

Gröbner bases in integer programming

BY

Serkan Hosten
School of
Operations
Research,
Cornell University

E-mail:
serkan@cs.cornell.edu

Rekha R. Thomas
Department of
Mathematics,
Texas A & M
University

E-mail:
rekha@math.tamu.edu

Introduction. Recently, the tools of commutative algebra and algebraic geometry have brought new insights to integer programming via the theory of *Gröbner bases*. A Gröbner basis of a *polynomial ideal* is a special generating set computed with respect to a speci-

fied weight vector on the variables (see [1], [8]). A fixed ideal may have different (but always finitely many) Gröbner bases as the weight vector is varied. In the 1960s, Bruno Buchberger provided an algorithm to compute these special generating tests, naming them Gröbner bases to acknowledge his thesis advisor Wolfgang Gröbner. The *Buchberger algorithm* [5], which can be viewed as a generalization of Gaussian elimination to systems of multivariate polynomial equations, can now be found in a number of computer algebra packages like MACAULAY [3] and MAPLE. Since Gröbner bases provide algorithmic solutions to a plethora of problems modeled using polynomial equations, the discovery of the Buchberger algorithm was the launching pad in the development of a number of algebraic algorithms. Recently, Gröbner bases have been applied extensively to study problems in convex geometry via certain special ideals called *toric ideals*. This is the topic of a forthcoming book by Bernd Sturmfels called *Gröbner Bases and Convex Polytopes* [20].

Gröbner bases and integer programming. In 1991, Pasqualina Conti and Carlo Traverso [6] gave an algebraic algorithm to solve the family of integer programs, denoted $IP_{A,c}(b)$,

$$\text{Minimize } cx : Ax = b, x \geq 0, \text{ integer,}$$

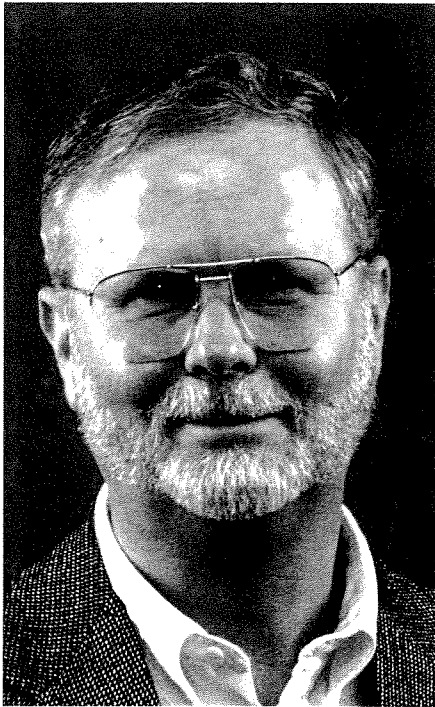
obtained by varying the right hand side vector $b \in \mathbf{Z}^m$ while keeping $A \in \mathbf{Z}^{m \times n}$ and $c \in \mathbf{R}^n$ fixed. Extensions to mixed integer programming have been worked out by David Bayer and Ian Morrison [2]. We will assume that all programs in the family $IP_{A,c}(\cdot)$ have bounded optimal solutions. Their algorithm works in two stages: Stage 1 is a preprocessing stage which computes G_c , the

PAGE THREE ▶

N U M B E R
48

the new chairman	2
software & computation	5
journals	7
conference notes	9
book reviews	13
gallimaufry	15

Rice University Professor John Dennis Is New MPS Chairman



Our new chairman, John Dennis, is the Noah Harding Professor at the Department of Computational and Applied Mathematics, Rice University, Houston, Texas, USA. His research interest is practical methods for optimization, in particular, parallel methods for nonlinear optimization described by coupled nonlinear simulations, and he has extensive experience with industrial applications. The 1983 book, Numerical Methods for Unconstrained Optimization and Nonlinear Equations, written with R.B. Schnabel is now available in the SIAM series "Classics in Applied Mathematics." He has been active within SIAM, served two terms on its Council and chaired the SIAM Activity Group for Optimization. He founded and served as first Editor-In-Chief of the SIAM Journal for Optimization, and he has been a co-editor of Mathematical Programming.

In the following article Dennis expresses his belief that the optimization community has not done a good marketing job but predicts that the future of scientific computing is going to see an increase in the application of optimization. He also discusses the current state of the Society and important issues for the future.

-KAREN AARDAL

AS engineers improve their ability to simulate airflow around a car body or the flow of fluids in porous media, they want to select decision variables that at least improve - if not optimize - the situation they are simulating. I think that the application of homemade 'voodoo optimization' techniques based on physical analogies of questionable relevance to problems like the TSP, where there are effective combinatorial techniques, is due largely to scientists and engineers who need optimization but who do not realize that we have some good techniques, so they do it themselves. I remember a comment from a physicist at the workshop Bob Bixby ran a few years ago, which Bob called the 'Great Texas TSP Shootout.' Everyone was to bring their software and examples to the meeting and compete. The combinatorial optimization techniques dominated the physical analogy codes in all the tests. The physicist complained that, since the optimizers never published in Scientific American, no one knew about all the work they had done. Well, the physicist had a point. We cannot expect prospective clients to search the optimization literature. I think it is time to branch out from adherence to our historical roots in operations research. We are just as important to the physical applications, but we must convince others of this. For example, I hope to see the MPS web pages linked to a cogent guide to optimization methods.

When the new officers took over in August 1995, the memory of the very successful Ann Arbor Symposium was still fresh in our minds, the journal had reduced its backlog to a manageable level, the plans for the Lausanne Symposium were in capable hands, and the treasury had a comfortable balance. Fortunately, in the few months the new slate has been in office, not much has changed. The Mathematical Programming Society is in great shape.

There is one fly in the ointment. Our dues have not increased in years, and we have reached the point where each member costs us each year about \$20 more than the dues he/she pays. We must raise dues. After some consideration, the Executive Committee has decided to ask the Council to approve an increase in dues by \$5 per year until we regain equilibrium. If the Council approves this increase, we will use the money in the treasury to make up the difference each year that is necessary. This decision was taken because we felt that some

of our members might find it more palatable than a large single jump in dues. Some sister organizations avoid this situation by raising dues a bit each year whether or not a raise is needed in a particular year. The whole issue of dues increases will be a topic for the Council meeting in Lausanne.

So, expect some increase in dues this year. MPS will remain a great bargain even when dues reach the break-even point. Rest assured that there is no move afoot to change the low-key, inexpensive, underadministered, research-oriented character of MPS. We do not aspire to be INFORMS, AMS, or SIAM. These are fine organizations, but MPS fills a different niche.

Bob Bixby (bixby@caam.rice.edu), the Chair of the Publications Committee, is Chair of an Ad Hoc Committee on Electronic Publishing. The committee consists of the Editors of MPS A&B as well as Irv Lustig, Clyde Monma, and Uwe Zimmerman. We do not plan to lead the charge into electronic publishing, but we do plan to position MPS to avoid being left behind. Contact members of the committee with your ideas.

Finally, some timely topics discussed at the recent MPS Executive Committee meeting: we are hoping for some strong proposals for the 2000 Symposium; we are trying to improve our membership services in light of persistent complaints, and OPTIMA has increased its pagination by adding interesting interviews and feature articles.

