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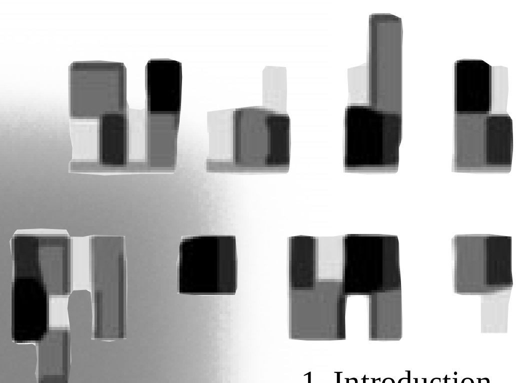
Combinatorial Online Optimization in Practice

Abstract *This paper gives a short introduction to combinatorial online optimization. It explains a few evaluation concepts for online algorithms, such as competitiveness, and discusses limitations in their application to real-world problems. The main focus, however, is a survey of combinatorial online problems in practice, in particular in large scale material flow and flexible manufacturing systems.*

Keywords: Online optimization, combinatorial optimization, real-world problems



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1. Introduction

In classical optimization, called *offline optimization* here, it is assumed that all input data of an instance are available before solution algorithms are applied. In many applications this is not realistic. Decisions have to be made before all data are known. Such situations are often termed *online*. They arise in particular in processes that are continuously running for a longer period of time. For instance, in material flow systems of companies, transportation tasks arise throughout the day and decisions have to be made before all jobs have been generated.

Online optimization is the task of finding "good," or "cheap," or "economic" decisions in online situations. An *online algorithm* provides such decisions. In practical applications online algorithms are typically subject to additional constraints. For example, they have to answer in real time, or must process a job (or request) within a given time frame, sometimes they have access to limited computing resources only.

Competitiveness. A common concept to evaluate online algorithms is *competitiveness*. Here, an online algorithm has to act as follows: it always has to serve a request of a sequence before the next request (or the next k different requests in a model with lookahead) becomes visible. The idea behind the variants of this notion is the following: compare the solution of the online algorithm under consideration with the solution that some adversary would produce on the same set of data.

The easiest case is the offline adversary, i.e., to him the complete sequence of requests is known in advance. For an algorithm X let C_Xs denote the cost X produces on input sequence s. We assume for notational convenience that C_Xs is positive for all s

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and that we want to minimize. A deterministic online algorithm A is *c-competitive* if for any sequence s of requests and any offline algorithm S

$$C_A s < c \cdot C_S s + a$$

holds for real numbers c,a, both not depending on~s.

The goal is to find online algorithms that are competitive in an optimal way, i.e., no other online algorithm can have a better performance ratio with the adversary. This concept is applicable both to deterministic and randomized algorithms, and it allows for provable statements about the performance of an online algorithm. A further advantage of this concept is that no information about the distribution of the input data is needed in order to make exact statements. Competitive analysis has been the subject of many investigations concerning (mainly elementary) online problems. (See, among others, Albers 1996; Albers 1997; Goemans 1994; Irani and Karlin 1997; Motwani, Raghavan 1995; and Ottmann et al. 1994 for more information.)

The competitiveness ratio is usually a pessimistic measure since the adversary is supposed to be a bad person trying to fool the algorithm by designing a particularly difficult sequence of requests. Moreover, competitive analysis is based on some hard restrictions to the model: one assumes that the next request does not become available before attending to the current request. In practice, however, there is often a dynamically growing and shrinking pool of requests of unknown size visible to the algorithm.

Sometimes competitive analysis provides absolutely no insight into the quality of an online algorithm. For instance, for a version of the greeting card commissioning problem to be discussed later, one can prove that all (reasonable) online algorithms have the same competitiveness factor K (the common capacity of the vehicles of the system) (see Kamin 1998).

Furthermore, the competitive analysis is not applicable if the decisions of the online algorithm have direct impact on the sequence of future requests.

Stochastic Optimization. Stochastic optimization uses a model that seems to be close to what we might call "reasonable acting under incomplete information." The decisions in the online algorithm are made based on the optimal solution of a *stochastic program* that has to be solved beforehand. Usually, this is a linear program where the objective function is the expectation of the cost function in the decision and request variables.

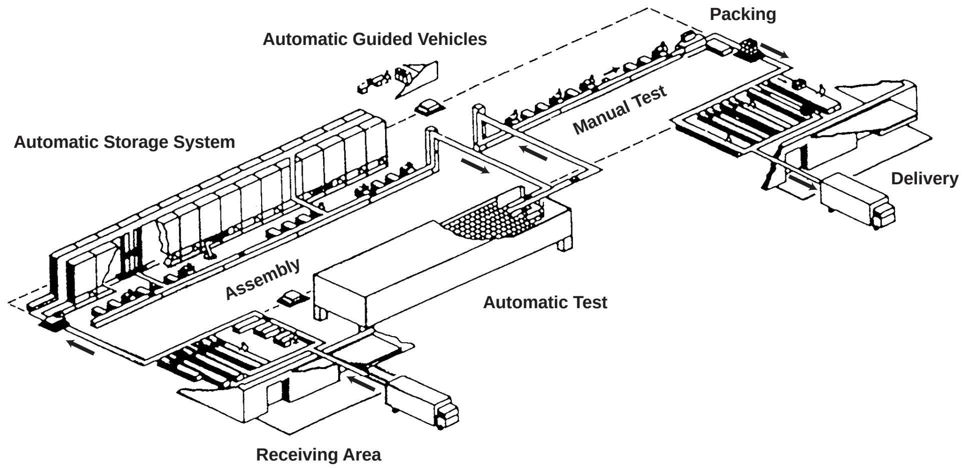

Figure 1. Sketch of the Factory Layout

This can only be done under the assumption that the request data has a certain distribution. Making decisions beforehand relying on statistical data can be viewed as an offline model of an online problem since the distribution allows for computing the expectations before the actual requests occur. There are also models that perform a certain number of alternating observation and optimization steps in order to adjust the information about the distribution. These models are, however, hard to evaluate in practice. Stochastic analysis is–as the name might suggest anyway–focusing on the average behavior of the algorithm. (See Prékopa 1995, and Kall and Wallace 1994 for more background and applications in this area.)

Since stochastic analysis requires an idea about what the distribution of the incoming requests may look like, it is not applicable if one cannot find any structure in the input data. Moreover, if the probability is not concentrated at the expectation, then the decisions that correspond to solutions from optimizing the expectation of the cost function may bear too large a risk of failure. Usually, there is no guarantee that the algorithm works well for any sequence of requests that might occur.

Simulation. This is the approach used in practice. A coarse mathematical model of the online problem is developed and implemented in the form of a simulation model. Several online algorithms are coded and experiments with real-world data are run on a computer to gain insight into the practical performance. In particular, experiments are made to analyze high load and failure situations. Usually those online algorithms that exhibit good average performance and can somehow cope with "catastrophes" are selected for use in practice.

A-posteriori Analysis. This approach uses history. The actual sequences that came up in the past are recorded and competitive analysis is made based on the gathered data only. This evaluation is often used to tune the online algorithms so that they perform better on these known input sequences. One hopes that the old input sequences are good representations of the typical data and that hence the modified algorithms will show improved performance also on future input sequences.

2. Application to Real-World Problems

Besides the theoretical attractiveness of online optimization problems, there is a broad variety of real-world problems that can be modeled as an online problem. In the sequel we outline problems we encountered in joint projects with industry that were aimed at optimizing the internal material flow within a flexible manufacturing system (FMS) and a distribution center. We just like to mention that, among others, there exist further applications in computer science, vehicle routing, scheduling, and telecommunication that will not be discussed in this paper.

2.1 Online Optimization of a Flexible Manufacturing System (FMS)

Siemens Nixdorf Informationssysteme AG (SNI) maintains a production plant where all their personal computers (PCs) and related products are assembled. (See Figure 1 for a sketch of the material flow within this FMS.) Parts that are used to produce PCs (PCB, floppy disk, cables, etc.) enter the FMS at the receiving area in normed containers. They are brought by automatic guided vehicles (AGV) into one of six automatic storage systems (AUSS). The AUSS serve as material buffer between the receiving area and the assembly lines located at each side of the AUSS. After assembly the PCs enter a test area where for up to 24 hours test programs are run in order to check the full functionality of the PCs. After a manual test the PCs are packed and delivered.

