N^o 61

OPTIMA

Mathematical Programming Society Newsletter

MARCH 1999

Planning Implants of Radionuclides for the Treatment of Prostate Cancer: An Application of Mixed Integer Programming

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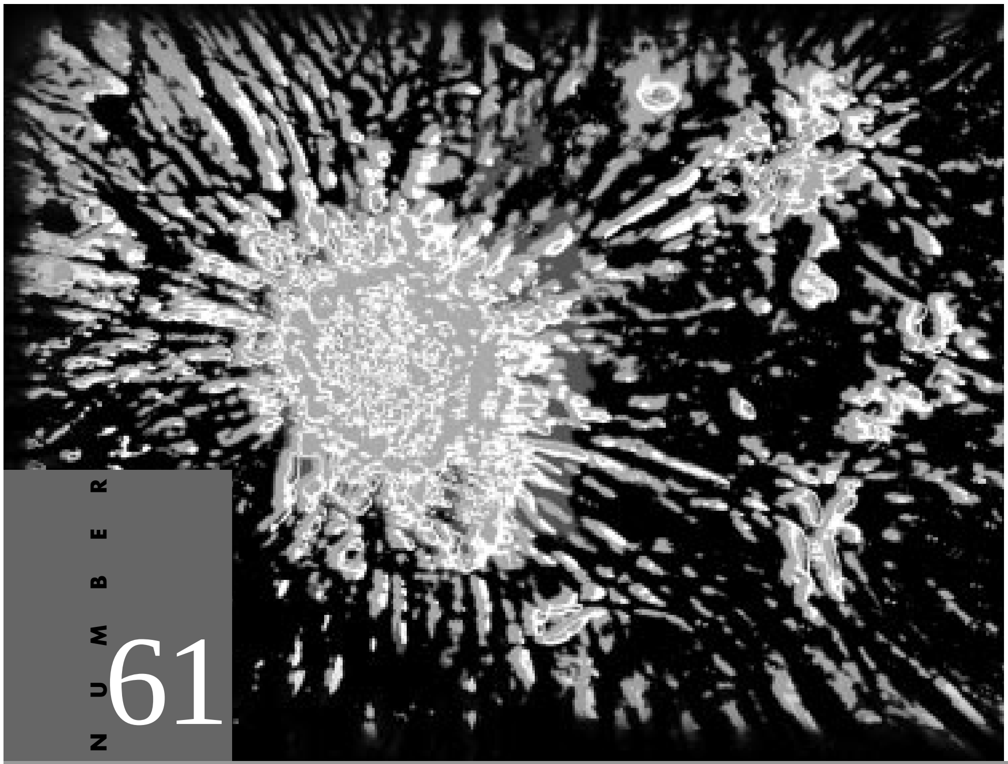
March 10, 1999



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Planning Implants of Radionuclides for the Treatment of Prostate Cancer: An Application of Mixed Integer Programming

1 Introduction

In recent years, technical advances in medical devices have led to the increasing use of radioactive implants as an alternative or supplement to external beam radiation for treating a variety of cancers. This treatment modality, known as brachytherapy, involves the placement of encapsulated radionuclides ("seeds") either within or near a tumor [4]. In the case of prostate cancer, seed implantation is typically performed with the aid of a transrectal ultrasound transducer attached to a template consisting of a plastic slab with a rectangular grid of holes in it. The transducer is inserted into the rectum and the template rests against the patient's perineum. A series of transverse images are taken through the prostate, and the ultrasound unit displays – superimposed on the anatomy of the prostate – the grid on the template. Needles inserted in the template at appropriate grid positions enable seed placement in the target at planned locations.

Despite the advances in devices that assist in accurate placement of seeds, deciding *where* to place the seeds remains a difficult problem. A treatment plan must be designed so that it achieves an appropriate radiation dose distribution to the target volume, while keeping the dose to surrounding normal tissues at a minimum. Moreover, a planning technique should enable controlling the dose at any given point in or near the implantation.

Traditionally, to design a treatment plan, several days (or weeks) prior to implantation the patient undergoes a simulation ultrasound scan. Based on the resulting images, an iterative process is performed to find a pattern of needle positions and seed coordinates along each needle which will yield an acceptable dose distribution. Adjustments are typically guided by repeated visual inspection of isodose curves overlaid on the target contours. This iterative manual process is lengthy, sometimes taking up to eight hours to complete. Moreover, the large number of possible source arrangements means that only a small fraction of possible configurations can actually be examined.

There have been a number of research efforts directed at developing computational approaches to aid in brachytherapy treatment planning. Among them, Silvern [12] and Yu and Schell [16] proposed genetic algorithm approaches, and Sloboda [14] proposed an approach based on simulated annealing. One shortcoming of these heuristic search methods is that they do not provide a mechanism for strictly enforcing clinically desirable properties within the models (e.g., strict lower and/or upper bounds on the dose delivered to specified points near the implantation).

In this article, various integer programming models for finding a good seed configuration in brachytherapy treatment planning are proposed and applied to the planning of permanent prostate implants. The basic model, described in Section 2, involves using 0/1 indicator variables to capture the placement or non-placement of seeds in a prespecified three-dimensional grid of potential locations. The dose delivered to each point in a discretized representation of the diseased organ and neighboring healthy tissue is modeled as a linear combination of these indicator variables. A system of linear constraints is imposed in an attempt to keep the dose level at each point to within the specified target bounds. Since it is physically impossible to satisfy all dose constraints simultaneously, each constraint uses a variable to either record when the target dose level is achieved, or record the deviation from the desired level. These additional variables are embedded into an objective function to be optimized. A description of this MIP approach and preliminary computational experiments with it have appeared in medical journals [3, 8, 13]. Although not the focus of this article, it is also noteworthy that for external beam radiation treatment planning, linear programming approaches have been proposed as far back as 1968 [1, 5, 10, 11].

Besides the likelihood of generating superior treatment plans to those generated via traditional manual methods, one potential advantage of using computational optimization approaches to treatment planning is speed and the consequent possibility of generating treatment plans immediately prior to implantation. It is often the case that the position of the diseased organ in the operating room differs from the position in the pre-implant simulation images. In such a case, there may be a need to change the plan in the operating room. One goal of an automated treatment planning system is to be able to assist physicians and radiation physicists in obtaining good treatment plans "on the fly." Hence, it is imperative that the optimization component of an automated system obtain good solutions quickly. The numerical results presented in

Section 3 indicate that “good” solutions can be obtained via the MIP approach within 5 to 15 minutes.