Please contact me (dennis@caam.rice.edu) or Executive Committee Chair Steve Wright (wright@mcs.anl.gov) with any comments on any society business. One of our great strengths in MPS is that our membership is scattered across the globe. Unfortunately, this also means that our business meetings are very infrequent. The internet has exciting possibilities for increased communication among us, and we invite you to use it to let us know what you are thinking.

Address:

John Dennis
187 CITI/Fondren MS 41
Rice University
Houston, Texas 77251-1892
Voice: (713)527-4094
Fax: (713)285-5318
dennis@caam.rice.edu
<http://www.ruf.rice.edu:80/~dennis/>

continued Gröbner

bases in
integer
programming

reduced Gröbner basis with respect to the cost vector c , of the toric ideal of A . (The toric ideal of A is a polynomial ideal constructed from A .) In Stage 2, we solve $IP_{A,c}(b)$ for a b of interest by "reducing" (with respect to G_c), an arbitrary feasible solution of the program to an optimal solution. A geometric interpretation of the above procedure, see [23], recognizes the reduced Gröbner basis G_c as a *test set* for the family of programs $IP_{A,c}(\cdot)$. A finite set of integral vectors $T_c \subset \mathbf{Z}^n$ is called a test set [18] for $IP_{A,c}(\cdot)$ if, for every non-optimal solution α to a program in $IP_{A,c}(\cdot)$, there is some vector $t \in T_c$ such that $\alpha - t$ is a solution to the same program with a smaller objective function value than that of α . In practice we refine the cost vector c by say the lexicographic order on the variables so that the resulting cost vector is a linear order on \mathbf{N}^n . This technical assumption guarantees finiteness of the Buchberger algorithm, uniqueness of the optimal solution for each program, and a strict decrease in cost value from α to $\alpha - t$. Clearly, if a test set for $IP_{A,c}(\cdot)$ is known, we have a trivial algorithm to solve all programs in the family, provided an initial solution is known for each program. In fact, the full Conti-Traverso algorithm has a "Phase I" and "Phase II" that allows one to start with an "artificial solution" to $IP_{A,c}(b)$ and arrive first at a feasible solution and finally the optimal solution. Once the reduced Gröbner basis G_c is obtained, Stage 2 of the Conti-Traverso algorithm can be seen as constructing a monotone

(with respect to c) path from the initial non-optimal solution of $IP_{A,c}(b)$ to the optimal solution, by using vectors in G_c to successively move from one feasible solution of the program to a better one. In effect, for every b such that $IP_{A,c}(b)$ is feasible, one can use G_c to build a directed graph in P_b^I the convex hull of all solutions to $IP_{A,c}(b)$, whose nodes are the lattice points in P_b^I and edges are the elements in G_c . Each such graph has a unique sink at the optimal solution to $IP_{A,c}(b)$.

Since test sets provide a very natural and intuitive method for solving an integer program, it is not surprising that one finds many test sets in the integer programming literature. In 1975, Jack Graver [13] showed the existence of a finite set of vectors that solve all programs in the family IP_A . Here IP_A denotes all integer programs of the form $IP_{A,c}(b)$ but for which both the cost and right hand side vectors are allowed to vary. Variants of the *Graver test set* appear both in [4] and [7]. In 1981, Herbert Scarf [17] introduced another test set called the *neighbors of the origin*. The relationships among all these test sets (including Gröbner bases) are discussed in [23]. In this context, a distinctive feature of the Gröbner basis is that it can be computed in practice via the Buchberger algorithm.

Universal test sets. A set of integral vectors $U_A \in \mathbf{Z}^n$ is called a *universal test set* for IP_A if, for any choice of c and b , U_A contains a test set for $IP_{A,c}(b)$. The Graver test set mentioned above is such a set. It can be shown that every reduced Gröbner basis G_w is contained in the Graver test set. Since the Graver test set is finite, it follows that there exist only finitely many distinct Gröbner bases associated with a fixed matrix A as the cost func-

tion is varied. The union of these reduced Gröbner bases, denoted UGB_A , is a minimal universal test set for IP_A called the *universal Gröbner basis* of IP_A . The size of an element in UGB_A could be exponential in the size of the data [19]. Let LP_A denote the family of linear programs that are the linear relaxations of programs in IP_A and let P_b denote the feasible region of $LP_{A,c}(b)$. The simplex method solves $LP_{A,c}(b)$ by starting at a vertex of P_b and moving monotonically along edges of P_b until an optimal vertex is reached. It is well known that the edge directions of the polyhedra P_b , as b varies, are precisely the minimal integral dependencies of the columns of A - also known as *circuits* of A . Hence the circuits of A form a universal test set for LP_A . Since we have rational data, for every right hand side vector b there exists a b' such that P_b^I is a multiple of $P_{b'}$. Hence the circuits of A are contained in the universal Gröbner basis UGB_A . In particular, for unimodular matrices, the circuits constitute UGB_A . Since universal Gröbner bases study integer programs using fundamentally different methods from the classical techniques, they provide many new insights into the structure of integer programs. One such result shows that the elements of UGB_A are precisely the set of all primitive edge directions in the polyhedra P_b^I as b varies [21]. Hence UGB_A does for integer programs what circuits do for linear programs. In a sense, the Gröbner basis method for integer programming is the "integer-analogue" of the simplex method for linear programming. Further analogies of this nature are established in [21].

Examples and GRIN. We saw earlier that in each polyhedron P_b^I the elements of G_c build a directed graph, which is a union of directed paths from every non-optimal lattice point in P_b^I to the optimal. If we disregard the direction of edges in this graph, then a reduced Gröbner basis can be thought of as providing a path between any two feasible lattice points in P_b^I . These two observations make way for a number of applications. Most simply one can use the first observation to enumerate and count all lattice points in P_b^I . This is accomplished by first computing a reduced Gröbner basis G_c and using this to build a graph (as above) in P_b^I whose unique sink is the optimal solution to $IP_{A,c}(b)$. In order to enumerate all lattice points in P_b^I , we reverse all edges in this graph and search the graph starting at the optimal solution of $IP_{A,c}(b)$ which now becomes the root of this graph. This "backtracking" procedure was adapted in [15] to solve a class of manufacturing problems modeled as integer programs with a probabilistic side constraint given by an oracle. In this case, it was possible to speed up computations by exploiting the structure of the matrices at hand.

A second application can be found in statistics [11]. One of the ways to check whether two attributes of a population, for instance, the eye color and hair color of its members, are correlated, is to construct contingency tables from samples of the population. A contingency table in our example can be viewed as a matrix whose ij th entry is the number of people from the sample with eye color i and hair color j . We study whether the attributes are correlated by comparing the given

continued Gröbner bases in integer programming

contingency table with another table selected at random from the set of all tables with the same marginal distribution (same row and column sums). All tables with a fixed marginal distribution can be described as the set of all solutions to a system $\{Ax = b, x \geq 0, \text{integer}\}$. Therefore, a Gröbner basis associated with A allows one to move from one contingency table to another with the same marginal distribution. This procedure can quickly generate enough tables which ensure that the table selected for the comparison is close to random.