Optimization Problems. A profound analysis of the system showed that it offers a variety of optimization problems, some of them of an online character. These mainly are: scheduling of transportation tasks within the AUSS; assignment of containers to storage locations; routing and scheduling of the AGV; assignment of locations in the test area; retrievals of PCs from the test area. Here, the first question will be discussed in more detail. Discussions of the other topics can be found in Abdelaziz 1994, Ascheuer 1995, and Krippner Matejka 1993.

2.1.1 Stacker Crane Routing in the Automatic Storage Systems

The AUSS are single-aisled with storage locations on both sides of the aisle. In the lower part there are buffer places where containers are provided to the assembly line. A single stacker crane has to fulfill all transportation tasks (jobs). So far, a certain priority was assigned to each task (storage, retrieval, buffer-refill, etc.). This priority was only dependent on the type of the task. Within one priority class, jobs were sequenced due to a FIFO-rule. Although easy to implement, this strategy resulted in a high percentage of unloaded travel time.

Since every algorithm has to process all the jobs, we can only control the unloaded moves of the stacker crane. We suggested sequencing the tasks in such a way that the total time needed for the unloaded moves between the jobs is minimized. This can be modeled as an *asymmetric traveling salesman problem* (ATSP) where each job that is not performed, together with the job that is currently processed by the stacker crane, is represented by a node in a complete digraph $D=(V,A)$. W.l.o.g. we assume that the current job corresponds to node 1. Each arc $(i,j) \in A$, $j \neq 1$, represents the unloaded move between jobs i and j. This arc is given a weight corresponding to the time needed for the unloaded move from the endpoint of job i to the starting point of job j. To all arcs $(i,1)$, $i \in V \setminus 1$, we associate weight 0. Now, an optimal tour through the nodes of $D=(V,A)$ corresponds to a sequence of the transportation tasks with minimal total unloaded travel time.

This is an online problem since not all transportation tasks (resp. nodes for the ATSP) are known in advance. They are generated during the production period and neither generation time nor start- and end-coordinates are known in advance, i.e., we have to solve an *online-ATSP*. A detailed discussion of this topic can be found in Ascheuer 1995. (See, e.g., Ausiello et al. 1994a, and Ausiello et al. 1995a for competitiveness results on the online ATSP.)

Solution Approach. We decided to simply ignore tasks that might be generated in the future and to solve a "static ATSP" as soon as a new job is generated. In order to avoid the stacker crane having to wait until we have finished our calculations, we have implemented a 3-phase process. Whenever a new job is generated we run the following optimization process:

- Phase 1. Simple insertion heuristic. Try to insert the new node as cheaply as possible into the current sequence;
- Phase 2. Run a more sophisticated heuristic. We have chosen a random insertion heuristic;
- Phase 3: Solve the ATSP to optimality. This is done using a branch & bound-implementation of Fischetti and Toth (Fischetti and Toth 1992).

Phase 1 runs in $O(n)$ time and is always completed. For the typical problem sizes that occur in our application ($n \leq 60$), the computations are done in fractions of a second. Even phase 3 was always completed within a few seconds.

After the completion of each phase, a sequence is available that can be improved by one of the subsequent phases. If, during the execution of phase 2 or 3, a new job is generated, then the whole process is stopped and restarted. If the stacker crane has finished a task and asks for a new one, the process is interrupted as well and the best sequence so far is passed to the control system of the stacker crane.

We tested several heuristics to be used in phase 2 (See Abdelhamid, Ascheuer and Gröetschel 1998). It is easy to construct examples where it does not always lead to the best solution if each ATSP is solved to optimality. The use of this strategy might construct sequences that are "not good" with respect to the nodes generated in the future. Nevertheless, phase 3 empirically gives the best results on the average.

Computational Results. SNI provided data for one week of production. During this period, each generated task and each move of the stacker crane was recorded at one AUSS. This data was used to validate the simulation model. Based on the SNI data we compared several strategies for sequencing the jobs within the simulation environment.

Extensive computational tests showed that it was possible to reduce the times needed for unloaded moves by approximately 30% in heavy load periods. As a result, this optimization package was put in use at five AUSS and the results were confirmed in everyday production. It showed that even in the production environment the optimization process could always finish with phase 3.

Quality of the Online Solutions. A scientific question that arises is: how good are the solutions in comparison to an optimal offline solution? To evaluate the quality we performed an *a-posteriori analysis*, i.e., we determined how we would have sequenced the tasks had we known which tasks were generated.

To this end we "collected" all jobs over a certain time period and sequenced them optimally. First note that the jobs cannot be sequenced earlier than they are generated. Moreover, the completion of the jobs cannot wait too long as, e.g., the production might be delayed. Thus, to each job a time window is associated and we only allow to visit a node within its time window (ATSP-TW) (See, among others, Desrochers et al. 1988 and Desrosiers et al. 1995). The ATSP-TW is a difficult combinatorial optimization problem where it is even strongly *NP*-complete to find a feasible solution (Garey and Johnson 1977, Savelsbergh 1985). We have developed a branch & cut-approach for the ATSP-TW (Ascheuer, Fischetti and Gröetschel 1997; Ascheuer, Fischetti and Gröetschel 1998) and a relaxation, namely the ATSP with precedence constraints (Ascheuer, Juenger and Reinelt 1997). The optimal solutions to these problems yield a lower bound to an optimal online strategy if the same sequence of nodes is generated. Computational tests based on the production data from SNI showed that there is still an online optimality gap of between 3-70%, with approximately 30% on average.

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We like to point out one important restriction of this a-posteriori analysis. This analysis is based on the fact that the same sequence of nodes is generated, independent of the way the jobs are performed. This is not necessarily the case for this application as the completion of a certain job may have an influence on other generated tasks. For example, consider the case that a container delivered to the assembly line (task A) contains parts that are not usable (e.g., they are broken). As a result, the workers generate a retrieval task B and order new parts (task C). The sooner task A is performed, the earlier tasks B and C are generated, and the earlier the time window for B and C will become active. Thus, there is no well defined *optimal offline solution* to which we can compare the online solution. As a consequence, competitive analysis cannot be applied.

2.2 Online-Optimization of a Distribution Center

The Herlitz PBS AG (WWW Herlitz) is the main manufacturing firm for office supplies in Germany. They maintain their Europe-wide distribution center in Falkensee, close to Berlin. A joint project is aimed at efficiently managing their complete internal material flow. In a first phase we have optimized one commissioning area. We are currently working on the optimization of the whole pallet transportation system consisting of a system of roller conveyors, ten elevator systems and 18 AUSS.

Online optimization questions arise in the following areas: routing of pallets; efficient control of elevator systems; routing within the AUSS; routing of commissioning vehicles.