2 Mixed Integer Programming Models

Our basic model involves using 0/1 variables to record placement or non-placement of seeds in a prespecified three-dimensional grid of potential locations. In the case of prostate cancer, the locations correspond to the projection of the holes in the template onto the region representing the prostate in each of the ultrasound images. If a seed is placed in a specific location, then it contributes a certain amount of radiation dosage to each point in the images. (The dose contribution to a point is proportional to the inverse square of the distance from the source.) Thus, once the grid of potential seed locations is specified, the total dose level at each point can be modeled. Let x_j be a 0/1 indicator variable for recording placement or non-placement of a seed in grid position j. Then the total radiation dose at point P is given by

$$\sum_j D\left(\|P - X_j\|\right)x_j, \tag{1}$$

where X_j is a vector corresponding to the coordinates of grid point j, $\|\ \|$ denotes the Euclidean norm, and $D(r)$ denotes the dose contribution of a seed to a point r units away. The target lower and upper bounds, L_P and U_P, for the radiation dose at point P can be incorporated with (1) to form constraints for the MIP model:

$$\begin{aligned} &\sum_j D\left(\|P - X_j\|\right)x_j \ge L_P \\ &\sum_j D\left(\|P - X_j\|\right)x_j \le U_P. \end{aligned} \tag{2}$$

Of course, not all points P in the images are considered. The images are discretized at a granularity that is conducive both to modeling the problem accurately and to enabling computational approaches to be effective in obtaining solutions in a timely manner. For discretizations that provide accurate modeling, it is typically not possible to satisfy desired dose constraints at all points simultaneously. This is due in part to the proximity of diseased tissue to healthy tissue. Also, because of the inverse square factor, the dose level contribution of a seed to a point less than 0.3 units away, say, is typically larger than the target upper bound for the point.

One approach of addressing this difficulty is to identify a *maximum feasible subsystem*. This is the essence of our first MIP model. By introducing additional 0/1 variables one can directly maximize the *number* of points satisfying the specified bounds. In this case, constraints (2) are replaced by

$$\begin{aligned} &\sum_j D\left(\|P - X_j\|\right)x_j - L_P \ge -N_P\left(1 - v_P\right) \\ &\sum_j D\left(\|P - X_j\|\right)x_j - U_P \le M_P\left(1 - w_P\right), \end{aligned} \tag{3}$$

where v_P and w_P are 0/1 variables, and M_P and N_P are suitably chosen positive constants. If a solution is found such that $v_P = 1$, then the right hand side of the first inequality in (3) is zero; and hence, the lower bound for the dose level at point P is not violated. Similarly, if $w_P = 1$, the upper bound at point P is not violated. In order to find a solution that satisfies as many bound constraints as possible, it suffices to maximize the sum of these additional 0/1 variables; i.e., maximize $\sum_P (v_P + w_P)$. In practice, achieving the target dose levels for certain points may be more critical than achieving the target dose levels for certain other points. In this case, one could maximize a weighted sum: $\sum_P (\alpha_P v_P + \beta_P w_P)$, where the more critical points receive a relatively larger weight. Using a weighted sum was important for the prostate cancer cases to be discussed in Section 3. Since there were significantly fewer urethra and rectum points compared to the number of points representing the prostate, to increase the likelihood that the former points achieved the target dose levels, a large weight was placed on the associated 0/1 variables.

The role of the constants N_P and M_P in (3) is to ensure that there will be feasible solutions to the mathematical model. In theory, these constants should be chosen suitably large so that if v_P or w_P is zero, the associated constraint in (3) will not be violated regardless of how the 0/1 variables x_j are assigned. In practice, the choice is driven by computational considerations of the optimization algorithm being used and/or by decisions by the radiation oncologist. For a branch-and-bound algorithm, it is advantageous computationally to assign values that are as tight as possible. The medical expert can guide the selection of the constants by either assigning absolute extremes on acceptable radiation dose levels delivered to each point (note that $U_P + M_P$ is the absolute maximum dose level that will be delivered to point P under the constraints in (3), and $L_P - N_P$ is the absolute minimum), or by estimating the number of seeds needed for a given plan. In the latter case, if the number of seeds needed is estimated to be between k_1 and k_2 ($k_1 \le k_2$), say, then the constant N_P can be taken to be L_P minus the sum of the smallest k_1 of the values D ($\|P - X_j\|$), and the constant M_P can be taken to be the sum of the largest k_2 such values minus U_P. Selection in this fashion will ensure that no plan having between k_1 and k_2 seeds will be eliminated from consideration.

An alternative model involves using continuous variables to capture the deviations of the dose level at a given point from its target bounds and minimizing a weighted sum of the deviations. In this case, the constraints (2) are replaced by constraints of the form

$$\begin{aligned} &\sum_j D\left(\|P - X_j\|\right)x_j + y_P \ge L_P \\ &\sum_j D\left(\|P - X_j\|\right)x_j - z_P \le U_P, \end{aligned} \tag{4}$$

where y_P and z_P are non-negative continuous variables. The objective for this model takes the form: minimize $\sum_P (\alpha_P y_P + \beta_P z_P)$, where α_P and β_P are non-negative weights selected according to the relative importance of satisfying the associated bounds. For example, weights associated with an upper bound on the radiation dose for points in a neighboring healthy organ may be given a relatively larger magnitude than weights associated with an upper bound on the dose level for points in the diseased organ.

One enhancement that we have not yet explored, but that could be incorporated into either of the above models, is the allowance of alternative seed types. There are a variety of radioactive sources that are used for brachytherapy, including palladium-103, iodine-125, cesium-137, iridium-192, and gold-198, each of which has its own set of exposure rate constants. (Pd-103 or I-125 are commonly used for treating prostate cancer.) Typically however, a single seed type is used in a given treatment plan. This fact is, in part, due to the difficulty of designing treatment plans with multiple seed types. The allowance of multiple seed types can easily be incorporated into the MIP framework – one need only modify the total dose level expression (1) as

$$\sum_j \sum_i D_i\left(\|P - X_j\|\right)x_{ij}. \tag{5}$$

Table 1. Lower and Upper Bound Specifications as Multiples of Target Prescription Dose

	Rectum	Urethra	Contour	Uniformity
Lower Bound	0	0.9	1.0	1.0
Upper Bound	0.78	1.1	1.5	1.6

Table 2. Problem Statistics

Model 1				Model 2		
Pt	Rows	Cols	0/1 Vars	Rows	Cols	0/1 Vars
1	4398	4568	4568	4398	4568	170
2	4546	4738	4738	4546	4738	192
3	3030	3128	3128	3030	3128	98
4	2774	2921	2921	2774	2921	147
5	5732	5957	5957	5732	5957	225
6	5728	5978	5978	5728	5978	250
7	2538	2658	2658	2538	2658	120
8	3506	3695	3695	3506	3695	189
9	2616	2777	2777	2616	2777	161
10	1680	1758	1758	1680	1758	78
11	5628	5848	5848	5628	5848	220
12	3484	3644	3644	3484	3644	160
13	3700	3833	3833	3700	3833	133
14	4220	4436	4436	4220	4436	216
15	2234	2330	2330	2234	2330	96
16	3823	3949	3949	3823	3949	126
17	4222	4362	4362	4222	4362	140
18	2612	2747	2747	2612	2747	135
19	2400	2484	2484	2400	2484	84
20	2298	2406	2406	2298	2406	108

Table 3a. Solution Statistics for Model 1 (Maximization)

Pt	Initial	First Heuristic		Best	Best
	LP Obj.	secs	Obj.	LP Obj.	IP Obj.
1	1888013.3	245.2	1752286	1873433.93	1766609
2	1809964.8	672.7	1736946	1796642.43	1736946
3	687448.6	43.6	587712	633843.00	593228
4	803564.9	338.7	753672	802134.58	765115
5	2855667.4	1345.9	2638679	2835825.38	2649950
6	2925181.3	1349.2	2805284	2907792.52	2805284
7	651682.5	54.7	582314	639160.50	598630
8	1132430.4	669.8	1062561	1112930.1	1075670
9	677253.3	194.7	639527	669073.94	641643
10	286986.4	25.5	252368	274188.69	257492
11	2585974.0	366.8	2453886	2529795.54	2462279
12	983328.6	70.7	804213	945400.35	817875
13	862373.4	52.0	744450	827676.91	795149
14	1611020.9	476.2	1509329	1590484.06	1531009
15	438667.7	29.5	376087	428376.60	396064
16	1273297.8	163.6	1170743	1248805.82	1204870
17	892239.9	52.2	747929	817014.30	757446
18	683918.1	71.0	581684	666083.02	592861
19	425871.6	14.9	341328	403235.91	376179
20	360474.3	14.1	288973	343623.43	309499

