, Dec. 10-14, 1995**
- ▶ **Conference on Network Optimization, Center for Applied Optimization, Gainesville, Florida, Feb. 12-14, 1996**
- ▶ **Workshop on SATISFIABILITY PROBLEM: THEORY AND APPLICATIONS Rutgers University March 11-13, 1996**
- ▶ **5th SIAM Conference on Optimization, Victoria, British Columbia, Canada, May 20-22, 1996.**
- ▶ **IPCO V, Vancouver, British Columbia, Canada, June 3-5, 1996**
- ▶ **IFORS 96 14th Triennial Conference, Vancouver, British Columbia, Canada, July 8-12, 1996**
- ▶ **International Conference on Nonlinear Programming, Beijing, China, Sept. 2-5, 1996**
- ▶ **XVI International Symposium on Mathematical Programming, Lausanne, Switzerland, Aug. 1997**

## Conference on Network Optimization

**Gainesville, Florida  
February 12-14, 1996**

Advances in data structures and computer technology and the development of new algorithms have made it possible to solve classes of network optimization problems that were formerly intractable. Among these are problems in airline scheduling, transportation, satellite communications, and VLSI chip design.

A conference on network optimization problems, hosted by the Center for Applied Optimization at the University of Florida, will bring together researchers working on many different aspects of network optimization and on diverse applications in fields such as engineering, computer science, operations research, transportation, telecommunications, and manufacturing. It will provide a unique opportunity for cross-disciplinary exchange of research.

*The conference has received endorsements from the Mathematical Programming Society and the Institute for Operations Research and Management Science, and is being held in cooperation with SIAM.*

Additional information is available from conference organizer *Panos Pardalos* of the University of Florida ([pardalos@ufl.edu](mailto:pardalos@ufl.edu); (904) 392-9011; fax: (904) 392-3537). Other organizers are *Don Hearn* ([hearn@ufl.edu](mailto:hearn@ufl.edu)) and *Bill Hager* ([hager@math.ufl.edu](mailto:hager@math.ufl.edu)).

## 8th Franco-Japanese and Franco-Chinese Conference Combinatorics and Computer Science

**Brest, France  
July 3-5, 1995**

*This joint meeting was attended by 80 participants from 10 countries (Austria, Canada, France, Germany, Italy, Japan, Sweden, Switzerland, Taiwan, USA). The scientific program consisted of 50 presentations centering around: Graph theory (graph-coloring, -decomposition, -generation, -recognition problems, homomorphisms, Slater's order, transversals, spectral characterizations and problems on trees); coding (p-adic, zigzag and block codes); linear and integer programming (gravitational and double description methods); combinatorial algorithms for LP and LCP, 3-index bottleneck assignment); polyhedral theory (Delauney and metric polytopes, edge-coloring and probability, crossing of hyperplanes on the torus); scheduling (jobshop with task intervals, combinatorics of scheduling optimization); approximation algorithms (for location and stable set problems); stochastic algorithms (GAs, simulated annealing e.g. to calculate Ramsey numbers); orders (contiguity orders, embedding of bipartite orders); and matroids (metric packings, pair-delta-matroids).*

A larger part of the contributions were focussed on the efficient solution of problems arising in important branches of computer science such as parallel algorithms and architectures (branch and bound, dynamic programming, perfect matchings in planar graphs, recognition of consecutive ones); distributed systems (task assignment using network flows, threshold graphs and synchronization, rerouting in DCS mesh networks); interconnection networks (embedding grids into de Bruijn graphs, gossiping in meshes, design of bus and lightwave networks); pattern matching (on the hypercube); and data compression.

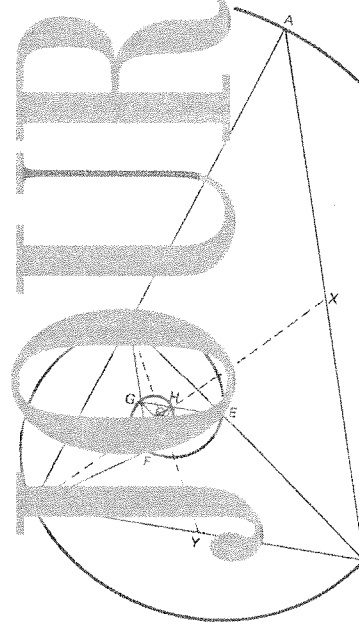
Selected topics did include optimal strategies without memory for playing Blackjack, 1-vehicle routing, the guard problem in spiral polygons, DNA sequencing and Motley gems.