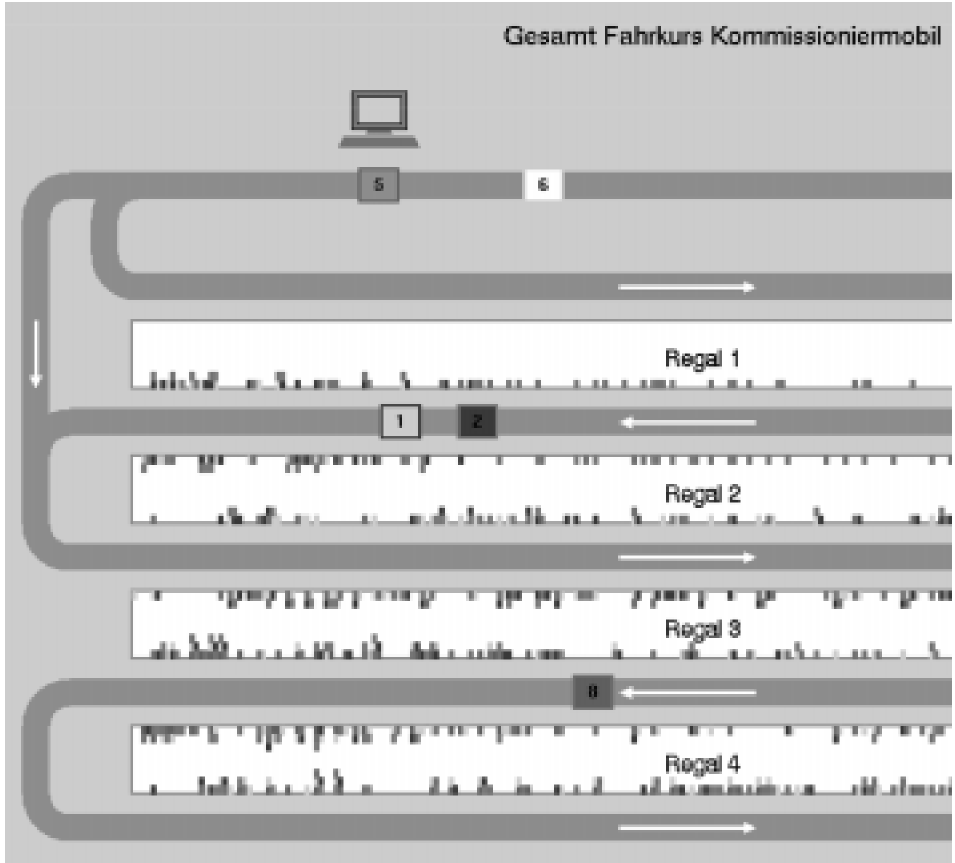

Figure 2: Commissioning Area for Greeting Cards *(screenshot from the simulation program)*

2.2.1 Commissioning of Greeting Cards

In this section we discuss one question in further detail, namely the efficient commissioning of greeting cards.

Description of the System. The cards are stored in four parallel shelving systems (see Figure 2). In accordance with the customers' orders, the different greeting cards have to be collected in boxes to be shipped to the customers. Order pickers on eight AGV collect the orders from the storage systems while following a circular course. The vehicles are unable to pass each other. Moreover, due to security reasons, only two vehicles are allowed to be in the middle aisles at the same time, whereas three are allowed in the first and last aisle.

At the loading zone each vehicle is "loaded" with up to 19 orders. Afterwards, a dispatcher decides when to send the vehicle onto the course. After leaving this area the vehicles automatically stop at a position where cards have to be picked from the shelf. Signal lights indicate the position from where and to which box the cards are picked.

The management was unhappy with the system since frequently vehicles ran into congestions and orders were completed late. For example, suppose that there is a vehicle that requires a lot of stops and the subsequent one only has a few stops. In case the dispatcher sends them onto the course, the fast vehicle will catch up with the slow one immediately, resulting in a congestion. As a consequence, the order pickers of fast vehicles often left the AGV to smoke a cigarette, etc., which resulted in further congestions.

Modeling. We suggested assigning the orders to the vehicles in such a way that whenever a vehicle stops the order picker can collect as many cards as possible; alternatively, for a given set of orders, minimize the total number of stops to fulfill these orders. In this way it is possible to avoid some time-consuming deceleration, fine adjustment, and acceleration phases for the vehicles. Besides minimizing the total number of stops, we aim at reducing the time vehicles spend in congestion. This can be modeled as a mixed integer program. First computational test showed that for some data sets provided by Herlitz it took several hours of CPU-time just to solve the linear relaxations of the MIP. Thus, an exact solution approach was unsuitable for a deployment in the distribution center.

It could be shown that already the problem of minimizing the total number of stops is *NP*-hard (Kamin 1998). Therefore, we implemented several heuristics that reduce the total number of stops required for the vehicles and evenly distribute these stops among them. We used variants of greedy- and best-fit-algorithms with an additional 2-exchange improvement heuristic. In addition, we used a coarse simulation to determine the best starting time for each vehicle. By this optimization-simulation approach, predictable congestions are shifted to the loading zone, where the order pickers can either have a break or can be assigned other tasks.

Results. We implemented a very detailed simulation model for the whole commissioning area in which we compared our approach to the one used so far. Herlitz provided production data from a period of about six weeks, which were the basis for the comparison. The main results are the following: a significant improvement with respect to the completion times of the orders can be achieved; the number of vehicles can be reduced from eight to six without any negative impact on the system performance; congestions can more or less be avoided completely. Vehicles run into congestions only for a few seconds.

More details can be found in Ascheuer, Gröetschel, and Kamin 1998; and Kamin 1998. A prototype of the simulation approach is currently tested by the support team of Herlitz for its use as a decision support tool for the dispatcher.

2.3 Challenges

Conveyor Modules in Large Scale Transportation Systems. In connection with another cooperation with Herlitz AG, Berlin, we are analyzing the following problem: the automated pallet transportation system in a large dispatch building of Herlitz in Falkensee has to take care of a congestion-free flow of pallets from/to ware-input, commissioning departments, shelf system, and ware-output. Among the building blocks for pallet transportation, the following seem to be the most complex ones: the automated shelf systems; the automated elevator systems. Modules of these types are found in many automated transportation systems.

One would like to describe how different modules of such a system must be controlled in order to work well together. The common practice is to run very simple heuristics with emphasis on avoiding congestion.

Prior to the investigation of the interplay between the modules it is necessary to understand the modules themselves. While for the automated shelf systems we can use our experience from the above mentioned project with SNI, the elevator control problem is not well understood so far. This is even the case in very elementary settings, let alone real-world layouts with additional restrictions to the flow from/to the elevators.

A generic simulation environment for elevator systems based on the event based simulation library (AMSEL 1997) was designed in order to test heuristic approaches to the problem (see Figure 3).