Here, x_{ij} is the indicator variable for placement or non-placement of a seed of type i in grid location j, and $D_i(r)$ denotes the dose level contribution of a seed of type i to a point r units away. In this case, a constraint restricting the number of seeds implanted at grid point j is also needed: $\;_i\, x_{ij}\;\;$ 1. It remains to be tested whether the added flexibility of allowing multiple seed types will have a substantial impact on the number of points at which target dose levels can be satisfied. Nevertheless, it is an intriguing possibility. Computationally, the optimization problem may prove to be more difficult due to the increased number of 0/1 variables.

Besides the basic dosimetric constraints, other physical constraints can be incorporated into our basic models. One could incorporate constraints to control the percentage of each tissue structure satisfying specified target bounds. Alternatively, one could – if desired – constrain the total number of seeds and/or needles used. Note also that one can ensure that target dose bounds at specific points are satisfied by fixing the associated “feasibility” variables (v_p, w_p, y_p, z_p) to appropriate values. In the numerical work reported in [8] we used this approach to ensure that the dose delivered to all points representing the urethra did not exceed a specified upper bound.

3 Computational Strategies and Clinical Experiments

We tested our MIP approach using data from twenty prostate cancer cases. In each case, iodine-125 was used as the radioactive source, and four separate categories of points, corresponding to distinct anatomical structures, were specified. *Contour points* defined the boundary of the diseased organ in each of the slices; the regions enclosed by each boundary were populated with uniformly spaced points, termed *uniformity points*; and points representing the positions of the *urethra* and *rectum* in each slice were also specified. For the 20 cases considered, the average numbers of points in each category were: uniformity 1305, contour 461, urethra 8, and rectum 9. The lower and upper bounds for each point type were specified as multiples of the target prescription dose. These are tabulated in Table 1.

Numerical tests were performed using two distinct models. Model 1 utilized constraints (3) and the associated objective max $\;(a_p v_p + b_p w_p)$; and Model 2 utilized constraints (4) and the objective min $\;(a_p y_p + b_p z_p)$. Various combinations of objective function weights for each of the two models were tested. Here, we present results based on one set of weights for each model. Detailed analysis of the two models and a study of the sensitivity of resulting plans to selected weights can be found in [7].

For both models, it is advantageous to place relatively large weights on the objective function variables associated with urethra, rectum, and

Table 3b. Solution Statistics for Model 2 (Minimization)

Pt	Initial LP Obj.	First Heuristic secs	First Heuristic Obj.	Optimal IP Obj.	bb nodes	Elapsed Time
1	29973430.5	21.7	440437196.1	93550763.6	377	9706.0
2	19921521.4	34.7	179171112.9	49156651.9	9184	378857.0
3	-11333869.7	5.2	97625273.7	50517325.3	4051	27724.0
4	2597572.3	18.7	189610043.6	21005621.8	1377	27485.0
5	73684327.8	112.4	467410325.8	93828192.8	1293	748292.3**
6	36902037.2	105.3	524058129.4	64216816.0	5293	1136221.7
7	45848681.6	6.5	302836935.1	118325071.3	712	4655.5
8	17614469.1	32.3	250057575.6	73399636.5	62373	1863362.0
9	14691002.3	17.3	344540093.9	57209440.5	1643	41212.1
10	28197622.0	2.1	90862556.4	55251869.2	883	2619.1
11	172211617.5	29.1	616562230.8	293530404.3	643	14741.8
12	292898229.2	11.5	785823995.0	518235776.6	1985	35718.5
13	-163007095.9	4.3	-21671699.9	-77173221.5	481	2817.6
14	40303495.4	27.1	378940132.7	119586431.2	1408	58654.2
15	89432119.5	5.5	236921860.0	191780731.4	10838	55913.8
16	78434032.7	14.1	244541089.6	148828362.1	1282	25969.0
17	-830974566.8	2.7	-717574515.4	-799657523.1	25	178.2
18	155505947.5	9.6	700452425.7	351076662.5	82118	554737.2
19	73628152.3	2.1	204208781.0	149604823.5	377	1207.8
20	-45968824.5	1.8	57904156.7	15635930.3	415	1222.5

** Not optimal

contour points. For the results reported herein, the objective function weights for the variables associated with uniformity points were set equal to 1; those associated with contour points were set equal to the ratio of the number of uniformity points to the number of contour points; and those associated with urethra and rectum points were set equal to the number of uniformity points. Selecting such large weights for the urethra and rectum points essentially ensures that the dose contribution to these points will lie within the specified bounds. The heavy weight for the contour points assists in achieving prescription isodose curves that conform well with the boundary of the diseased prostate.

The dosimetric constraint matrix in both models is completely dense and has coefficients ranging in magnitude from tens to tens of thousands. Table 2 shows the problem statistics for Models 1 and 2. Here, *Rows* and *Cols* indicate the number of rows and columns, respectively, in the constraint matrix; and *0/1 vars* indicates the number of 0/1 variables.

Although the problem size is only moderate, even solving the initial linear programming relaxation is memory-taxing, often resulting in a process having a total size of 300MB (including text, data, and stack), and total resident memory approaching 300MB. Computational experience with these instances has demonstrated that they are extremely difficult to solve to optimality, requiring strenuous computational effort to improve the objective value by a marginal amount. Even obtaining good feasible solutions which are clinically acceptable is difficult.

The numerical work reported herein is based on a specialized branch-and-bound MIP solver which is built on top of a general-purpose mixed integer research code (MIPSOL) [6], using CPLEX 6.0 as the intermediate LP solver. The general-purpose code, which incorporates preprocessing, reduced-cost fixing, cut generation, and fast heuristics, has been quite effective in solving the instances reported in MIPLIB3 [2]. For the prostate cancer instances reported in this paper, specialized heuristics and branching schemes were implemented to quickly obtain good feasible solutions [7]. We also experimented with a reduction and approximation approach in an attempt to devise efficient computational strategies to improve the solution process for these instances [9].

None of the instances for Model 1 were solved to proven-optimality, whereas for Model 2, all except one were solved to optimality. In Tables 3a and 3b the solution statistics for both models are given. The instances were solved on an UltraSparc-II 168 Mhz workstation. We set the running time limit to be 10,000 CPU seconds for Model 1. In each table, the column labeled *Pt* denotes the patient case; the column labeled *Initial LP Obj.* lists the optimal objective value of the initial LP relaxation; and the columns *First Heuristic (secs, Obj.)* list the elapsed time when the heuristic procedure is first called and the objective value corresponding to the feasible integer solution returned by the heuristic. For Table 3a, the columns *Best LP Obj.* and *Best IP Obj.* report, respectively, the LP objective bound corresponding to the best node in the remaining branch-and-bound tree and the incumbent objective value corresponding to the best integer feasible solution upon termination of the solution process (10,000 CPU seconds). In Table 3b, the columns *Optimal IP Obj., bb nodes*, and *Elapsed Time* report, respectively, the optimal IP objective value, the total number of branch-and-bound tree nodes solved, and the total elapsed time for the solution process.