Gröbner bases can be used to compute *primitive partition identities* (ppi's) as shown in [10]. For a given $n \in \mathbf{N}$ a ppi is any identity of the form

$$a_1 + a_2 + \dots + a_k = b_1 + b_2 + \dots + b_l$$

where $0 < a_i, b_j \leq n, a_i, b_j \in \mathbf{N}$ with no proper subidentity

$$a_{i_1} + a_{i_2} + \dots + a_{i_r} = b_{j_1} + b_{j_2} + \dots + b_{j_s}$$

where $1 \leq r + s \leq k + l - 1$. One can think of ppi's as the generalization of the identity $1+1=2$. It is not hard to recognize the ppi's for a given n as the Graver test set of the matrix $A = [1, 2, \dots, n]$. This set can be computed using Buchberger's algorithm. As n increases, the cardinality of the set of ppi's grow very fast; for $n = 12$ and $n = 13$ there are 9285 and 18900 ppi's respectively. Until recently, these could be computed efficiently up to $n = 13$ by

MACAULAY. This has been extended up to $n = 22$ using a Hilbert basis computation [16]. The Graver test set of the matrix $A = [(1,1), (1,2), \dots, (1,n)]$ corresponds to the *homogeneous ppi's* (hppi) which are partition identities of the above form with the same number of summands on the left and right sides of both equations. The associated ideal is the ideal of the projective monomial curve of degree $n-1$, an important curve in algebraic geometry. The champion again is Buchberger's algorithm; MACAULAY can compute hppi's for $n \leq 12$. For $n = 13$, there are 16968 hppi's, and they were found by GRIN after a lengthy computation.

GRIN (GRöbner bases for INteger programming) is an experimental software package which computes Gröbner bases of ideals arising from integer programs. It is intended to be a tool for combinatorial optimization and computational algebra and for problems that lie in the intersection of these fields. GRIN exploits the special structure of toric ideals, which are the ideals that occur in this context. Due to their special nature, these ideals allow simple data structures and also an implementation of the Buchberger algorithm that is easier, and more efficient, than in general. A main feature is a built-in option for computing the reduced Gröbner basis of a given ideal by making "short" Gröbner basis computations successively, a new approach in the area. Other state-of-the-art algorithms, such as an algorithm due to Fausto DiBiase and Rüdiger Urbanke [9], are also implemented. In [14], one can find a comparison

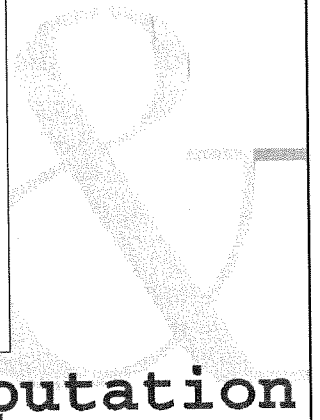
of GRIN to existing integer programming software (CPLEX).

The future. The connections between toric ideals and integer programming point toward many exciting future directions of research. We indicate some of them below.

An important practical issue is the computability of Gröbner bases for integer programs. Devising both theoretical and programming tricks to speed up the Buchberger algorithm is an obvious step in this direction. Like in the classical techniques for integer programming, computations can often be fine tuned by exploiting the structure of the problems. If one is interested in solving $IP_{A,c}(b)$ for a fixed b , then often a small subset of the Gröbner basis \mathcal{G}_c will suffice as a test set. Therefore, methods that incorporate b into the Buchberger algorithm (permitting shortcuts in computations) to produce a sufficient test set for $IP_{A,c}(b)$ are very worthwhile in this respect. An idea in this direction can be found in [24] and [25] where the concept of a *truncated Gröbner basis* is introduced. This procedure produces a test set for the family of integer programs whose right hand side vector is "smaller than or equal to" b in a specific sense. In the case of 0-1 programs, truncation dramatically cuts down computational effort. Algorithms that generate Gröbner basis elements as needed and decide whether a given set is a Gröbner basis are some other interesting issues that fall in this general area of research.

We close by indicating some connections between the Gröbner basis method and Ralph Gomory's group theoretic approach [12] to solve integer programs, which we refer to as the *group problem*. (See [22] for details about this connection.) The group problem can be interpreted as the symmetric analog to the usual linear relaxation of an integer program: instead of relaxing the integrality constraints, we remove the non-negativity constraints on the variables while keeping the integrality requirements. This can be seen as a "localization" of the ideal associated with the problem [20]. This localization leads to an even simpler ideal, which might provide a test set that solves the original problem. As in every relaxation procedure, this method does not guarantee to find the optimal solution at the first attempt. Here, it would be interesting to know whether there are classes of integer programs for which we can guarantee that the localization will give the optimal solution immediately.

The Gröbner basis technique for integer programming is still very much in its infancy. Hopefully, the access this technique provides to the powerful tools of algebra and algebraic geometry will help shed new light on the structure and complexity of integer programs.



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In this issue we feature a versatile and robust nonlinear programming software package that has been available to the academic and research community as a Beta release for several years. While a detailed user's guide has also been available, this guide will be integrated into a SIAM book that also includes theory and real world applications.

The following article has been adapted from the first few pages of the *User's Guide* and from correspondence with the authors.

FSQP: A Versatile Tool for Nonlinear Programming - User's Guide, by C.T. Lawrence, A.L. Tits, and J.L. Zhou, SIAM Press, 1996 (forthcoming). This book is intended as a detailed user's guide to the two feasible sequential quadratic programming (FSQP) packages, CFSQP (C version) and FFSQP (FORTRAN version). It also presents an in-depth explanation of the FSQP algorithm and includes examples of diverse applications from science and engineering. The algorithm can be used to solve three families of problems: (P1) find a solution to a system of nonlinear equations (feasibility problem); (P2) find a solution to a system of nonlinear constraints (both equality and inequality) that optimizes a given function (standard nonlinear programming problem); and (P3) find a solution to a system of nonlinear constraints that minimizes the maximum (or maximizes the minimum) of a set of functions (constrained minimax (or maximin) problem). The functions are all assumed to be continuously differentiable; however, it frequently works well when this assumption is violated. Moreover, special techniques are employed to exploit the structure of linear constraints for added efficiency.

The Method: Sequential quadratic programming (SQP) is a superlinearly convergent algorithm for finding approximate local solutions to nonlinear programming problems (see P.E. Gill, W. Murray, and M. Wright, *Practical Optimization*, Academic Press, New York, 1981). In general, without additional safeguards, the approximate solution

obtained can be locally suboptimal or superoptimal and feasible or infeasible, depending on the functions in the problem and the feasibility and optimality tolerances used. Feasibility is only guaranteed in the limit. This approach has enjoyed wide success on applications whose constraints are not rigid. In many important applications, however, violating some or all of the constraints is not acceptable. This is the case when the objective function is not defined outside the feasible set. For example, dynamical systems must be stable in order for certain steady state errors to be well-defined. Another reason for generating feasible iterates is the requirement in some applications (such as certain real-time control problems) that certain "hard" constraints must be satisfied after a prescribed amount of time.