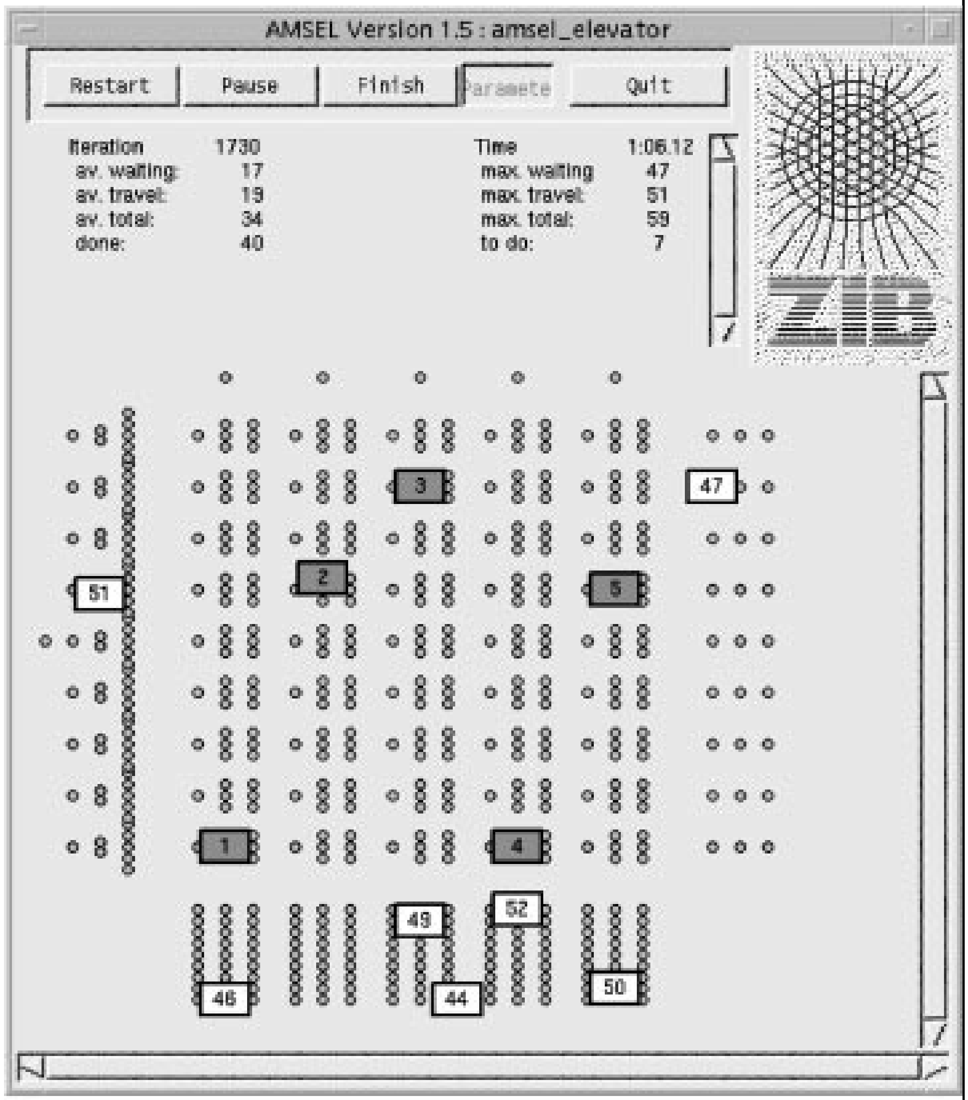

Figure 3: A Snapshot of the Animated Simulation of an Elevator System

Conceptual Problems. Competitive analysis is a mathematical performance measure of online algorithms. It has the advantage of not being dependent on the knowledge of the probability distribution of the requests. However, the online model that it is based on is too restrictive for many real-world problems. For an elevator system, e.g., there are usually many requests available at the same time, and the elevator has the opportunity to make an offline schedule based on the known information. Moreover, not yet processed requests may be rescheduled by the algorithm. This *dynamic look ahead* should be integrated into a generalization of competitive analysis.

A very hard problem occurs if there is no corresponding offline problem at hand. In these cases even the definition of what should be an optimal solution to the online control problem is problematic. In control theory one computes an optimal control at each point in time. Usually one cannot ensure that these local optima combine to a globally “optimal” solution if the problem is discrete because the objectives are not continuously dependent on the decisions.

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3 Conclusion and Outlook

Online problems show up almost everywhere in industrial production, logistics, etc. We have illustrated this by means of a few relevant examples from practice. Online optimization problems have, however, not received too much attention from the mathematical programming community yet.

The field lacks "good" mathematical concepts for decision support. From a practical point of view, competitive analysis as well as similar approaches rarely yield results that can guide decision makers in the selection of which online algorithm to use. Simulation experiments are still the state of the art.

Nevertheless, by using the tools that have been developed in combinatorial optimization over the years, such as combining and modifying various heuristic and exact approaches for associated offline problems, it is still possible to improve considerably on what is currently done in practice, as our examples show.

Acknowledgments

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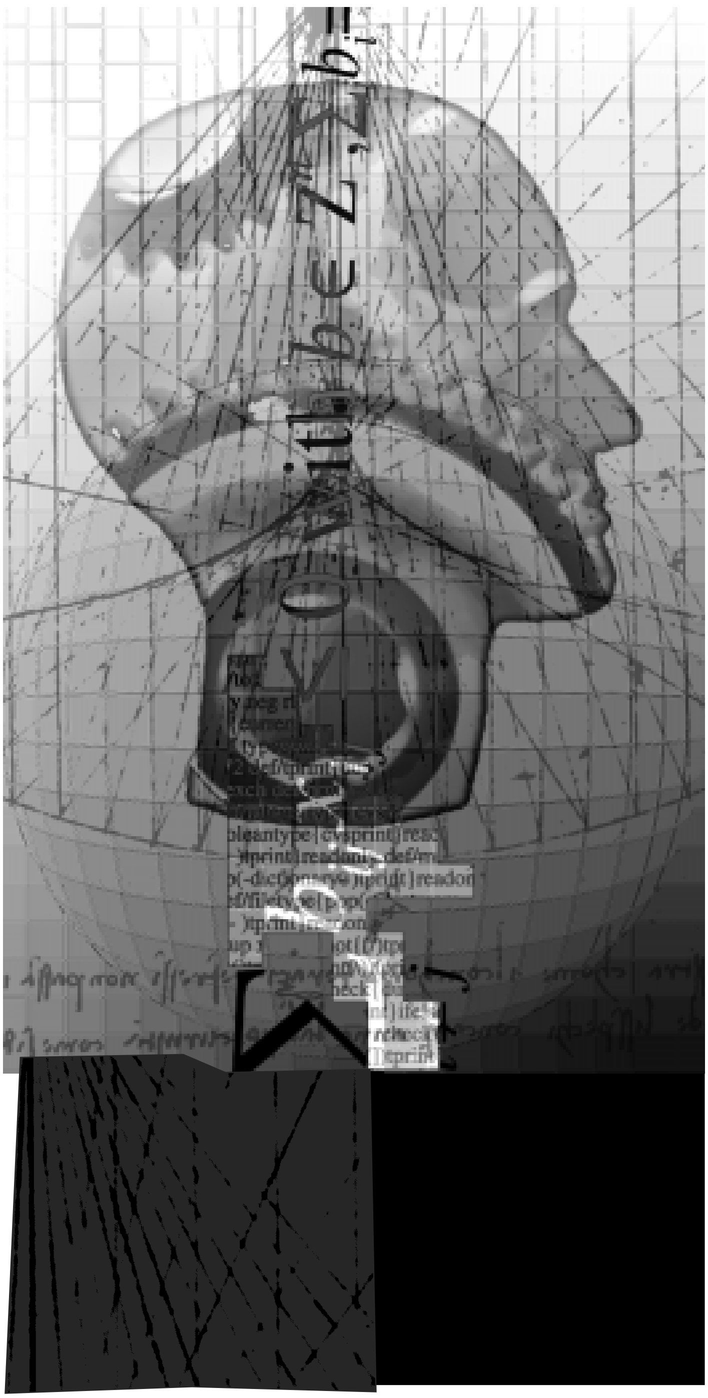
My Favorite THEOREM & Open Problem in *Mathematical Programming*



WHEN I was asked to write this article, I thought immediately of the amazing advances of the last dozen years relating to interior-point methods, from Karmarkar's projective-scaling algorithm to the discovery and exploration of the concept of self-concordant barrier function by Nesterov and Nemirovskii. (I don't think anyone will argue with the significance of these developments; however, what follows definitely exhibits my own prejudices. I hope the reader will find it interesting, even while deploring my taste!) As exciting as these new approaches have been, I believe it's enlightening just to consider how fascinating plain vanilla linear programming is, in all its multi-faceted glory (pun intended).

I have long regarded the term "linear programming" as a misnomer, not just because of the connotations of the word "programming," but also since the subject is so much more complicated (and beautiful) than other linear computational problems such as linear equations and linear least squares; of course, this is due to the presence of inequalities. Inequalities lend a piecewise-linear and nonsmooth element to a problem defined by linear functions. Should we exploit the piecewise-linear nature of the problem and use combinatorial methods like the simplex method, or approximate the nonsmooth feasible region using a smooth but nonlinear function, the barrier function, and use ideas from unconstrained (or linear-equality-constrained) optimization? Geometrically, what do typical high-dimensional polyhedra look like: are they like the mirrored balls hanging in discos, for which edge-following methods like the simplex algorithm seem very inefficient, while an interior-point approach appears very suitable; or more like quartz crystals, with long edges running from one side of the polyhedron to the other, where the reverse conclusion seems plausible?

As we celebrated the 50th birthday of the simplex method in Lausanne last year, I was reminded again of how counterintuitive it is that such an algorithm is so efficient on large-scale problems. Of course, as George Dantzig has remarked, our intuition about high-dimensional polyhedra can be singularly misleading. Different viewpoints can give very contrasting indications: every student of mathematical programming should know the simplex interpretation of the simplex method (Dantzig 1963), which led Dantzig to believe that it could be efficient. But a vast amount of successful computational experience over many years has perhaps left us blasé about the remarkable efficiency of edge-following algorithms.