Using the reduction approach alluded to earlier, the running time for Model 2 decreased by 5 to 100 times for the 20 instances, with an average decrease of 28.3 times. The readers are referred to [7, 9] for more details regarding this approach.

To contrast the performance of our solver on these instances and to illustrate their level of difficulty, in Table 4 we provide a solution profile for case Pt 1 using the MIP solver of CPLEX 6.0 (with pseudo-cost branching, which appears to be the best among the possible options). We note that none of the instances were solved to optimality using CPLEX 6.0. For Model 1

Table 4. Solution Statistics for Pt 1 Running on CPLEX 6.0

CPU secs elapsed	Best IP obj.	Best LP obj.	bbnodes searched
Model 1			
6784.82	914006	1888012.12	2100
14196.93	914006	1888011.90	8100
25141.08	914006	1888011.66	15100
35972.33	995417	1888011.59	21600
62448.75	995417	1888011.58	27400
Cuts added 13173			
Model 2			
106.60	1047338492.9	3.5015e+07	61
5008.38	440437196.1	7.3056e+07	2241
10037.33	108100907.2	8.0022e+07	6241
15185.81	93550763.5	8.3096e+07	15001
20357.77	93550763.5	8.4342e+07	25001
32736.21	93550763.5	8.5909e+07	50001
45911.94	93550763.5	8.6919e+07	77321
46884.43	93550763.5	8.6987e+07	79341

Table 5. Clinical Significance of the MIP Generated Plans

Pt	Prostate Vol. (cc)	Activity (mCi)	conformity	coverage	No. Seeds
100Gy					
1	49.1	0.592	1.20	.973	40
2	53.6	0.450	1.16	.994	51
3	34.2	0.334	1.18	.945	51
4	31.0	0.400	1.17	.985	42
5	68.7	0.590	1.21	.985	50
6	68.1	0.450	1.20	.986	64
7	26.7	0.400	1.25	.970	39
8	40.8	0.450	1.21	.983	44
9	28.9	0.500	1.28	.988	32
120Gy					
10	16.6	0.468	1.29	.939	28
160Gy					
11	66.1	0.520	1.12	.964	85
12	38.3	0.544	1.23	.951	58
13	39.9	0.450	1.22	.986	70
14	48.2	0.450	1.17	.989	76
15	24.3	0.550	1.18	.980	42
16	45.3	0.592	1.15	.975	57
17	50.7	0.463	1.11	.874	72
18	26.4	0.500	1.29	.970	51
19	25.4	0.450	1.15	.964	48
20	25.6	0.400	1.16	.977	57

instances, great computational effort was exerted, only to yield marginal improvement in the objective value. Instances for Model 2 were slightly more manageable, although the objective value improvement eventually stalled (e.g., after 80,000 nodes for Pt 1). The columns *CPU secs elapsed, Best IP obj., Best LP obj.,* and *bbnodes searched* record, respectively, the time elapsed within the solution process, the incumbent objective value corresponding to the best integer feasible solution, the corresponding best LP value from the remaining branch-and-bound nodes, and the number of nodes solved. For Model 1, we report the solution process up to 62448.75 CPU seconds and for Model 2, the solution process is observed up to 46884.43 CPU seconds.

It is noteworthy that using our reduction approach we solved the Model 2 instance for Pt 1 in 1012.8 CPU seconds. In addition, this same instance was solved in 1312.24 CPU seconds when CPLEX 6.5 was used, running on an UltraSparc-II 296 Mhz machine.

Despite the computational difficulty, high-quality clinically desirable treatment plans were obtained using both models. In Table 5 we report some clinically relevant statistics. For the results reported in the table, the treatment plans are those associated with the first feasible solution obtained from our specialized solver applied to Model 1. The cases are categorized according to the target prescription dose (100Gy, 120Gy, or 160Gy). *Prostate Vol (cc)* records the volume of the prostate, *Activity (mCi)* is the activity rate of the implanted seeds, and *conformity* and *coverage* are measures of the quality of the generated plans. Conformity is defined as the ratio of the volume of the prescription isodose surface determined by the plan to the portion of the target volume within this surface. Coverage measures the ratio of the target volume within the prescription isodose surface to the entire target volume. For an ideal plan both the conformity and coverage indices should be 1. A conformity index greater than 1 provides a measure of the amount of healthy normal tissue receiving the prescription dose or greater. In particular, a smaller conformity index implies that nearby healthy tissue is exposed to less radiation, thus reducing the probability of complications. Compared to traditional manual planning methods, plans derived via the MIP approach use fewer seeds (20-30 fewer) and needles, and provide better coverage and conformity indices [7, 8].

4 Concluding Remarks

The computational work presented herein demonstrates that a mixed integer programming approach to brachytherapy treatment planning can produce high-quality treatment plans for prostate cancer cases. The MIP models provide the flexibility to enforce clinically critical dosimetric conditions, and to prioritize dose level achievement for vital organs and tissues near the diseased structure.

Although the mixed integer programming problem instances are difficult to solve to optimality, with our specialized heuristic procedure, good treatment plans were returned within 15 CPU minutes. This suggests that incorporation of an MIP-based optimization module into a comprehensive treatment planning system for use in the operating room is feasible. With this in mind, work is now in progress to interface the optimization module with standard clinical evaluation tools (e.g., tools for displaying isodose curves and cumulative dose-volume-histograms). A major advantage of such an online system is that it would allow the generation of an alternative plan in the event that unforeseen circumstances arising during implantation prevent strictly following the pre-implant simulation plan. In this scenario, the system must be able to accommodate the fact that some seeds may have already been implanted based on the pre-implant plan, and that other pre-selected seed locations must now be disallowed. With respect to the MIP models, this only requires fixing selected binary variables to zero or one.

This work has the potential to have a direct positive impact on treatment success, as well as in eliminating the time-consuming task of generating treatment plans via traditional manual approaches. Interested readers can refer to the publications [3, 8, 13, 15] for additional medical and clinical insights regarding this research. Studies detailing the integer programming aspects, including computational strategies for solving the associated MIP instances, are reported in [7, 9].

5 Acknowledgments

The first author was supported in part by National Science Foundation CAREER grant 9501584/9796312. The second author was supported in part by National Library of Medicine Training Grant LM07079. The computational work for this research was performed on SUN equipment made available through SUN Microsystems Academic Equipment Grant SUN AEG EDUD-US-970311. The authors thank Professor Ellis Johnson and Professor George Nemhauser for their comments on an earlier version of this article. In addition, we acknowledge CPLEX, a division of ILOG Inc., for providing CPLEX 6.0 for this research. We also thank Professor Robert Bixby for profiling the Model 2 instance for Pt 1 using the recently released CPLEX 6.5. Finally, the authors would like to thank Dr. David Silvern for his assistance in providing the data used in the numerical work presented in Section 3.

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OPTIMIZATION

at GSIA, Carnegie-Mellon University

This is the first of a series of articles on the history of some of the leading academic programs in optimization throughout the United States. For the first article I chose the Operations Research Group at the Graduate School of Industrial Administration in Carnegie Mellon University.