Another situation where feasibility of successive iterates is imperative for some constraints arises in the interactive process used for designing engineering systems (see W.T. Nye and A.L. Tits, "An Application-Oriented, Optimization-Based Methodology for Interactive Design of Engineering Systems," *International Journal of Control* 43 (1986) 1693-1721). In such problems, some of the specifications can be relaxed, but others (such as stability or physical realizability) cannot. There are usually tradeoffs for violating the "soft" constraints (specifications) which can be explored by the designer during the design process. This tradeoff exploration, however, is only meaningful after the "hard" constraints (specifications) are satisfied. Since each iteration of an optimization algorithm involves one or more function evaluations, and since in a typical design environment function evaluations call for computationally expensive system simulations, it is essen-

continued software & computation

tially required that hard constraints be satisfied *at each iteration*.

First order methods of feasible directions have been proposed since the late 1950s by Zoutendijk, Polak, Pshenichnyi, and others, but these early procedures all had linear rates of convergence. Motivated by the need for feasible iterates in large applications, a group of researchers at the University of Maryland set out to modify the SQP method to generate feasible iterates without sacrificing its superlinear rate of convergence. The algorithm's main architect was Dr. Eliane R. Panier when she was still at the University of Maryland. Development of the methodology and its implementations has been ongoing at the University's Institute for Systems Research. The key feature that distinguishes FSQP from other SQP algorithms is the concept of a "semi-feasible" point. In solving nonlinear programming (P2) and constrained maximax (P3) problems, the FSQP algorithm first determines a point that satisfies all inequality constraints and linear equality constraints but violates some or all of the nonlinear equality constraints. Such a point is called semi-feasible. The algorithm subsequently generates a sequence of semi-feasible points while striving to satisfy all nonlinear equality constraints and to optimize the objective function.

On the other hand, FSQP makes use of a classical variable metric scheme to estimate the Hessian of the Lagrangian. As a consequence, it is not well-suited for problems that involve a "very large" number of decision variables.

The core of the FSQP algorithm only deals with inequality constraints (and linear equality constraints). Given a point x satisfying the constraints, the basic SQP search direction d^0 may not be a feasible direction; i.e., even short steps along this direction may yield points that do not satisfy the constraints. Yet d^0 is at worst "tangent" to the feasible set X . Thus, in FSQP, d^0 is slightly "tilted" toward the interior of X , to yield the search direction d . The amount of tilting is closely monitored in order to preserve the quasi-Newton convergence properties of the SQP direction.

A further adjustment is needed in order to prevent a Maratos-like effect. In a nutshell, the Maratos effect stems from the near conflict between the need to possibly reduce the step length along the search direction, to prevent oscillation or divergence in the early iterations, and the need to allow a full unit step when a solution is approached, to allow fast convergence to take place. In the context of FSQP, reduction of the step length in early iterations is necessary not only to avoid divergence, but also to ensure feasibility of the successive iterates. Two schemes are available in FSQP to ensure that a full unit step will always be accepted close to a solution: (i) a second order correction, with "bending" of the search direction, and (ii) a nonmonotone line search. On the average, the latter allows marginally faster progress towards the solution as it often yields a larger step size. On the other hand, the former has the property that the value of the objective function improves at each iteration, which can be an important advantage in certain applications.

Finally, nonlinear equality constraints are dealt with by means of a technique first suggested by Mayne and Polak (see D.Q. Mayne and E. Polak, "Feasible Direction Algorithms for Optimization Problems with Equality and Inequality Constraints," *Mathematical Programming* 11 (1976) 67-80) in the context of first order methods of feasible directions. The idea is as follows: split each equality constraint into two inequalities (" \leq " and " \geq "). Include one of these (the one satisfied by the initial iterate, say, $h(x) \leq 0$) with the other nonlinear inequality constraints and penalize violations of the other one ($h(x) \geq 0$) by means of a simple penalty function: a multiple $-ch(x)$ (for $c > 0$) of the violation is added to the objective function (or to the maximum among the objective functions). The advantages of this scheme over the more common quadratic penalty function are that (i) convergence to a feasible point is achieved without driving c to infinity (i.e., this is an *exact* differentiable penalty function) and (ii) one side of the constraints (here $h(x) \leq 0$) is satisfied throughout the optimization process ("semi-feasibility"), which is a desirable feature in certain applications.

The Software: The first version of the FORTRAN implementation (by the third author of this book) was released in 1989. The first C implementation (by the first author of this book) was released in 1993. The successive versions of both packages were made widely available on the Internet. As a result, the authors received voluminous feedback over the years from the user community. This led to elimination of bugs and improved implementations and enhancements.

The C version (CFSQP) includes a special scheme to efficiently handle problems with a large number of either (minimax) objective functions or inequality constraints relative to the number of decision variables. Such problems often arise in engineering applications. CFSQP is especially tuned for the case where groups of such constraints or objective functions are identified by the user as being "sequentially related," i.e., as consisting of "linear lists" of functions where each one is nearly identical to its predecessor and to its successor. This is the case, for instance, with groups of constraints arising from the discretization of a continuum of constraints (semi-infinite optimization). Intuitively, if d is a direction of descent for one constraint in a "sequentially related" list, it is also a descent direction for nearby constraints in the list. The CFSQP implementation of FSQP exploits this observation by computing successive search directions based on a small but significant subset of the objectives/constraints, with an ensuing reduced computing cost per iteration and a decreased risk of numerical difficulties. This subset is updated at each iteration in such a way that global convergence is ensured. This scheme dramatically accelerates execution times of problems with "sequentially related" constraints over the FORTRAN version.

Both FFSQP and CFSQP run on just about any platform. The authors incorporate modifications into new releases whenever they are notified of system compatibility problems. The software has been designed to be a valuable research and development tool and has enjoyed a good reputation as a reliable and versatile engine that can be integrated into other packages. One user has interfaced it to MATLAB and is reporting good results. Another user is developing an interface to Octave.

In closing, I include a list of applications that have used the package with success:

1. Minimizing reconstruction noise in Magnetic Resonance Imaging (MRI). Uses "sequentially related" constraints feature of CFSQP to reduce computational effort.
2. Obtaining the "best" description of clutter noise in over-the-horizon radar. The feasibility feature of FSQP is required for some of the models.
3. Dynamic Manipulation. Robotic manipulation planners that exploit dynamic effects rather than ignoring them or attempting to cancel them out.
4. Optimization-based design of hub-and-shaft assemblies for dual-wheel excavators.
5. Optimal Protein Separation. Using ionic strength as a control variable, a piece-wise constant optimal control problem is solved as a sequence of optimal parameter selection problems.
6. Parametric Surface Polygonization. A polygonal mesh representative of a surface is constructed.
7. "Estimating the dose effect for the analysis of intermediately lethal tumors," A. J. Rossini and L. Ryan, preprint, Pennsylvania State College of Medicine, Hershey, PA 17033.
8. "Synthesis of hierarchical traffic control systems," Ludmil Mikhailov, Technical report, Universite Libre de Bruxelles, Laboratoire d'Automatique, February 1993.
9. "Design of redundancy relations for failure detection and isolation by constrained optimization," Michel Kinnaert, preprint, Universite libre de Bruxelles, Laboratoire d'Automatique, July 1992.
10. Screening multi-purpose reservoir systems - optimization model for sizing and selecting among several potential reservoir sites.