One approach to explaining this efficiency is of course the probabilistic analyses of the simplex method carried out by Borgwardt, Smale, Haimovich, Adler, Karp, Megiddo, Shamir, and me in the early '80s (Borgwardt 1987). But I want to highlight here a much older result on polytopes, due originally to Carathéodory and rediscovered several times since (see Grünbaum's classic book, Grünbaum 1967, for the history, or the recent book by Ziegler, Ziegler 1995, which discusses the result in its 0th chapter). This theorem challenges our intuition on polyhedra, giving credence to the "many long edges" view that suggests the efficiency of the simplex method. It also intriguingly points out behavior that starts in dimension four, just beyond our power to visualize. Finally, the proof is very simple!

Theorem. For every $d \geq 4$ and $n > d$, there is a d-dimensional polytope with n vertices which is 2-neighborly–every pair of vertices is connected by an edge.

I'll give the proof for $d = 4$ for simplicity; it clearly generalizes. Consider the moment curve $x(t) := (t; t^2; t^3; t^4) \in \mathbb{R}^4: t \in \mathbb{R}$. Note that the intersections of this curve with the hyperplane $x \in \mathbb{R}^4: a^T x = a_0$ correspond to the roots of the polynomial $-a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4$, and conversely any quartic (polynomial of degree at most four) corresponds to a hyperplane.

Choose any $t_1 < t_2 < \cdots < t_n$, $n \geq 5$, and let the polytope P be the convex hull of the corresponding n points on the moment curve. First note that this is indeed a 4-dimensional set, as any hyperplane (corresponding to a

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quartic) can contain at most four of the n points. Next, each of the n points is indeed a vertex of P, since the hyperplane corresponding to $(t - t_i)^2$ contains the ith point $x(t_i)$, with all others strictly on one side. Thus this hyperplane supports the polytope precisely at the ith point. Finally, for any two vertices, say $x(t_i)$ and $x(t_j)$, consider the hyperplane corresponding to the quartic $(t - t_i)^2 (t - t_j)^2$. This contains the two specified vertices, with all others strictly on one side, so it supports P along the edge joining these two vertices. The proof is complete.

Before leaving this example, we note two things. First, for $d \geq 4$, a similar proof shows the existence of $\lfloor d/2 \rfloor$-neighborly polytopes with arbitrarily many vertices; here the convex hull of *any* set of $\lfloor d/2 \rfloor$ vertices forms a simplicial face of the polytope. Second, the presence of edges linking every pair of vertices merely hints at the simplex method's efficiency. While there is an edge from any initial vertex to the optimal vertex, both are highly degenerate, and many pivots may be necessary to reach an optimal basis. Moreover, all vertices when projected onto the (x_1,x_2)-plane lie on the parabola $x_2 = x_1^2$, and hence they are all "shadow vertices" under this projection; thus an unluckily chosen parametric objective simplex method might go through every vertex on its way to the optimal vertex.

I chose this result because it is simple, counterintuitive, insightful, and I believe not widely known. Here is a list of some of the "runners up." I have been very impressed with two results using convex programming to attack discrete optimization problems: Lovász's bound for the Shannon capacity problem (Lovász 1979) and Goemans and Williamson's .878-approximation algorithm for the max-cut problem (Goemans and Williamson 1995), both using semidefinite programming. I am very fond of the result, proved independently by Adler, Karp, and Shamir, by Adler and Megiddo, and by me, that a certain lexicographic parametric variant of the simplex method requires an expected number of pivots growing only quadratically with the smaller dimension of the problem, to solve an LP instance generated by a particular class of probability distributions. (It may seem strange that Adler is a member of two of these groups; however, at the time it was thought that these two papers addressed different algorithms–only later was it realized that they were in fact the same.) For a discussion, see again Borgwardt's book; unfortunately, the class of distributions is rather unsatisfactory. I stand in awe at the intellectual achievement of Nesterov and Nemirovskii in describing the class of convex programming problems for which efficient interior-point methods can be derived, although it is hard to point to a specific encapsulating theorem (Nesterov and Nemirovskii 1994). Finally, I value very highly the recent results of Kalai on subexponential bounds on the expected number of pivots for a certain randomized pivot rule (Kalai 1997).

Let me turn now to open problems. Here again the field of interior-point methods presents many possible choices: what is the "best" infeasible-interior-point algorithm (a method that starts with an infeasible solution and works toward both feasibility and optimality, or possibly toward detecting primal or dual infeasibility)? What are the "right" search directions to use in a primal-dual method for semidefinite programming (there seem to be at least four or five candidates, see Todd 1997), or for general nonlinear programming? Is there an easily computed self-concordant barrier function for a d-dimensional polyhedron with n facets, $n \gg d$, with parameter close to d? (Existence is known due to results of Nesterov and Nemirovskii, but the only known explicitly computable barriers have parameters n or $O(\sqrt{nd})$.) And of course, explaining the good behavior of the simplex method still requires a definitive resolution: is there a polynomial simplex method, is the bounded Hirsch conjecture true?

My favorite open problem is in fact the analogous question for interior-point methods for LP. This may seem paradoxical, since the methods are provably polynomial-time. But the bounds on the worst-case number of iterations to attain a given precision are $O(n)$ or $O(\sqrt{n})$, where n is the number of inequalities, whereas the performance in practice is much better. Indeed, early computational experience led some to believe that there might even be a constant bound, whereas results of Lustig et al. (Lustig, Marsten and Shanno 1990) suggest that the growth in practice may be of order $\ln n$ (they solve "slices" of a fixed problem, with n varying up to about two million). In terms of $\ln n$, this is an exponential gap, as occurs for most variants of the simplex method! And if the methods really took $\Omega(n)$ or $\Omega(\sqrt{n})$ iterations, they would be hopelessly inefficient on very large problems. So we still need to explain why primal-dual methods work as well as they do.

Some probabilistic analysis has been performed, but it does not explain the drop from polynomial to logarithmic in n. On the negative side, we now have results suggesting that there really is a large gap, i.e., that the gap is not just due to our inadequate analysis, but that pathological problems exist. Results of mine, extended in joint work with Ye (Todd and Ye 1996), show that for many primal-dual interior-point methods quite close to those used in practice, there exist instances with n inequalities for which the methods require $\Omega(n^{1/3})$ iterations to achieve a very modest reduction in the duality gap. So, at least for these methods, it is not possible to improve the worst-case bound by much. Thus we are left with the challenging question: can we explain the very slow growth rate in practice? The answer is left as an exercise for the reader!

–M. J. TODD

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CONFERENCE NOTES

▶ **International Conference on Interval Methods and Their Application in Global Optimization (INTERVAL '98)**
April 20-23, 1998, Nanjing China.
URL: http://cs.utep.edu/interval-comp/china.html.

▶ **INFORMS National Meeting**
April 26-29, 1998, Montréal, Quebec, Canada.
URL: http://www.informs.org/Conf/Montreal98/.

▶ **IPCO '98 — Sixth Conference on Integer Programming and Combinatorial Optimization**
June 22-24, 1998, Houston, TX.
URL: http://www.hpc.uh.edu/~ipco98.

▶ **INFORMS International Meeting**
June 28-July 1, 1998, Tel Aviv, Israel.
URL: http://www.informs.org/Conf/TelAviv98/.

▶ **4th International Conference on Optimization**
July 1-3, 1998, Perth, Australia.
URL: http://www.cs.curtin.edu.au/maths/icota98.

▶ **APPROX 98 — 1st International Workshop on Approximation Algorithms for Combinatorial Optimization Problems**
July 18-19, Aalborg, Denmark.
URL: http://www.mpi-sb.mpg.de/~approx98/.

▶ **Optimization 98**
July 20-22, 1998, Coimbra, Portugal.
URL: http://www.it.uc.pt/~opti98.

▶ **2nd WORKSHOP ON ALGORITHM ENGINEERING, W A E '98**
August 19-21, 1998, Saarbruecken, Germany.
URL: http://www.mpi-sb.mpg.de/~wae98/.

▶ **INFORMS National Meeting**
October 25-28, 1998, Seattle WA.
URL: http://www.math.org/seattleinforms.html.

▶ **Sixth SIAM Conference on Optimization**
May 10-12, 1999, Atlanta, GA.