A refereed post-conference proceedings volume will be published by Springer in the series LNCS.

The respective subsequent meetings will be held in Japan in 1996 and in Taiwan in 1997.

-REINHARDT EULER

# JOURNALS



## Volume 69, No. 1

**B-issue: Nondifferentiable and Large-Scale Optimization**

Guest Editors: J.-P. Vial and J.-L. Goffin

D.S. Atkinson and P.M. Vaidya, "A cutting plane algorithm for convex programming that uses analytic centers."

O. Bahn, O. du Merle, J.-L. Goffin and J.-P. Vial, "A cutting plane method from analytic centers for stochastic programming."

D. den Hertog, F. Jarre, C. Roos and T. Terlaky, "A sufficient condition for self-concordance, with application to some classes of structured convex programming problems."

K.C. Kiwiel, "Proximal level bundle methods for convex nondifferentiable optimization, saddle-point problems and variational inequalities."

C. Lemaréchal, A. Nemirovskii and Yu. Nesterov, "New variants of bundle methods."

Yu. Nesterov, "Complexity estimates of some cutting plane methods based on the analytic barrier."

Yu.E. Nesterov and A.S. Nemirovskii, "An interior-point method for generalized linear-fractional programming."

L. Vandenberghe and S. Boyd, "A primal-dual potential reduction method for problems involving matrix inequalities."

D.S. Hochbaum and S.-P. Hong, "About strongly polynomial time algorithms for quadratic optimization over submodular constraints."

R.D.C. Monteiro and S.J. Wright, "Superlinear primal-dual affine scaling algorithms for LCP."

M.X. Goemans, "Worst-case comparison of valid inequalities for the TSP."

## Vol. 69, No. 3

J. Miao, "A quadratically convergent  $O((k+1)+nL)$ -iteration algorithm for the  $P(k)$ -matrix linear complementarity problem."

S.R. Tayur, R.R. Thomas and N.R. Natraj, "An algebraic geometry algorithm for scheduling in presence of setups and correlated demands."

J.-M. Cao, "Necessary and sufficient condition for local minima of a class of nonconvex quadratic programs."

S.L. van de Velde, "Dual decomposition of a single-machine scheduling problem."

M. Bellare and P. Rogaway, "The complexity of approximating a nonlinear program."

Z.B. Zabinsky, G.R. Wood, M.A. Steel and W.P. Baritompa, "Pure adaptive search for finite global optimization."

A.I. Barvinok, "New algorithms for linear  $k$ -matroid intersection and matroid  $k$ -parity problems."

H. Hamers, P. Borm and S. Tijs, "On games corresponding to sequencing situations with ready times."

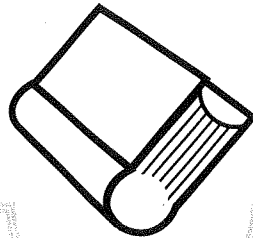
## Vol. 69, No. 2

B. Chen and P.T. Harker, "A continuation method for monotone variational inequalities."

A. Ebiefung, "Nonlinear mappings associated with the generalized linear complementarity problem."

# BOOK

## R E V I E W S



### *Numerical Approximation of Partial Differential Equations*

by Alfio Quarteroni and Alberto Valli

Springer Series in Computational Mathematics 23

Springer-Verlag, Berlin, 1994

ISBN 3-540-57111-6

This book is devoted to the numerical solution of three important classes of second order partial differential equations: elliptic, parabolic, and hyperbolic ones. Also relevant PDEs of mixed type, like advection-diffusion, Stokes, and Navier-Stokes receive considerable attention. In particular, the latter ones play a central role in Computational Fluid Dynamics. Main emphasis is on finite element approximations, but other techniques, such as collocation methods, are discussed as well.