If you are interested in more information about the software or the applications, please contact:

Prof. André L. Tits, Dept. of Electrical Engineering and Institute for Systems Research, University of Maryland, College Park, MD 20742, USA

Phone (301) 405-3669

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-FAIZ AL-KHAYYAL

JOURNALS

Volume 70, No. 1

A. Prékopa and W. Li, "Solution of and bounding in a linearly constrained optimization problem with convex, polyhedral objective function."

N. Garg and V.V. Vazirani, "A polyhedron with all s-t cuts as vertices, and adjacency of cuts."

K.G. Murty and S.-J. Chung, "Segments in enumerating faces."

J.E. Falk and J. Liu, "On bilevel programming, Part I: General nonlinear cases."

R. Schultz, "On structure and stability in stochastic programs with random technology matrix and complete integer recourse."

A.L. Dontchev, "Implicit function theorems for generalized equations."

G. Zhao and J. Zhu, "The curvature integral and the complexity of linear complementarity problems."

Volume 70, No. 2

J.F. Bonnans and A. Sulem, "Pseudopower expansion of solutions of generalized equations and constrained optimization problems."

A. Shapiro, "Directional differentiability of the optimal value function in convex semi-infinite programming."

D. Ralph and S. Dempe, "Directional derivatives of the solution of a parametric nonlinear program."

J.A. Hoogeveen and S.L. van de Velde, "Stronger Lagrangian bounds by use of slack variables: Applications to machine scheduling problems."

S. Schaible and J.-C. Yao, "On the equivalence of nonlinear complementarity problems and least-element problems."

A. Frank and Z. Szigeti, "A note on packing paths in planar graphs."

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Volume 70, No. 3

J.P. Dussault and Y. Gningue, "Unification of basic and composite nondifferentiable optimization."

P.E. Gill, W. Murray, D.B. Ponceleón and M.A. Saunders, "Primal-dual methods for linear programming."

J. Renegar, "Linear programming, complexity theory and elementary functional analysis."

Volume 71, No. 1

Y.T. Ikebe and A. Tamura, "Ideal polytopes and face structures of some combinatorial optimization problems."

S. Kim, K.-N. Chang and J.-Y. Lee, "A descent method with linear programming subproblems for nondifferentiable convex optimization."

M. Laurent and S. Poljak, "One-third-integrality in the max-cut problem."

C. Chen and O.L. Mangasarian, "Smoothing methods for convex inequalities and linear complementarity problems."

J. Brimberg, "The Fermat-Weber location problem revisited."

A. Auslender and M. Haddou, "An interior-proximal method for convex linearly constrained problems and its extension to variational inequalities."

E. Carrizosa and F. Plastria, "On minquantile and maxcovering optimisation."

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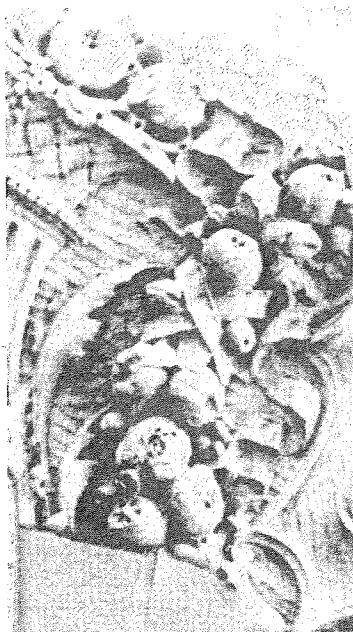
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Conference Notes

- ▶ **Third Workshop on Global Optimization, Szeged, Hungary, Dec. 10-14, 1995**
- ▶ **Conference on Network Optimization, University of Florida, Feb. 12-14, 1996**
- ▶ **Workshop on SATISFIABILITY PROBLEM: THEORY AND APPLICATIONS Rutgers University March 11-13, 1996**
- ▶ **5th SIAM Conference on Optimization, British Columbia, May 20-22, 1996.**
- ▶ **18th Symposium on Mathematical Programming with Data Perturbations, George Washington University, 23-24 May 1996.**
- ▶ **IPCO V, Vancouver, British Columbia, Canada, June 3-5, 1996**
- ▶ **Fifth International Symposium on Generalized Convexity Luminy-Marseille, France June 17-21, 1996**
- ▶ **IFORS 96 14th Triennial Conference, Vancouver, British Columbia, Canada, July 8-12, 1996**
- ▶ **IRREGULAR 96 Santa Barbara, California Aug. 19-23, 1996**
- ▶ **International Conference on Nonlinear Programming, Beijing, China, Sept. 2-5, 1996**
- ▶ **XVI International Symposium on Mathematical Programming, Lausanne, Switzerland, Aug. 1997**



forthcoming

conferences

The **EIGHTEENTH Symposium on Mathematical Programming with Data Perturbations** will be held at **George Washington**

University's Marvin Center on 23-24 May 1996. This symposium is designed to bring together practitioners who use mathematical programming optimization models and deal with questions of sensitivity analysis, with researchers who are developing techniques applicable to these problems.

Contributed papers in mathematical programming are solicited in the following areas:

1. Sensitivity and stability analysis results and their applications.
2. Solution methods for problems involving implicitly defined problem functions.
3. Solution methods for problems involving deterministic or stochastic parameter changes.
4. Solution approximation techniques and error analysis.

Call for Papers

"Clinical" presentations that describe problems in sensitivity or stability analysis encountered in applications are also invited.

Abstracts of papers intended for presentation at the Symposium should be sent in triplicate to Professor Anthony V. Fiacco. Abstracts should provide a good technical summary of key results, avoid the use of mathematical symbols and references, not exceed 500 words, and include a title and the name and the full mailing address of each author. The deadline for submission of abstracts is 17 March 1996.

Approximately 30 minutes will be allocated for the presentation of each paper. A blackboard and overhead projector will be available.

ANTHONY V. FIACCO, Organizer
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Irregular Workshop

The series of workshops on Parallel Algorithms for Irregularly Structured Problems - IRREGULAR'9X - started in Geneva in August 1994 and was held in Lyon in September 1995. These workshops address issues related to deriving efficient parallel solutions to irregularly structured problems.

In fact, efficient parallel solutions have been found to many problems. Some of these solutions can be obtained automatically from sequential programs using compilers. However, there still exists a large class of problems, known as irregular problems, that lack efficient solutions.

The aim of the IRREGULAR series is to foster cooperation among practitioners and theoreticians in the field. It covers such topics as approximated and randomized methods, automatic synthesis, branch and bound, combinatorial optimization, compiling, computer vision, load balancing,

parallel data structures, scheduling and mapping, and sparse matrix and symbolic computation.

The papers presented in Geneva were published in a book by Kluwer Academic Publishers. In 1995, the proceedings were published in *Lecture Notes in Computer Sciences* (LNCS) by Springer-Verlag.

IRREGULAR'96 will take place in Santa Barbara, California, from August 19-23, 1996. Its proceedings will be published again in LNCS and will be available at the workshop.