Workshop on Non-Standard Methods for Integer Programming
Utrecht University,
Utrecht, The Netherlands
April 20, 1998

The Worskshop is sponsored by ALCOM-IT, ESPRIT Long Term Research Project No. 20244, and by the graduate school IPA - Institute for Programming Research and Algorithmics.

Several problems that occur in areas such as telecommunication network design, routing, planning and scheduling can be modeled as integer programming problems. During the past decade impressive algorithmic results have been obtained for problems where the variables can take values zero or one. In general integer programming, much less is known about practical optimization algorithms. During this workshop some recent approaches (e.g., test sets, basis reduction, and group relaxations) to solving general integer programming problems will be discussed.

The meeting will be held at the "Uithof Campus" of Utrecht University, Centrumgebouw Zuid, Heidelberglaan 1, lecture hall F125. Directions to the workshop site are included online (http://www.cs.ruu.nl/docs/ipworkshop/directions.html).

You can register by filling out the form on the Workshop homepage (http://www.cs.ruu.nl/docs/ipworkshop/index.html), or by sending an e-mail to Karen Aardal (aardal@cs.ruu.nl). If you register via e-mail, please include your address, phone number and fax number.

The registration fee is NLG 30,00, and can be paid to postal giro account number 1443360 in the name of Karen Aardal. Please mention "IP Workshop" on the payment form. Each registered participant will be offered lunch, and coffee and tea during the break. On-site registration is possible, but we can only guarantee lunch for participants that register before April 13, 1998.

The program includes lectures by Karen Aardal, Imre Barany, Milind Dawande, Matteo Fischetti, Robert Weismantel and Laurence Wolsey.

The full program can be found on the conference web page. For further information, please contact Karen Aardal (aardal@cs.ruu.nl).

–Karen Aardal and Laurence Wolsey

Ettore Majorana Centre for Scientific Culture
International School of Mathematics
"G. Stampacchia" Workshop
Nonlinear Optimization and Applications
Erice, Italy
June 23 - July 2, 1998

Objectives

The workshop aims to review and discuss recent advances and promising research trends concerning theory, algorithms and innovative applications in the field of nonlinear optimization. Both the finite and the infinite dimensional case will be of interest.

Topics

Topics include, but are not limited to: Constrained and unconstrained optimization; Convex analysis; Global optimization; Interior point techniques for linear and nonlinear programming; Large scale optimization; Linear and nonlinear complementarity problems; Nonsmooth optimization; Neural networks and optimization; Applications of nonlinear optimization.

Lectures

As usual, the course will be structured to include invited lectures and contributed lectures. Proceedings including the invited lectures and a selection of contributed lectures will be published. The following is the list of invited lecturers: V.F. Demyanov, St. Petersburg State University, St. Petersburg, Russia; M. Fukushima, Kyoto University, Kyoto, Japan; N.I.M. Gould, Rutherford Appleton Laboratory, England; A. Ioffe, Technion University, Haifa, Israel; O.L. Mangasarian, University of Wisconsin, Madison, WI, USA; P. Marcotte, Universite de Montreal, Montreal, Canada; J.J. Moré, Argonne National Laboratory, Argonne, IL, USA; J. Nocedal, Northwestern University, Evanston, IL, USA; J.-S. Pang, Johns Hopkins University, Baltimore, MD, USA; P.M. Pardalos, University of Florida, Gainesville, FL, USA; E. Polak, University of California, Berkeley, CA, USA; L. Qi, University of New South Wales, Kensington, NSW, Australia; T. Rapcsak, Hungarian Academy of Sciences, Budapest, Hungary; S.M. Robinson, University of Wisconsin, Madison, WI, USA; R.T. Rockafellar, University of Washington, WA, USA; Ph. L. Toint, FUNDP, Namur, Belgium; P. Tseng, University of Washington, WA, USA; M.H. Wright, AT&T Bell Laboratories, NJ, USA; S. Wright, Argonne National Laboratory, Argonne, IL, USA; J. Zowe, University of Erlangen-Nuernberg, Germany.

How to Participate

Persons wishing to attend the workshop should write to: Prof. Gianni Di Pillo, Dipartimento di Informatica e Sistemistica, Universitá di Roma "La Sapienza" via Buonarroti 12, 00185 Roma, Italy (E-mail: erice@dis.uniroma1.it).

They should include date and place of birth, together with present nationality, affiliation, address and e-mail address. If they want to contribute a lecture, they should also include the title and abstract of the proposed lecture. Young persons with only limited experience should enclose a scientific curriculum vitae and a letter of recommendation from the head of their research group or from an experienced person in the field. The total fee, which includes full board and lodging (arranged by the School), is US $800.

Closing date for application is April 30, 1998. Application by e-mail is strongly encouraged.

Availability is limited. If necessary, admission to the workshop will be decided in consultation with the Advisory Committee of the School comprised of Professors F. Giannessi, G. Di Pillo and A. Zichichi and will be communicated shortly after the closing date for application.

Participants must arrive in Erice on June 23 no later than 3 p.m. and will leave on July 2.

How to Reach Erice

Erice is situated in the northwest corner of Sicily in the south of Italy. The easiest way to reach it is to take a plane to Palermo or Trapani. The Majorana Centre will then provide transportation to Erice. More details will be given to successful applicants.

Further Information

More information about the workshop and a sample data sheet for participants can be found at the web site (http://www.dis.uniroma1.it/pub/OR/erice/page.html).

–Franco Giannessi, School Director
Gianni Di Pillo, Course Director

19th IFIP TC7 Conference on System Modeling and Optimization
Cambridge, England
July 12-16, 1999

Further information about this meeting is now available on the web (www.damtp.cam.ac.uk/user/na/tc7con/). This web site includes a list of conference topics, the names of the plenary speakers, and a call for submitted papers. Special attention will be given to general algorithms for optimization calculations by reserving a parallel session throughout the meeting for talks on this subject.

The following people have accepted invitations to present the plenary lectures: Martin P. Bendsoe, Technical University of Denmark; Robert E. Bixby, Rice University, USA; Asen L. Dontchev, American Mathematical Society, USA; Nicholas I.M. Gould, Rutherford Appleton Laboratory, GB; Julia L. Higle, University of Arizona, USA; Rolf H. Möhring, Technical University of Berlin, Germany; Fadil Santosa, University of Minnesota, USA; Jan C. Willems, University of Groningen, The Netherlands; Henry H. Wolkowicz, University of Waterloo, Canada; Stephen J. Wright, Argonne National Laboratory, USA.

The deadline for submitted papers is January 31st, 1999 (next year). If you wish to receive future announcements and reminders about deadlines, please send an e-mail message to tc7con@amtp.cam.ac.uk.

–M.J.D. Powell

ICM98
International Congress of Mathematicians
Berlin, Germany
August 18 - 27, 1998
Second Announcement

The Second Announcement of ICM'98 has been printed and shipping has already started. It will be mailed to all those who have provided mailing addresses.

The Second Announcement contains up-to-date information about ICM'98, although some information is not yet available, such as details about the social program (e.g., the footloose tours) and the scientific program (e.g., the list and schedule of the invited speakers). The ICM'98 E-Mail Information Service will distribute this and other additional information about ICM'98 as soon as possible.

For those who want to retrieve the Second Announcement electronically, Postscript and LaTeX versions of the Second Announcement can be obtained from the web site (http://elib.zib.de/ICM98/Second_Announcement), or by anonymous ftp from elib.zib.de in the subdirectory pub/IMU/HTML/ICM98/Second_Announcement. The files of interest are scndannc.ps (Announcement, Postscript, DIN A4);us_scnda.ps (Postscript, US-paper); scndannc.tex (LaTeX, no maps); reg-form.ps (Registration Form, Postscript, DIN A4); us_regf.ps (Postscript, US-paper); wordregf.doc (MS-Word 6.0).

Note that the above versions of the Second Announcement differ somewhat from the printed version (e.g., some pictures are missing).