When I started to read this book, I was a bit skeptical; after all, some 500 pages does not seem too many if one attempts to cover the ins and outs of numerical evaluation of PDEs. One has to consider many special cases in order to obtain as well as to analyze efficient and reliable computational schemes. The relevant literature in this field is enormous. However, gradually I began to like this book very much. Almost everything that one needs to know when attacking these types of PDEs in a scientific computing environment has been considered and has been presented in a very clear and understandable way. The authors state in their introduction: "A sound balancing of theoretical analysis, description of algorithms, and discussion of applications is our primary concern. Many kinds of problems are addressed: linear and non-linear, steady and time-dependent, having either smooth or non-smooth solutions. Besides model equations, we consider a number of (initial-) boundary value problems of interest in several fields of application."

Although the authors have set high goals for themselves, I must admit that they have succeeded well in their task. The book gives an impressive mix of theory, applications, and implementation aspects. This is all nicely illustrated by well-chosen computational examples, relevant for large scale realistic problems.

O P T I M A



**"The theory is consistently presented; theorems are motivated by the preceding discussions: *what are we going to see in the next theorem?*, and *what is it good for?* This makes it worthwhile also for a novice in the field to study the relevant theory."**

The discussed and analyzed techniques are relevant for modern three-dimensional modeling of physical problems. For instance, much attention has been paid to the iterative solution of the linear finite element systems, besides the more traditional direct methods which were more or less the methods of choice in classical two-dimensional finite element computations. As far as I can judge, the described techniques represent the state of the art: not only have methods been treated that were published as recently as in 1992, they have also been implemented and the discussions by the authors seem to be supported by their computational experience as well. Together with all the further references, this makes the book a very valuable source of information. Possibly I am slightly biased in my judgment because of the elaborate treatment of Bi-CGSTAB (published in 1992), but I can only conclude that the presentation of all relevant methods is very much to the point. Discussions are supported by actual computational examples that help the reader to get some feeling for the selection of methods for a particular given problem. This is necessary since on more than one occasion there is a variety of approaches that may lead to an acceptable solution. The choice of a particular method or approach usually determines the efficiency of the computational work, but often the actual behavior of a method depends on parameters that are not explicitly available to the user when solving realistic problems.

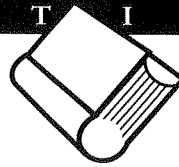
The book is also excellent for teaching; the only disadvantage is that exercises are missing. The positive point for students is that unsolved problems are mentioned as well which prevents the student from receiving the misleading impression that virtually everything in this field is well understood. The theory is consistently presented; theorems are motivated by the preceding discussions: *what are we going to see in the next theorem?*, and *what is it good for?* This makes it worthwhile also for a novice in the field to study the relevant theory. Both style and presentation are very helpful for attacking practical problems as well as for further research. Of course, there are places in the book where the expert might occasionally frown, but the limits of the acceptable are never overstepped.

What I liked in particular is the attempts made by the authors to point out analogies between approaches in widely different applications. For instance, Uzawa's scheme for the Stokes problem is at the discretized level recognized as a preconditioned Richardson scheme for linear systems. This kind of parallel is very appealing to me; not only does it contribute to more insight, it also helps to clear up the apparent chaos of, at first sight, very different approaches and techniques. It is also very helpful for memorizing some of the relevant approaches.

The authors state that "the book is addressed to graduate students as well as to researchers in the field of numerical simulation of partial differential equations." I strongly believe, as should be clear from my review, that Quarteroni and Valli have succeeded in their mission, and I recommend the book to anyone who has interest in numerical mathematics, a central field in large scale scientific computing.

-H.A. VAN DER VORST

O P T I M A



## *Interior Point Approach to Linear, Quadratic and Convex Programming*

by D. den Hertog  
 Mathematics and its Applications  
 Kluwer Academic Publishers  
 Dordrecht, The Netherlands, 1994

ISBN 0-7923-2734-9.

This excellent book deals with recent developments in interior point algorithms for linear, quadratic and convex programming. Since the publication of Karmarkar's algorithm for linear programming [1], this exciting area has been intensively and extensively studied by many researchers. Most of the work is focused on specific algorithmic improvement and linear optimization application. This book presents a general and rigorous foundation for solving nonlinear convex optimization problems. This foundation theory is mainly due to Nesterov and Nemirovskii [2], but den Hertog simplifies and finalizes some of the results. Thus, it seems much better to read this book before reading [2].