For the call for papers and further information, please contact one of the IRREGULAR co-chairs:

Afonso Ferreira
ferreira@lip.ens-lyon.fr

or Jose Rolim
rolim@cui.unige.ch

-PANOS PARDALOS

*First Announcement***Fifth International Symposium on Generalized Convexity
Luminy-Marseille, France
June 17-21, 1996**

After the NATO Advanced Study Institute on *Generalized Concavity in Optimization and Economics* in Vancouver, Canada (1980), and similar symposia in Canton, NY (1986), Pisa, Italy (1988) and Pécs, Hungary (1992), we are glad to announce Generalized Convexity 5. This symposium will be held at Centre International de Rencontres Mathématiques (CIRM), Luminy, near the Mediterranean Sea. In addition to CIRM, sponsors are the Mathematical Programming Society (MPS) and the recently founded Working Group on Generalized Convexity (WGC) within MPS.

This symposium attempts to solve open problems related to theoretical, algorithmic, computational and modeling issues in connection with generalizations of convexity, as they arise in mathematical programming, economics, management science, engineering, applied sciences, numerical mathematics, etc. An emphasis will be placed on the discussion of generalized monotonicity, a new area of research in the '90s, relevant to variational inequalities and equilibrium problems.

Furthermore, a special effort will be made to relate generalized convexity more closely to neighboring fields such as nonsmooth analysis, economic theory, complementarity theory/variational inequalities and stochastic programming. The following scholars have tentatively agreed to participate through tutorials in these areas: F. Clarke, A. Mas-Colell, J.S. Pang and R. Wets.

The International Scientific Committee of WGC serves as Program Committee and consists of S. Schaible, USA (chair), C.R. Bector, Canada, B.D. Craven, Australia, J.-P. Crouzeix, France, J.B.G. Frenk, The Netherlands, S. Komlosi, Hungary, J.E. Martinez-Legaz, Spain, and P. Mazzoleni, Italy.

A Second Announcement will be sent to those who preregister. If possible, use e-mail. Please contact the chair of the Organizing Committee:

Jean-Pierre Crouzeix (gcv5), Applied Mathematics, Université Blaise Pascal, E-63177 Aubière Cédex, FRANCE

E-mail:
crouzeix@ucfma.univ-bpclermont.fr

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**First International Conference on Complementarity Problems
Johns Hopkins University
Baltimore, MD**

The first international conference on complementarity problems was held at the Johns Hopkins University in Baltimore from November 1-4, 1995. The meeting was attended by over 50 researchers from around the world, including attendees from Australia, Belgium, Canada, the Czech Republic, Denmark, Finland, Germany, Great Britain, India, Italy, Japan, The Netherlands, Norway, Portugal, Sweden and the United States. The meeting was organized with the aim of bringing together researchers of the mathematical programming aspects of complementarity problems and experts in a variety of applications areas. The purpose of the meeting was to create increased cross fertilization and communication between these areas. In particular, a better understanding and appreciation of the different aspects that each of these areas considers is expected to be beneficial for the effective solution of practical complementarity problems arising from applied disciplines. As such, we believe that the meeting was a great success.

With more than three decades of research, the subject of complementarity problems has become a well-established and fruitful discipline within mathematical programming. Sources of complementarity problems are diverse and include many problems in engineering, economics, and the sciences. Several monographs and surveys have documented the basic theory, algorithms, and applications of complementarity problems and their role in optimization theory.

The meeting started with an overview of the complementarity field at which stage a new web site, CPNET, <http://www.cs.wisc.edu/cpnet/> was announced. When completed, this site will eventually contain up-to-date information on upcoming conferences in the area, a list of active researchers and pointers to work on algorithms, applications and software.

Currently, the page contains a list of all the researchers present at the conference, along with papers and software that outline some developments in the area. This includes the growing collection of test problems for MCP, MCPLIB, and the COMPLEMENTARITY TOOLBOX, a suite of programs and routines for use in conjunction with MATLAB. It is hoped that CPNET will allow this collection to grow considerably to include many new algorithms and application problems. There are some pointers to extensions of modeling software that allow real applications to be formulated in standard modeling languages.

There were various themes that developed during the meeting. Several speakers introduced new extensions of the basic framework and cited applications that needed such extensions. Some new theoretical results were outlined relating to vertical, horizontal and extended linear complementarity problems, along with several ideas to unify these areas. Other speakers considered noncooperative and stochastic game theory and outlined existence results and algorithms for their solution. Variational and bimatrix inequalities also drew the attention of several talks. Merit functions and smoothing techniques were also popular topics.

One extension that received considerable attention was the Mathematical Program with Equilibrium Constraints (MPEC). Several algorithms were given for the solution of these problems, and lots of discussion resulted during application talks relating to reformulating problems into this framework. This appears to be a very fruitful area for future research.

Contact problems are a rich source of complementarity problems. For these problems, complementarity is the result of the contact condition which stipulates that the gap between two objects in contact is either zero, or the pressure between them is zero. Classical obstacle problems were extended to include the effects of convection and diffusion. An interesting use of complementarity in contact mechanics arises in robot design, and key features of the problem that can be modeled in the new framework include sliding, friction and rigid body properties. Structural mechanics has also used complementarity models in studies of material elasticity and plasticity. Several very informative and interesting talks opened up these areas to the field in general.

Complementarity has been used in economics for a long time. The renowned Walrasian law of supply and demand in general equilibrium theory states that either there is excess supply or the price of the corresponding good is zero. Several extensions of this basic idea were outlined in talks that dealt with oligopolistic equilibria, integrated assessment for problems in energy modeling, relocation effects due to the European Common Market and the National Energy Modeling System (NEMS). The use of similar models for traffic assignment was also outlined. In this area, dynamic models are becoming important, and several new ideas for tolling and congestion analysis were presented at the meeting.

Several new algorithmic developments were outlined. Some of these involved the traditional simplicial and pivotal based techniques while others used novel reformulations of the complementarity problem both as smooth and nonsmooth systems of nonlinear equations. A very popular approach takes systems of nonsmooth equations and applies a smoothing so that traditional Newton based techniques could be applied. Still other methods were based on quadratic programming and proximal points formulations. New computational extensions were also outlined. Several talks introduced new merit functions that will prove useful in error analysis and future algorithmic design.

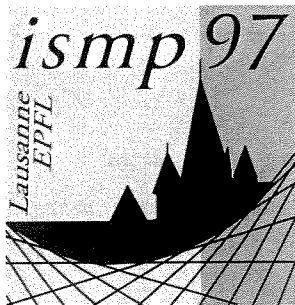
In conclusion, the meeting showed that the field of complementarity research is a burgeoning area. There are already many interesting algorithms for solving complementarity problems, along with fairly sophisticated techniques for analysis and computation. The growth in the number of new application areas that use this framework will require even more sophisticated solution techniques. Furthermore, it is clear that even more applications will be developed that use complementarity modeling in some form or other, a significant portion of which was made possible by this meeting.

A refereed proceedings of this meeting will be published in 1996 by SIAM. Further developments in this area will undoubtedly be reported at the next International Conference on Complementarity Problems. Planning is already underway, and the conference is tentatively set for July 1998 to be held in Madison, Wisconsin.