Registration

Registration for ICM'98 is possible from now through the date of the conference. You can register by filling out the registration form in the Second Announcement, but it is also possible to register and submit abstracts for ICM'98 presentations (short communications, contributions to poster sessions, invited and plenary addresses) via the WWW (http://elib.zib.de/ICM98) by clicking on "register" or "abstracts." We strongly encourage electronic registration and abstract submission.

PLEASE NOTE: Registration, hotel reservations, and the handling of fees is carried out by DER CONGRESS, a professional congress organization company. When you register, book a room, or request tickets, you will obtain an "official" confirmation message by standard mail from DER CONGRESS. Abstracts are handled by U. Rehmann (Bielefeld), who will acknowledge submissions.

Web Site Redesign

The ICM'98 web site (http://elib.zib.de/ICM98) has been redesigned completely. The new version basically follows the structure of the Second Announcement and will undergo further changes in upcoming months to reflect the progress of the organization.

If you click on "info" or on the center of the logo on the new homepage, or go to http://elib.zib.de/ICM98/info.html directly, you will find that information is now grouped into three main categories (plus two additional sections): General Information, Detailed Information, and Information Regarding the Organization of ICM'98.

In "General Information" you will find the most commonly sought items pertaining to the congress, including: important addresses; the registration form; the Second Announcement (and also the First Announcement); instructions on how to submit to short communications and poster sessions, the Video Math Festival and the Session on Mathematical Software; the list of prospective participants; and a complete list of deadlines.

The new category on the web site, "Detailed Information," is structured along the lines of the printed version of the Second Announcement in order to facilitate your navigation in both documents. However, the subsections will soon contain significantly more up-to-date information than the corresponding items in the Second Announcement.

Finally, the section "Information Regarding the Organization of ICM'98" will give some "Meta-Information" about the various committees and institutions involved in the preparation of the Congress, as well as some items on the historical development of ICM'98.

We hope that the new structure will enable you to quickly find the information for which you are looking from now through the end of ICM'98, August 18-27, in Berlin.

—Martin Gröetschel, President of the ICM'98 Organizing Committee

Reviews

Developments in Global Optimization

edited by Immanuel E. Bomze, Tibor Csendes, Reiner Horst and Panos M. Pardalos
Kluwer Academic Publishers, Dordrecht 1997

ISBN 0-7923-4351-4

The objective of global optimization (GO) is to find the "absolutely best" solution of (potentially) multiextremal optimization problems. In recent years, GO has found numerous applications in the sciences, engineering and economics. Nonlinear approximation, information retrieval, engineering design, extremal energy models (in physics, chemistry, biology), and econometric equilibrium models may serve as illustrative examples from a dynamically expanding list of areas in which GO has imminent relevance. (Several chapters of the book reviewed discuss further examples.)

The book is Volume 18 in the fast growing Kluwer series on Nonconvex Optimization and Its Applications. It is based on refereed contributions submitted by participants of the Third Workshop on Global Optimization (Szeged, Hungary, December 1995). The volume consists of 19 articles which are very briefly annotated below.

- Neumaier discusses NOP, a compact input format to formulate nonlinear optimization problems
- Dallwig, Neumaier and Schichl describe GLOPT, a program system developed for solving constrained GO problems
- Vrahatis, Sotiropoulos and Triantafyllou suggest a new approach to solve "noisy" (i.e., imprecisely given) GO problems
- Ratz provides new results related to extended interval Newton Gauss Seidel steps applicable in rigorous GO
- De Angelis, Pardalos and Toraldo describe global optimality conditions and computational approaches to the (indefinite) quadratic programming problem (QP) under box constraints
- Bomze, Pelillo and Giacomini present a new algorithm to solve the maximum clique problem (in a form leading to general QP over a simplex)
- Stephens discusses finding lower and upper bounds on the Hessian of a function over (box) search (sub)domains, again in an interval GO context
- Strekalovsky and Vasiliev consider non-convex optimal control problems, related to the maximization of a convex function of the terminal state
- Price presents a multistart clustering (linkage) algorithm which exploits gradient information
- Hichert, Hoffmann and Phu analyze the convergence speed of an integral approach for computing the essential supremum of an objective function
- Zabinsky and Kristinsdottir provide a Markov chain analysis for combining pure adaptive search (an "ideal" algorithm) with passive random search
- Pintér describes a model development and solver system for continuous and Lipschitz GO
- Sergeyev presents an algorithm for solving one-dimensional GO problems in which the objective has a Lipschitzian gradient
- Dill, Phillips and Rosen propose a convex global underestimation procedure, subsequently applied to molecular structure prediction
- Pfening and Telek analyze renewal policies for slowly degrading ("aging") mass service systems
- Bollweg, Maurer and Kroll study the numerical prediction of crystal structures by applying a simulated annealing approach
- Garcia, Ortigosa, Casado, Herman and Matej describe a parallel stochastic GO method applied to image reconstruction
- Holmquist, Migdalas and Pardalos study a greedy randomized adaptive search heuristic to solve location problems with economies of scale
- Imreh, Friedler and Fan provide an improved bounding procedure to solve complex process network synthesis problems by branch-and-bound

As can be seen even from this very sketchy description, the book discusses a broad spectrum of GO topics, encompassing theoretical results, algorithm development, decision support systems, as well as a variety of challenging practical applications.

This volume can be recommended to researchers and graduate students working on the areas of mathematical programming, operations research, computer science, applied sciences, economics and engineering.
–JÁNOS D. PINTÉR

Linear Programming 1: Introduction

George B. Dantzig and
Mukund N. Thapa
Springer, 1997
ISBN 0-387-94833-3

This introductory book on linear programming is designed to be used in an undergraduate course. It surveys linear programming and network flow problems, especially the simplex algorithm and variants of it. The text is the first of three volumes; the second will cover theory and implementation and the third will cover structured linear programs and planning under uncertainty. It comes with a CD-ROM, which contains implementations of many of the algorithms described in the text. One of the authors needs no introduction! The other author was a Ph.D. student under Professor Dantzig in the 1970s, and now runs his own company and also teaches at Stanford University.

There is a twelve page preface by George Dantzig discussing the early history of linear programming. This is entertaining and interesting, and should provide good motivation for the student. It also includes a list of recent applications of linear programming and its extensions, and shows its growing importance, and how it has benefited by and contributed to the development of ever-faster computers.

Chapter 1 is an introductory chapter, with several examples of formulating linear programming problems. Row and column approaches to developing formulations are discussed, and the student is taken through them step by step.

Chapter 2 discusses the solution of simple linear programming problems. It considers graphical solution of problems with two variables or two constraints. The approach for the two-constraint case gives motivation for a definition of the dual problem. The dual problem is not solved graphically, but is used as a check of optimality. Fourier-Motzkin elimination is also discussed, perhaps surprisingly for an undergraduate text, since it is really a theoretical procedure, although it is easy to understand conceptually.

The simplex method is the subject of Chapter 3. This is well handled and comprehensive, including treatment of upper and lower bounds and the revised simplex method. The case of a linear program with an unbounded optimal value is described by a theorem; it would have been useful to also include an example. The case where some artificial variables remain basic at the end of Phase I is discussed thoroughly. To emphasize the integration of the two phases, the first fully worked example in the chapter requires both Phase I and Phase II.

Interior point methods are discussed in Chapter 4. This consists of a short treatment of the primal affine, or Dikin's, method. There is only one rudimentary picture. The treatment emphasizes the algebra of a single iteration. It would have been nice to have seen, for example, a discussion of the fact that Dikin's step is equivalent to minimizing the objective function over an inscribed ellipsoid. The choice of step length for which the algorithm is guaranteed to converge should have been discussed. There is no discussion of obtaining a dual solution when using this method. It would be good to have more treatment of interior point methods, especially the polynomial time and practical potential reduction and path following methods, since these methods require only slightly more motivation than Dikin's method.

Chapter 5 covers duality. Finding the dual of any system is described. The remaining material in the chapter constitutes only four pages plus a bibliography and exercises. This material should have been expanded, with more examples and more illustration of its importance and mathematical structure. For example, the section on obtaining a dual solution from the final tableau is very brief and only considers the case where all the artificial variables are in the final tableau. Theorems of the alternative are not considered. This chapter would have been improved by the inclusion of Section 7.1, which considers shadow prices and provides more motivation for duality, including discussion by means of two long examples.