There are several groups at Carnegie Mellon University that have exceptional expertise in the topic of Optimization. The most important and undoubtedly better known is the Operations Research Group at the Graduate School of Industrial Administration (GSIA). Within GSIA, faculty members at the Operations Management Group keep in close contact with their colleagues in Operations Research (OR), and often share teaching responsibilities and research projects. Other researchers within Carnegie Mellon include faculty at the Chemical Engineering Department, the Department of Mathematics, the School of Computer Science, and the Heinz School of Urban and Public Affairs. Collectively, Carnegie Mellon has one of the best groups of researchers in Optimization in the whole world.

The group at GSIA was the place of origin of the first industrial applications of OR in the early 1950s. It was the kind of atmosphere where research that crossed conventional boundaries was very much encouraged, and unconventional approaches were enthusiastically supported. In those days the Operations Research Faculty consisted mainly of Bill Cooper and Gerry Thompson. Bill Cooper was one of the leading authorities in Linear Programming in the fifties, and had significant influence on the first applications of OR to solving applied problems. Among other achievements, Cooper is credited with the invention of Goal Programming and Data Envelopment Analysis. His book with Charnes is still a classic in the early applications of Linear Programming. Gerry Thompson, who is still among the faculty at GSIA, pioneered the use of computers and quantitative methods in business education. He often used classical methods to apply OR technology to problems in economics, marketing and other general disciplines of business.

In the late sixties the group welcomed a Romanian immigrant, who would later become one of the most influential leading researchers in the area of Optimization. When Egon Balas arrived to GSIA, he had already published what would be later recognized as “the most frequently cited paper in OR between 1954 and 1981.” The paper described an additive algorithm for solving zero-one problems and it was one of the early prototypes of implicit enumeration or branch-and-bound algorithms, which used logical tests not unlike those underlying the constraint propagation approach of our days. Egon Balas has one of the most interesting life stories I have ever heard (or read), a rather stormy life that included action in the

Hungarian anti-Nazi underground movement, followed by arrest, prison, escape and hiding, a brief tenure as a Romanian diplomat in London, soon followed by arrest and solitary confinement in the horrible prisons of Bucharest. He had managed to emigrate to the West in 1966, with a brief stay in Rome, a summer in Stanford's OR group, and the final arrival to GSIA. His life story is the subject of a new book, which should reach the press later this year.

Balas helped build a prosperous group with an important concentration in integer programming. He attracted the late Bob Jeroslow, and strengthened the doctoral program that had already fostered notorious Ph.D. students such as Fred Glover and M.R. Rao. During his first years at GSIA, Balas worked with a brilliant Ph.D. student, Manfred Padberg. Padberg's work on the knapsack polyhedron and set-partitioning problems would later become one of the seminal studies in modern Integer Programming. Reading Padberg's Ph.D. thesis 25 years later is an interesting experience since it is an excellent compendium of ideas to come in the late seventies and early eighties. It is during this time that Balas completed his first paper on Disjunctive Programming. Surprisingly, this paper was never published, because Balas refused to write it according to the referee's suggestion. The paper is being published now, 25 years later, as an invited paper with an introductory note appraising its (original) merits. During the seventies the group at GSIA became a power house in Integer Programming, as Egon Balas and others completed several papers on the structure of basic polyhedra, disjunctive programming, intersection cuts, and general cutting plane theory.

Jong Shi Pang joined the faculty in the mid-seventies, and concentrated his research on the solution of linear-complementarity problems and the study of variational inequalities. In the late seventies two young and promising stars joined the faculty: Gerard Cornuéjols and Dorit Hochbaum. Gerard Cornuéjols joined the group after completing his dissertation in Cornell under the supervision of George Nemhauser. His joint work with Nemhauser and Fisher on the solution of facility location problem had been recognized with the Lanchester Prize, and is now considered one of the seminal papers in theory and computation for Facility Location. As a graduate student, Cornuéjols had also completed several papers on graph theoretical problems. Dorit Hochbaum was a rising star in the area of approximation theory. She had completed her dissertation under the supervision of Marshall Fisher at Wharton. During the late seventies and early eighties, the structure of the group was solidified under the leadership of Balas and the influence of the young researchers that had joined the group, either as faculty or as Ph.D. students. Balas was extremely supportive of the young faculty and interceded on many occasions to shield them from the ongoing debates on OR education within a business school.

In the eighties, the interests of the group in other areas increased. Balas was involved in several projects that would leave a mark in other areas of optimization. He worked on heuristics for general 0-1 programs (pivot-and-complement), heuristics for scheduling problems (the shifting-bottleneck procedure), the structure of set-partitioning and set-covering polyhedra, representation and projection of general polyhedra, and the traveling salesman problem. Thompson developed his pivot-and-probe version of the simplex algorithm, and Cornuéjols wrote a number of papers on a variety of topics, like the structure of the traveling salesman polytope, a comparison of Lagrangian relaxations for Facility Location Problems, and the first steps in the structure of 0-1 matrices. The group lost Pang and Hochbaum in the early eighties, but received two very interesting and different newcomers in the eighties, John Hooker and Michael Trick. Hooker had completed two doctorates, one in philosophy and another one in OR. He had an interest in logic and operations research, and a diverse range of interests in several philosophical aspects of the discipline. Michael Trick joined the group after graduating from Georgia Tech under the supervision of John Bartholdi, and spending a year as a postdoc in Bonn. The two newest hires are R. Ravi and Javier Pena, from Brown University and Cornell University, respectively. Ravi specializes in approximation algorithms for combinatorial optimization problems, and Pena in nonlinear programming.

The academic record of the Operations Research Group at GSIA is more than impressive. Gerald Thompson is the IBM Professor of Systems and Operations Research and a Senior Fellow at the IC2 Institute at the University of Texas. Egon Balas is University Professor and the Thomas Lord Professor of Operations Research. He was recently recognized with the John von Neumann Theory Prize of INFORMS. Gerard Cornuéjols is a recipient of the Lanchester Prize and Editor-in-Chief of Mathematics of Operations Research. John Hooker recently received the INFORMS Award for the Best Paper in the OR/Computer Science Interface, and Michael Trick is the Founding Editor of INFORMS Online and the Director of the Bosch Institute for International Management. His web site (mat.gsia.cmu.edu) is a must-see destination for academics and practitioners with an interest in OR. Finally, R. Ravi is an NSF Career Awardee.

The Operations Research Group is in charge of the Ph.D. program in Operations Research at GSIA, and has joint responsibilities for the program in Algorithms, Combinatorics and Optimization (ACO). The ACO program was created in 1989, in collaboration with the Discrete Math Group in Mathematics and the Theory Group at Computer Science. This program has been so successful that it has been used as a model for other leading institutions throughout the world.