The main concept is the self-concordant barrier function on an open convex set, introduced in Chapter 2 and analyzed in Chapter 3. This is a Lipschitzian smoothness condition of the Hessian with respect to a local Euclidean metric, plus a barrier for the underlying convex set. The authors prove that Newton's method is effective on self-concordant barrier functions. Thus, the problem of developing an efficient path-following or potential-reduction algorithm "reduces" to constructing a self-concordant barrier for the constraint set (see Appendix A).

The authors devote the next chapter, Chapter 4, to reducing the complexity for linear programming. I believe that his technique on adding and deleting constraints has an important impact in practice as well. Chapter 5 is special; it unifies several popular interior-point algorithms extremely well. It also presents a clear framework on how these algorithms are related and brings mathematical insights to understand these algorithms.

The book is well and clearly written. It is comprehensive and well-balanced on various topics. It can make an excellent text for an advanced or seminar course on optimization, primarily addressed to graduate students in mathematics, pure or applied, computer science and engineering schools. On the other hand, researchers will also find it a valuable reference because the theorems contained in many of its sections represent the current state of the art. In fact, the extensive bibliographic section is another strong point of the book, quite complete and up to date. I believe this work will remain a basic reference for whomever is interested in convex optimization for years to come.

### References

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2. Yu, E. Nesterov and A. Nemirovskii, "Interior Polynomial Algorithms in Convex Programming," *SIAM*, Philadelphia, 1994.

-YINYU YE

**"...an excellent text for an advanced or seminar course on optimization, primarily addressed to graduate students in mathematics, pure or applied, computer science and engineering schools."**

# ACADEMIC Position

*Catholic University of Louvain  
Department of Mathematical  
Engineering.*

Louvain-la-Neuve, Belgium

The Department of Mathematical Engineering invites applications for an academic appointment in mathematical engineering, with preference for one of the following topics:

- numerical analysis and scientific computing
- stochastic modelling
- discrete mathematics, combinatorics, graphs and algorithms
- mathematical system theory
- optimisation and variational calculus.

The applicant will have a teaching load in applied mathematics. Applicants should demonstrate both breadth of interest and promise in research and teaching. Tenured positions will be considered but appointment rank will depend on the candidates and their records of accomplishment. Applicants will be expected to teach in French, possibly after a certain transition period.

The 550-year-old Catholic University of Louvain is located on a campus created in 1972 in Louvain-la-Neuve. The new campus now has more than 20,000 students and about 5,000 staff members.

The Mathematical Engineering Department hosts research programs in the different topics listed above. The department also has close ties with the Center for Operations Research and Econometrics (CORE); the Center for Systems Engineering and Applied Mechanics (CESAME) and the Institute for Pure and Applied Mathematics (MAPA).

The applicant should have a doctoral degree in engineering (or equivalent), postdoctoral experience, and a recognized scientific publications record. The appointment will start October 1996.

The applicants should furnish a curriculum vitae, a list of publications, an abstract of five selected publications, the name and address of four referees including two Belgian referees who are not members of UCL, before 15 December 1995 to the Rector of the university:

Recteur de l'Université Catholique de Louvain  
1, Place de l'Université  
B-1348 Louvain la Neuve  
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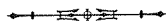
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**OCTOBER** 1995

OPTIMA is expanding to four issues per year with publication dates keyed to the academic semesters. The new schedule will have issues in October, December, March and June, with due dates of Sept. 15, Nov. 15, Feb. 15 and May 15, respectively. As of August, John Dennis ([dennis@caam.rice.edu](mailto:dennis@caam.rice.edu)) is the Chair of MPS, Clyde Monma ([clyde@bellcore.com](mailto:clyde@bellcore.com)) is Treasurer and Steve Wright ([wright@mcs.anl.gov](mailto:wright@mcs.anl.gov)) is Chair of the Executive Committee. **Deadline** for the next OPTIMA is November 15, 1995.

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