Chapter 6 discusses problem reformulations, including handling free variables, goal programming problems, and problems with piecewise linear objective functions. The authors do not mention that splitting variables is not a good technique with an interior point method because the set of optimal solutions becomes unbounded. Section 6.7 contains one notable error: it is stated that a function is strictly convex if and only if its Hessian is positive definite everywhere.

The remaining sections of Chapter 7 are concerned with sensitivity analysis. Unusually, the effect of a change in the entries in the constraint matrix is discussed, with analysis using the Sherman-Morrison-Woodbury formula. One of the weaknesses of the text is that the dual simplex method is not discussed—it is instead delayed to the second volume. The sensitivity analysis of the case when a constraint is added is therefore somewhat cumbersome, requiring the use of a Phase I method.

The last two chapters cover network problems and together constitute 110 pages, approximately a third of the main body of the text. The use of figures is extensive and helpful, and the text explains the various algorithms well. Chapter 8 discusses the transportation problem. A Phase I procedure for the capacitated transportation problem is also considered. Chapter 9 considers general networks. It covers a broad range of topics, including augmenting path algorithms, the max flow/min-cut theorem, the shortest path problem, the spanning tree problem, and the network simplex algorithm. The discussion of network simplex, in particular, is extensive. The definition of "strongly connected" mistakenly uses chains rather than directed paths, a potentially confusing typographical error.

There are two appendices covering background material in linear algebra and solving systems of equations. Material on numerical implementation of the solution methods is delayed to the second volume.

There are lots of exercises at the end of each chapter, and also sprinkled throughout each chapter. Many of these make extensive use of the CD-ROM, and there are plenty of more traditional exercises. Some of the exercises are quite challenging, having been drawn from various Stanford Ph.D. comprehensive exams.

The CD-ROM contains implementations for a PC of the primal simplex method, Dikin's algorithm, Fourier-Motzkin elimination, network simplex, Dijkstra's algorithm, and other network algorithms. It is easy to use and works well for small instances. The only problem I noticed with it is that the implementation of Dikin's algorithm may give incorrect answers for badly scaled examples.

There are selected bibliographies at the end of each chapter. These generally contain lots of references to seminal work in the '40s, '50s, and '60s, with more limited coverage of more recent work. The References section is somewhat lacking in selectivity. Not every paper in this section is mentioned in the text; for example, nine papers by M. J. Todd are listed, but his name does not appear in the index. The References section

▶

contains sixteen pages of papers where Professor Dantzig is the primary author.

The tone of the text is somewhat mathematical, and the students must be comfortable with matrix notation and introductory linear algebra. However, most proofs are delayed until the second volume, as are details of modifications required to handle large scale problems. There are a number of examples, and these are often used to illustrate theorems. Nonetheless, the text would have benefited from more examples in Chapters 5 through 7.

This text covers the main topics of linear programming, with supplementary material on networks. More material should have been included on interior point methods, since these are becoming ever more important. The text is clearly written, in a style accessible to undergraduates with some mathematical sophistication. The CD-ROM provides a useful accompaniment to the text. Some instructors will find this text very suitable for an introductory course in linear programming.

–JOHN E. MITCHELL

Theory and Algorithms for Linear Optimization: An Interior Point Approach

by C. Roos, T. Terlaky and J.-Ph. Vial

Wiley, Chichester, 1997

ISBN 0-471-95676

Interior point methods applied to Linear Programs have reached a high level of sophistication, with new scientific publications slowing down. This is a clear indication that the field is ready to be digested in book form. It is therefore no surprise that several books on the topic have recently been published; for instance, the books by Saigal, Vanderbei, and Wright. The present book is nevertheless an interesting addition to those books. It consists of 20 chapters, grouped into four parts.

The first part covers the basics of Linear Optimization (LO). Duality and polynomial solvability are introduced using the skew-symmetric model for Linear Programs. Theoretical results, like the existence of strictly complementary solutions, are derived through properties of the *central path*. The complexity is obtained in an elementary way through the *Dikin direction*.

Part 2 of the book takes a closer look at various ways to solve LO through barrier methods. Chapters 6 and 7 are similar in structure and provide a thorough analysis of the *Dual Logarithmic Barrier Method* and the *Primal-Dual Logarithmic Barrier* respectively. The Newton-direction is introduced, and sufficient conditions for local quadratic convergence with full Newton steps are investigated. The predictor-corrector approach is analyzed in detail.

The second half of the book is devoted to more advanced topics. In Part 3, the general *Target Following Approach* is presented in depth, with an analysis of the Primal, the Dual and the Primal-Dual Newton Methods. The last part, *Miscellaneous Topics*, consists of seven chapters. Here one finds Karmakar's Projective Method, a discussion of differentiability properties of the central path, higher order search directions, and parametric and sensitivity analysis. The book closes with a chapter on implementation issues.

The book is carefully written, with an emphasis on mathematical detail. It can serve as a basis for a modern treatment of Linear Optimization at both graduate and undergraduate level. The most distinguishing feature of the book lies in the convergence proofs of all the interior point variants addressed. These are elementary, and at the same time elegant. The most prominent topic not addressed is infeasible interior point methods. In summary, the book is recommended for anyone doing research or teaching in the field of interior point methods.

–FRANZ RENDL

Optimization on Low Rank Nonconvex Structures

Hiroshi Kono, Phan Thien Thach and Hoang Tuy

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The last decade has seen immense progress of global optimization. Several general concepts and algorithmic paradigms have been developed which proved useful in solving small to medium sized global optimization problems. A real breakthrough, however, has occurred in solving global optimization problems with special structures like low rank nonconvex programming problems. Such problems are characterized by the fact that they become convex when a vector of the form Bx is fixed where B is a low rank matrix. Such problems occur in many engineering or economic applications like location problems, design centering and others.

This monograph describes the theoretical foundations, methods and algorithms as well as selected applications in this field in a very thorough and profound way. The first six chapters are devoted to the foundations of the field. Though self-contained, some knowledge of convex analysis is helpful. After a general introduction, the notion of quasi-convexity is discussed, including quasi-conjugacy and quasi-subdifferentials. Starting from typical examples, Chapter 3 develops the theory of differences of convex functions and sets (d.c.functions, d.c.sets). The authors continue with a duality framework for important classes of nonconvex problems and set the foundations for low-rank nonconvex structures. The sixth and last chapter of Part 1 deals with global search methods and basic algorithms for d.c.optimization.

The second part of this monograph develops numerical methods and algorithms for solving typical special classes of global optimization problems by exploiting their low rank structure. In particular it deals with parametric approaches for solving low rank nonconvex quadratic programs and concave minimization problems, with multiplicative programming problems, and with monotonic problems. In addition, a price-directive decomposition approach is developed and dynamic programming algorithms in global optimization are discussed.

The third part deals with selected applications. It starts with a chapter on low rank nonconvex quadratic programming problems; then continuous location problems are discussed; and finally, design centering and related geometric problems are described. It closes with a chapter on multiobjective and bilevel programming.

Throughout the text numerous new results are given. Moreover, the extensive bibliography of more than 400 titles will prove to be very useful. All in all, this monograph is a highly welcome enrichment of the mathematical programming literature which gives a comprehensive insight in a very active field.

–RAINER E. BURKARD

Tom Magnanti, who recently became Institute Professor at MIT, has also been awarded the degree of doctor honoris causa by IAG, Université Catholique de Louvain, Louvain-la-Neuve, Belgium… *Ravindra Ahuja* (IIT, Kanpur), *Stanislav Uryasev* (Brookhaven) and *Joe Geunes* (Penn State) will join the ISE Department and Center for Applied Optimization at the University of Florida in August, 1998… *Peter Hammer* (RUTCOR) received an Honorary Doctorate (Laurea Honoris Causa) from La Sapienza University in Rome on March 23, 1998… *Panos Pardalos* has been awarded a three-year appointment to a University Research Foundation Professorship at the University of Florida.

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