Michael Ferris
Computer Sciences Department,
University of Wisconsin, Madison

ferris@cs.wisc.edu

Jong-Shi Pang
Department of Mathematical Sciences
The Johns Hopkins University, Baltimore
jsp@vicp.mts.jhu.edu



International Symposium on Mathematical Programming

Lausanne, Switzerland, August 24-29, 1997

First Announcement

The International Symposium on Mathematical Programming is the triennial scientific meeting of the Mathematical Programming Society. The 16th Symposium will be held at the Swiss Federal Institute of Technology in Lausanne, Aug. 24-29, 1997, the year of the 50th anniversary of George Dantzig's Simplex Method for Linear Programming.

Organizing Committee: Chair: Th.M. Liebling, D. de Werra, K. Frauendorfer, K. Fukuda, H. Gröflin, A. Haurie, A. Hertz, P. Kall, J. Kohlas, B. Lara, H.-J. Lüthi, D. Naddef, P. Neusch, F.-L. Perret, A. Prodon, P. Stähly, J.-P. Vial, M. Widmer. D. Müller (Head coordinator). **International Advisory Committee:** Chair: D. de Werra, R. Ahuja, M. Akgul, K. Al-Sultan, E. Allgower, K.M. Anstreicher, J. Araoz, M. Avriel, A. Auslender, A. Bachem, E. Balas, M. Balinski, A. Ben-tal, D.P. Bertsekas, C. Berge, R. E. Bixby, P. Bod, A. Buckley, R. E. Burkard, V. Chandru, S.Y. Chang, S.J. Chung, V. Chvátal, A. R. Conn, R. Correa, G.B. Dantzig, M.A.H. Dempster, J.E. Dennis Jr., L. C.W. Dixon, J. Dupacova, B. C. Eaves, Y. Ermoliev, S.C. Fang, R. Fletcher, A. Frank, S. Fujishige, S. Gass, F. Giannessi, Ph. Gill, J.-L. Goffin, D. Goldfarb, C.C. Gonzaga, N. I.M. Gould, R.L. Graham, M. Grötschel, H.W. Hamacher, P.L. Hammer, A.J. Hoffman, K.L. Hoffman, M. Iri, A.N. Iusem, E. L. Johnson, J. Judice, S.N. Kabadi, R. Kannan, N. Karmarkar, R.M. Karp, A.V. Karzanov, L. Khachiyan, V. Klee, M. Kojima, H. Konno, B. Korte, J. Krarup, H.W. Kuhn, C. Lemarechal, J. K. Lenstra, P.O. Lindberg, F. Louveaux, L. Lovász, Th.M. Liebling, F.M. Maffioli, T.L. Magnanti, S. Maya, F. McDonald, N. Megiddo, K. Mehlhorn, G. Mitra, S. Mizuno, S.R. Mohan, B. Murtagh, G.L. Nemhauser, J. Nocedal, M.W. Padberg, J.-S. Pang, K. Paparizos, P. Pardalos, C. Perin, B. Polyak, M.J.D. Powell, A. Prekopa, W. R. Pulleyblank, L. Qi, M.R. Rao, A.H.G. Rinnooy Kan, R.T. Rockafellar, J.B. Rosen, H.E. Scarf, R.B. Schnabel, A. Schrijver, N.Z. Shor, J. Stoer, E. Tardos, J. Tind, M.J. Todd, Ph. L.M.J. Toint, P. Toth, A. Tucker, H. Tuy, S. W. Wallace, A. Weintraub, R.J.-B. Wets, H.P. Williams, P. Wolfe, L.A. Wolsey, M.H. Wright, S. Wright, Y. Ye, M.Y. Yue, J. Zowe. **Symposium Advisory Committee:** Chair: B. Korte, J.R. Birge, C.C. Gonzaga, A. Schrijver.

Tentative list of topics

Sessions on the following topics are planned.

Suggestions for further areas to be included are welcome.

1. Linear, integer, mixed-integer programming
2. Interior-point and path-following algorithms
3. Nonlinear, nonconvex, nondifferentiable, global optimization
4. Complementarity and fixed point theory
5. Dynamic and stochastic programming, optimal control
6. Real-time optimization
7. Game theory and multicriterion optimization
8. Combinatorial optimization, graphs and networks, matroids
9. Computational complexity, performance guarantees and quantum computation
10. Approximation methods, heuristics
11. Local search, simulated annealing, tabu search, etc.
12. Computational geometry, VLSI-design
13. Computational biology
14. Implementation and evaluation of algorithms and software
15. Large-scale mathematical programming
16. Parallel computing in mathematical programming
17. Expert, interactive and decision support systems, neural networks, fuzzy logic
18. Simulation, optimization in discrete event simulation
19. Mathematical programming on personal computers
20. Teaching in mathematical programming
21. Applications of mathematical programming in industry, government, economics, management, finance, transportation, engineering, energy, environment, agriculture, sciences and humanities

Site

The Symposium will take place at the Swiss Federal Institute of Technology (EPFL) in Lausanne, Switzerland. Hotels of various categories as well as low price accommodations will be available.

Preregistration

It is necessary to preregister in order to be included in our mailing list. This procedure is free of charge. The second announcement, due to appear in 1996, will be sent only to those who preregister.

(See reverse side)

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(We strongly encourage you to use it).

Mailing address

ISMP 97
c/o Prof. Thomas M. Liebling
EPFL-DMA
CH-1015 Lausanne (Switzerland)
phone: + 41 21 693 2595
fax: + 41 21 693 4250

Dates & deadlines

Aug. 31, 1996: Preregistration

Sept. 1, 1996: Second announcement

April 30, 1997: Submission of titles and abstracts and early registration

Aug. 24-29, 1997: The Symposium

Call for papers

Papers on all theoretical, computational and practical aspects of mathematical programming are welcome. The presentation of very recent results is encouraged. All abstracts will be available via WWW.

Structure of the meeting

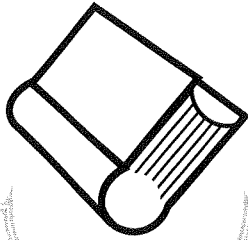
A large number of people will be invited to organize sessions. Other people may, from their own initiative, propose themselves as session organizers. If they do so, they must write us and once our program committee has agreed, they will be included in the list. During the plenary opening session, the following prizes will be awarded: Fulkerson Prizes (for outstanding papers in discrete mathematics), Orchard-Hays Prize (for excellence in computational mathematical programming), A.W. Tucker Prize (for an outstanding paper by a student).

Social program

In addition to the official program, social activities will be organized for the participants, their family members and friends. This will include a lake cruise and banquet, visits to museums, a concert, an excursion to a cheese factory, wine-tasting, sightseeing, hiking, etc....

BOOK

R E V I E W S



Dynamic Policies of the Firm

by O. van Hilten, P. Kort,
P.J.J.M. van Loon
Springer Verlag, Berlin, 1993

ISBN 3-540-56125-0

DURING the last 25 years optimal control theory has developed as a standard tool in dynamic economics. In particular, *the dynamics of the firm* is now one of the applications *par excellence* of the maximum principle.