–SEBASTIAN CERIA

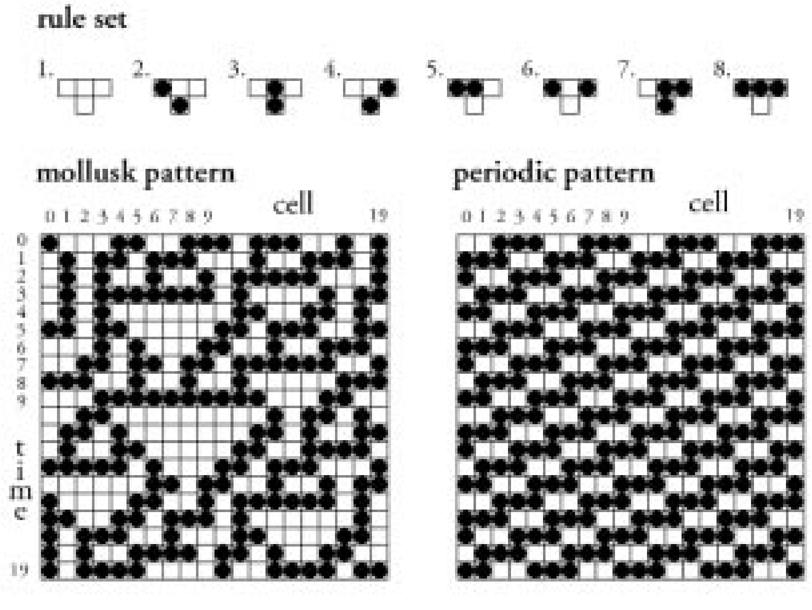
We invite OPTIMA readers to submit solutions to the problems to Robert Bosch (bobb@cs.oberlin.edu). The most attractive solutions will be presented in a forthcoming issue.

Maximizing Vitality

Robert A. Bosch

February 26, 1999

Figure 1 gives the rule set of a simple one-dimensional cellular automaton, first investigated by Stephen Wolfram, that when properly initialized produces “evolution patterns” that look remarkably like the natural patterns found on certain mollusk shells. (See [2, p. 71-73].)



This particular cellular automaton, or CA, consists of *n* cells arranged in a horizontal line. We refer to the leftmost cell as cell 0 and the rightmost cell as cell *n*–1. We consider each cell *i* to have two neighbors: a left neighbor *l*(*i*) and a right neighbor *r*(*i*). For each 0<*i* *n*–1, we set *l*(*i*) = *i*–1; for each 0 *i* < *n*–1, we set *r*(*i*) = *i*+1. To allow the cells to “wrap around,” we set *l*(0) = *n*–1 and *r*(*n*–1) = 0.

At each point in time, each cell is either alive or dead. To start the CA, we must decide which cells will be alive at time 0 and which ones will be dead then. (To generate the mollusk pattern, we decided that cells 0, 4, 5, 8, 9, 10, 12, 13, 14, 17, and 19 would be alive at time 0.) To run the CA, we simply apply the rule set over and over again. The first application of the rule set determines the states of the cells at time *t*=1. The second application determines the states of the cells at time *t*=2. And so on. (In the mollusk pattern, cell 0 is dead at time 1 because of rule 5, which states that if cells *l*(*i*) and *i* are alive at time *t* and cell *r*(*i*) is dead at time *t*, then cell *i* must be dead at time *t*+1. Cell 1 is alive at time 1 because of rule 2. Cell 2 is dead at time 1 because of rule 1.)

The Maximum Average Vitality Problem

We define the *vitality* of a cell over a time interval to be the fraction of the time the cell is alive during that time interval. (In the mollusk pattern, cell 0 has a vitality of 0.45 over the displayed time interval.) In addition, we define the average vitality of the CA over a time interval to be the average of its cells' vitalities over that time interval. (In the mollusk pattern, the average vitality over the displayed time interval works out to be 0.4975.) And finally, we define the maximum average vitality problem to be the problem of finding an initial assignment of states to cells that maximizes the average vitality of the CA over a given time interval [*a*,*b*].

An IP Formulation

It is easy to model the maximum average vitality problem as an IP. For each $0 \le i \le n-1$ and each $0 \le t \le b$, let

$$x_{i,t} = \begin{cases} 1 & \text{if cell } i \text{ is alive at time } t, \\ 0 & \text{if not.} \end{cases}$$

Clearly, our objective is to maximize

$$\sum_{i=0}^{n-1} \sum_{t=a}^{b} x_{i,t} .$$

To enforce rule 1 for cell *i* during the transition from time *t* to time *t*+1, we can impose the constraint

$$-x_{l(i),t} - x_{i,t} - x_{r(i),t} + x_{i,t+1} \le 0, \qquad \begin{array}{c|c|c|c} & l(i) & i & r(i) \\ t & -1 & -1 & -1 \\ t+1 & & 1 & \end{array}$$

which works by prohibiting the only "configuration" that violates rule 1 (i.e., the configuration that has cells *l*(*i*), *i*, and *r*(*i*) dead at time *t*, and cell *i* alive at time *t*+1). Similarly, to enforce rule 6 for cell *i* during the transition from time *t* to time *t*+1, we can impose the constraint

$$x_{l(i),t} + x_{r(i),t} + x_{i,t+1} \le 2, \qquad \begin{array}{c|c|c|c} & l(i) & i & r(i) \\ t & 1 & 0 & 1 \\ t+1 & & 1 & \end{array} \le 2$$

which prohibits the only configuration that violates rule 6 (and, in addition, the only configuration that violates rule 8). To enforce the remaining rules, we can impose similar constraints.

An Upper Bound on Vitality

Using the constraints of the formulation described above, it is relatively easy to prove that the maximum average vitality *v*(*a*,*b*) of the CA over the time interval [*a*,*b*] satisfies

$$v(a,b) \le \frac{3}{5} + \frac{2}{5(b-a+1)}.$$

Note that as the length of [*a*,*b*] increases, the upper bound gets closer and closer to 3/5. As a consequence, it follows that the periodic evolution pattern displayed in Figure 1 is the "most vital" of all periodic evolution patterns.

Problems

Interested readers may enjoy trying to solve the following problems:

1. Find constraints that enforce rules 2-5 and 7.
2. Prove that the upper bound on *v*(*a*,*b*) is correct. *Hint:* Prove that the following inequality is valid for the IP formulation:

$$\begin{array}{c|c|c|c} & l(i) & i & r(i) \\ t & 1 & 1 & 1 \\ t+1 & & 1 & 1 \end{array} \le 3$$

3. The IP formulation has *n*(*b*+1) binary variables. But only *n* of them – the variables for time 0 – are really decision variables in the truest sense. (Once we have the values of the time-0 variables, the values of the remaining ones are completely determined. In fact, we could run the CA to obtain their values.) Devise a solution strategy for the maximum average vitality problem that exploits this "property."

We will present solutions in a future issue of OPTIMA. Please send solutions or comments to bobb@cs.oberlin.edu. See [1] for a detailed discussion of how integer programming can be used to find interesting patterns in another cellular automaton, Conway's game of Life.

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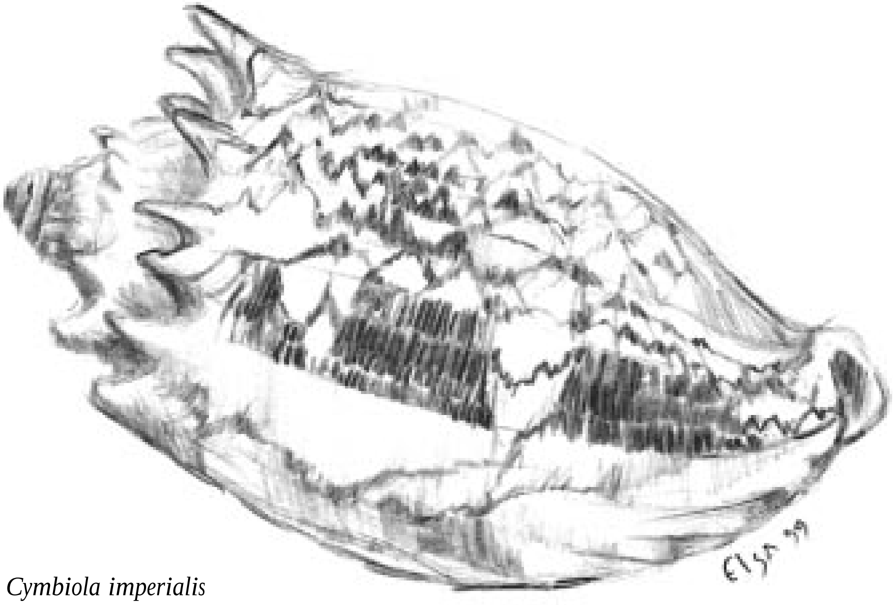

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Conference Calendar

▶ Canadian Operational Research Society Conference CORS '99
June 7-9, 1999, Windsor, Ontario, Canada
URL: http://www.cors.ca/windsor/

▶ Seventh Conference on Integer Programming and Combinatorial Optimization - IPCO '99
June 9-11, 1999, TU Graz, Graz, Austria
URL: http://www.opt.math.tu-graz.ac.at/ipco99

▶ Fourth Workshop on Models and Algorithms for Planning and Scheduling Problems MAPSP '99
June 14-18, 1999, Renesse, The Netherlands
URL: http://www.win.tue.nl/~mapsp99/index.html (general information);
http://www.win.tue.nl/~mapsp99/regis.html (registration)

▶ Workshop on Continuous Optimization
June 21-26, 1999, Rio de Janeiro
URL: http://www.impa.br/~opt/

▶ Computational Mathematics Driven by Industrial Applications
June 21-27, 1999, Martina Franca, Apulia, Italy
URL: http://www.math.unifi.it/CIME/

▶ Paul Erdos and His Mathematics
July 4-11, 1999, Hungarian Academy of Sciences, Budapest, Hungary
E-mail: erdos99@math-inst.hu
URL: http://www.math-inst.hu/~erdos99

▶ Fourth International Conference on Industrial and Applied Mathematics
July 5-9, 1999, Edinburgh, Scotland
URL: http://www.ma.hw.ac.uk/iciam99

▶ First ASMO UK/ISSMO Conference on Engineering Design Optimization
July 8-9, 1999, Ilkley, West Yorkshire, UK
URL: http://www.brad.ac.uk/staff/vtoropov/asmo_uk/asmoukc.htm

▶ 19th IFIP TC7 Conference on System Modeling and Optimization
July 12-16, 1999, Cambridge, England
URL: http://www.damtp.cam.ac.uk/user/na/tc7con

▶ Third Workshop on Algorithm Engineering W A E '99
July 19-21, 1999, London, UK
E-mail: wae99@dcs.kcl.ac.uk
URL: http://www.dcs.kcl.ac.uk/events/wae99/

▶ Second International Workshop on Approximation Algorithms for Combinatorial Optimization Problems,
and
Third International Workshop on Randomization and Approximation Techniques in Computer Science
August 8-11, 1999, Berkeley, CA, USA
URL: http://cuiwww.unige.ch/~rolim/approx; http://cuiwww.unige.ch/~rolim/random

▶ Sixth International Conference on Parametric Optimization and Related Topics
October 4-8, 1999, Dubrovnik, Croatia
URL: http://www.math.hr/dubrovnik/index.htm

▶ INFORMS National Meeting
November 7-10, 1999, Philadelphia, PA, USA
URL: http://www.informs.org/Conf/Philadelphia99/

▶ 7th INFORMS Computing Society Conference on Computing and Optimization: Tools for the New Millenium
January 5-7, 2000, Cancun, Mexico
URL: http://www-bus.colorado.edu/Faculty/Laguna/cancun2000.html

ATLANTA

International Symposium on Mathematical Programming

The 17th International Symposium on Mathematical Programming will be held August 7-11, 2000, on the campus of Georgia Institute of Technology in Atlanta, Georgia, USA. A brochure with more information and a call for papers will be issued soon. The official web page of the symposium is currently under construction, but watch for it at http://www.isye.gatech.edu/ismp2000.

Call for Proposals to Host

ISMP 2003

The time has come for all interested parties to make proposals for hosting the 2003 International Symposium on Mathematical Programming. Following tradition, a university site outside the US will host the 2003 Symposium.

All proposals are welcome and will be examined by the Symposium Advisory Committee, composed of Karen Aardal, John Dennis, Martin Grötschel and Thomas Liebling (Chair). It will make its recommendation based on criteria such as professional reputation of the local organizers, facilities, accommodations, accessibility and funding. Based on the recommendations of the Advisory Committee, the final decision will be made and announced by the MPS Council during the 2000 Symposium in Atlanta.

Detailed proposal letters should be addressed to: Prof. Thomas M. Liebling, DMA-EPFL, CH-1015 Lausanne, Switzerland (E-mail: Thomas.Liebling@epfl.ch).

First Announcement and Call for Papers

Seventh International Workshop on Project Management and Scheduling (PMS 2000)

April 17-19, 2000, University of Osnabrueck, Germany

Following the six successful workshops in Lisbon (Portugal), Como (Italy), Compiegne (France), Leuven (Belgium), Poznan (Poland), and in Istanbul (Turkey), the Seventh International Workshop on Project Management and Scheduling is to be held in Osnabrück, a small, charming city located halfway between Cologne and Hamburg.

The main objectives of PMS 2000 are to bring together researchers in the area of project management and scheduling in order to provide a medium for discussions of research results and research ideas and to create an opportunity for researchers and practitioners to get involved in joint research.

Another objective is to attract new recruits to the field of project management and scheduling to make them feel a part of a larger network. For this aim there will be special sessions on railway scheduling, timetabling, batch scheduling in the chemical industry, and robot scheduling.

Program Committee Peter Brucker, Chair (University of Osnabrueck), Lucio Bianco (IASI, Rome), Jacek Blazewicz (Poznan University of Technology), Fayez Boctor (Laval University), Jacques Carlier (Université de Technologie Compiegne), Eric Demeulemeester (Katholieke Universiteit Leuven), Andreas Drexl (Christian-Albrechts-Universität zu Kiel), Salak E. Elmaghraby (North Carolina State University), Selcuk Erenguc (University of Florida), Willy Herroelen (Katholieke Universiteit Leuven), Wieslaw Kubiak (Memorial University of Newfoundland), Chung-Yee Lee (Texas A & M University), Klaus Neumann (University of Karlsruhe), Linet Ôzdamar (Istanbul Kultur University), James Patterson (Indiana University), Erwin Pesch (University of Bonn), Marie-Claude Portmann (Ecole des Mines de Nancy, INPL), Avraham Shtub (Technion Israel Institute of Technology), Roman Slowinski (Poznan University of Technology), Luis Valadares Tavares (Instituto Superior Technico, Lisbon), Gunduz Ulusoy (Bogazici University, Istanbul), Vicente Valls (University of Valencia), Jan Weglarz (Poznan University of Technology), Robert J. Willis (Monash University).

Preregistration If you are interested in participating, please visit our web site (http://www.mathematik.uni-osnabrueck.de/research/OR/pms2000/) and complete the pre-registration form, or contact us by e-mail (pms2000@mathematik.uni-osnabrueck.de).