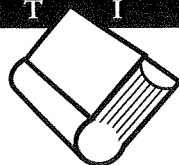
The aim of the dynamics of the firm is to study the growth and contraction of representative firms in a microeconomic framework. Among its main issues are optimal investment, financing, production policies, impact of taxation, technological progress and environmental constraints. Early researchers, like Albach (1976), have stressed that the development of the firm over time can be divided into different stages or regimes. In order to understand these regimes in a proper way, optimal control theory delivers a useful framework.

According to Lesourne and Leban (1982) optimal control of the dynamics of the firm is an indispensable instrument to improve the understanding of management policies, to enable government to assess the impact of its policy on the firm and to provide academic teachers with a tool to outline the essentials of the firm. One of the aims of the theory lies in management training. There is a discussion of how debt financing can facilitate expansion and how unit costs, prices of capital goods and investment grants influence investment decisions. Today, it is important for the man-

ager to know how optimal investment decisions have to take into consideration not only taxation and technological progress but also business cycles and environmental pollution.

As indicated above, the mathematical tool used to derive optimal policies of the firm is Optimal Control Theory. The maximum principle formulated by Pontryagin, and proved by his coworkers Boltyanskii, Gamkrelidze and Mishchenko in the 50s, yields necessary (and sometimes also sufficient) optimality conditions which allow a characterization of the optimal solution trajectories. Two observations are important. First, the approach is qualitative, i.e. the model functions and solutions are characterized qualitatively rather than quantitatively. Second, based on these optimality conditions, an iterative solution procedure, the so-called path-connecting procedure, is developed. This provides the possibility for constructing and interpreting the optimal solution for the entire planning period in an analytical way.

This book is divided into five parts. Part A provides a survey of dynamic theories of the firm and describes several predecessors of the models presented. In Part B the basic model is explained and used to discuss optimal investment and financing behaviour. Part C deals with production and activity analysis. The representative firm has to choose between production techniques with different characteristics. In particular, Chap. 8 discusses the 'hot topic' on environmental pollution and cleaning activities. In Part D the firm is faced with an 'outside world' changing over time. These changes are encapsulated in technological progress, a business cycle and a stochastic demand function. The rest of the book contains six appendices, mainly an economic interpretation of the maximum principle and technical



details of the solution procedures of previous chapters. The book uses the so-called direct method in dealing with pure state constraints. The last part of the book in which management problems in a dynamic environment are analyzed seems to open interesting new possibilities for further research.

This book is the outgrowth of what today might be called the 'Netherlands school' of the applications of optimal control theory to dynamic economics. Piet Verheyen, Paul van Loon, Geert-Jan van Schijndel, Peter Kort, Raymond Gradus, Onno van Hilten and others are members of this school concentrated at Tilburg University. Van Loon (1983) also wrote the first text book on the dynamics of the firm. These scholars continued the work of Bensoussan et al. (1974), Ludwig (1978), Lesourne and Leban (1982).

The reviewer of this book is reminded of the first encounter with the 'dynamics of the firm.' It was at a seminar at the Institute for Advanced Studies in Vienna given by Horst Albach in the late '70s. He presented the 'path connection method' as the pedagogical method for management training. Had it not been for this seminar, the book by Feichtinger and Hartl (1986) (published in German), in which several models of the dynamics of the firm are described might not have been written. In management training, control theory models are excellent for showing students or junior managers how to combine policies through time to define an efficient strategy. This book by van Hilten, Kort and van Loon is the main reference in this field.

If you had a graduate student who would like to learn the core theory of optimal control theory applied to economics, what single book would you recommend? In my view the book by van Hilten et al. covers central aspects of dynamic economics at a level accessible to applied scientists. Theory is presented systematically, and only a modest mathematical background is required in keeping with the target audience.

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-GUSTAV FEICHTINGER

Discrete and Fractional Programming Techniques for Location Models

by Anna Isabel Martins Botto de Barros
Tinbergen Institute Research Series 89,
Amsterdam, 1995

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IN DISCRETE LOCATION, one is interested in locating a facility or facilities on a finite set of points. In fractional programming, one is interested in optimizing a program where the objective function is a ratio of a numerator and denominator function (such functions may also occur in the constraints). In this monograph, the author integrates these two concepts to study a variety of models of interest in location theory.

Chapter 2, entitled Discrete Location Models, reviews the well-known uncapacitated facility location model and its extensions to two levels and two echelons. In the two level case, fixed costs are associated with each level; in the two echelon case the fixed cost in the second echelon is jointly determined by the first and the second echelon. It is shown that the submodularity property does not hold for the two echelon case. The fixed cost structure in both the above cases is then aggregated into a new model which generalizes the above. A variety of bounds using both linear and Lagrangian

relaxation are established. Together with heuristics they are used in a branch and bound algorithm, and extensive computational results are given.

Chapter 3 integrates ideas in fractional programming with the location models discussed in Chapter 2. It is explained that the traditional criterion of maximizing profit is sometimes replaced by a profitability index defined by the ratio of the present value of a project and the investment made. A brief overview of fractional programming is then made with emphasis on Dinkelbach's algorithm and its extension to integer programming. At this point the author reformulates the location models in Chapter 2 with a ratio objective as described above. Using basic ideas from Dinkelbach, some structural results are given.

The author moves her focus from discrete optimization models in location to generalized fractional programming for continuous variables in Chapter 4. The chapter begins by reviewing ideas of Crouzeix et al. that extend Dinkelbach's approach to generalized fractional programs. An example involving allocation to a distributed service network is then given. This motivates the study of generalized fractional programming with a constraint on the denominator. A Dinkelbach-type algorithm is developed. The author then turns to the dual problem as an alternative vehicle to solve generalized fractional programs. A new algorithm is given along with various convergence results, once again extending Dinkelbach's parametric ideas. A further generalization to nonlinear convex constraints is made. I was particularly pleased to see that algorithms in this chapter rely heavily on versions of the dual program. The chapter concludes with extensive computational testing of ratios of positive definite quadratic functions and affine functions.

In summary, the book should be interesting to those in fractional programming who are interested in seeing where the theory can be applied. It should also be interesting to those in location who may not be aware of the potential of theoretical and computational ideas from fractional programming in their field. Overall, it is a valuable addition to the literature of the field.

-CARLTON SCOTT

O P T I M A

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Karen Aardal, FEATURES EDITOR
Utrecht University
Department of Computer Science
P.O. Box 80089
3508 TB Utrecht
The Netherlands
e-mail: aardal@cs.ruu.nl

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Dolf Talman, BOOK REVIEW EDITOR
Department of Econometrics
Tilburg University
P.O. Box 90153
5000 LE Tilburg
The Netherlands
e-mail: talman@kub.nl

Elsa Drake, DESIGNER

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48

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At the fall INFORMS meeting, the Lanchester Prize was awarded to **Richard W. Cottle, Jong-Shi Pang** and **Richard E. Stone** for their book, *The Linear Complementarity Problem*. See OPTIMA N°45 for a review. ¶There will not be a separate mailing of the first announcement for the 1997 International Symposium in Lausanne. It is found on Pages 11 & 12 of this issue. Members are urged to register for subsequent announcements. ¶Deadline for the next OPTIMA is Feb. 15, 1996.

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