Pre-registration does not involve any obligations, but helps us to plan the schedule and keep you informed. In your e-mail please include your surname, first name(s), affiliation and e-mail address, and whether or not you intend to give a talk. Presentations will be selected on the basis of a three-page extended abstract to be submitted no later than September 15, 1999.

Important Dates Abstract submission: September 15, 1999; Notification of acceptance: November 1, 1999; Workshop registration deadline: December 15, 1999.

Registration costs include the conference fee, a welcoming party, coffee breaks, and three lunches. The following prices are provisional: Early registration fee, DM 300; Late registration fee, DM 350; Excursion and dinner, to be announced.

The deadline for early registration is December 15, 1999. Please consult the conference web site to register.

Information Sources For up-to-date information, including information on hotels and the city of Osnabrück, please visit our web site (http://www.mathematik.uni-osnabrueck.de/research/OR/pms2000/).

First Announcement
6th International Symposium on Generalized Convexity/Monotonicity

Karlovassi, Samos, Greece, August 30 - September 3, 1999
and
Summer School on Generalized Convexity/Monotonicity
Karlovassi, Samos, Greece, August 25-28, 1999

Scope Various generalizations of convex functions have been introduced in areas such as mathematical programming, economics, management science, engineering, stochastics and applied sciences. Such functions preserve one or more properties of convex functions and give rise to models that are more adaptable to real-world situations than convex models. Similarly, generalizations of monotone maps have been studied recently. A growing literature in this interdisciplinary field has appeared, including the proceedings of the five preceding international symposia since 1980. The Symposium is organized by the Working Group on Generalized Convexity and aims to review the latest developments in the field.

Invited Speakers J. Jahn (University of Erlangen, Germany), H. Konno (Tokyo Institute of Technology, Japan), P. Pardalos (University of Florida, USA), A. Prekopa (Rutgers University, USA)

Program Committee
S. Komlosi (Pecs, Hungary), Chair; C.R. Bector (Winnipeg, Canada); R. Cambini (Pisa, Italy); B.D. Craven (Melbourne, Australia); J.P. Crouzeix (Clermont-Ferrand, France); J.B.G. Frenk (Rotterdam, The Netherlands); N. Hadjisavvas (Samos, Greece); D.T. Luc (Hanoi, Vietnam); J.E. Martinez-Legaz (Barcelona, Spain); P. Mazzoleni (Milan, Italy); J.P. Penot(Pau, France); S. Schaible (Riverside, USA)

Organizing Committee N. Hadjisavvas (Samos, Greece), Chair; R. Cambini (Pisa, Italy); A. Daniilidis (Pau, France); J.B.G. Frenk (Rotterdam, The Netherlands); S. Schaible (Riverside, USA)

Symposium Information

General information The Symposium will be hosted by the Department of Mathematics at the University of the Aegean, located in Karlovassi on the island of Samos. Samos, the birthplace of Pythagoras and Aristarchus, is one of the biggest and most picturesque islands in the Aegean Archipelago. Its unique natural and archeological sites make it a distinct resort and a historical treasure.

Registration The Symposium fee is USD 150 (USD 75 for students submitting verification of their status) for registration until March 31, 1999 and USD 200 (USD 100 for students) after this date and until June 30, 1999. The fee includes: admission to all sessions, transportation from and to the airport for those participants who will come at the beginning and leave at the end of the Symposium or the Summer School, a one-day excursion around the island, a banquet, and a copy of the proceedings when published. For more information, please send an e-mail to gc6@math.aegean.gr.

Important dates

Please note the following deadlines: Early registration: March 31, 1999; Late registration (after March 31, 1999): June 30, 1999; Final manuscripts of invited papers: At the Symposium; Titles and abstracts of talks: June 30, 1999; Submission of manuscripts for publication in the Symposium proceedings: September 30, 1999

Important addresses/More information
Additional information about the Symposium may be obtained by writing to Mrs. Thea Vigli-Papadaki, Department of Mathematics, University of the Aegean, Karlovassi 83200, Samos, Greece. Phone (+30-273-33914, 34750; Fax: +30-273-33896; e-mail: gc6@math.aegean.gr) or by visiting the web page of the Symposium: http://kerkis.math.aegean.gr/~gc6/GC6.htm.

More information on the activities of the Working Group on Generalized Convexity can be obtained at the URL address http://www.ec.unipi.it/~genconv/.

Proceedings The symposium proceedings will be published by Springer-Verlag in the series "Lecture Notes in Economics and Mathematical Systems." The editors of this volume, responsible for the refereeing process, are: N. Hadjisavvas (nhad@aegean.gr); J.E. Martinez-Legaz (JuanEnrique.Martinez@uab.es); and J.P. Penot (jean-paul.penot@univ-pau.fr). Manuscripts should be written in plain LaTeX (with AMS symbols if necessary). Those not familiar with LaTeX may use an interface such as Scientific Word (choose style: standard LaTeX article). Contributors must send the electronic file and one hard copy to one of the editors by September 30, 1999.

Summer School

A Summer School will precede the Symposium. It will be held from August 25 to August 28, 1999, at the same place as the Symposium (Karlovassi, Samos, Greece). The Summer School aims to introduce graduate and Ph.D. students, as well as scientists from other fields, to the subject of Generalized Convexity and Generalized Monotonicity. Topics include: Introduction to convexity and generalized convexity (J.B.G. Frenk), Uses of generalized convexity in economics (J.E. Martinez-Legaz), Fractional programming (S. Schaible), 1st and 2nd order characterizations (J.P. Crouzeix), Generalized convexity and nonsmooth analysis (J.P. Penot), Duality and application to economics (J.E. Martinez-Legaz), Algorithmical aspects (J.B.G. Frenk), Vector optimization (D.T. Luc), Introduction to global optimization and its applications (P. Pardalos), Generalized monotonicity (S. Schaible), Variational inequalities and equilibrium problems (N. Hadjisavvas).

No fees are required for participation, but graduate and Ph.D. students wishing to attend should send a brief CV and a letter of recommendation to: Professor Nicolas Hadjisavvas, Department of Mathematics, University of the Aegean, 83200 Karlovassi, Samos, Greece. You may also e-mail the above to gc6@math.aegean.gr (CV and recommendation letters are not required for established scientists). An effort will be made to cover, at least in part, local expenses of a number of participants.

From the Nordic Section

The next meeting of the Nordic Section of the Mathematical Programming Society will be held at Mälardalen University in Västerås, Sweden, on September 25-26, 1999. For more information, please look on the web (http://www.ima.mdh.se/tom).

See you there.

–KAJ HOLMBERG

Workshop on Continuous Optimization

Rio De Janeiro, June 21-26, 1999
Special Issue of Annals of
Operations Research

An agreement has been reached with the editors of Annals of Operations Research concerning the publication of a special issue of this journal devoted to papers submitted to our workshop. Papers submitted to the special issue will be subject to the standard refereeing process, and the issue will be a regular one (i.e., it will not be the proceedings of the workshop, but rather it will consist of those papers submitted to the workshop which the referees assess as deserving publication).

We request that all potential participants let us know if they intend to submit a paper to be considered for this special issue. This does not represent a full commitment to submit the paper, or even to attend the workshop, but it will allow us to estimate an upper bound on the number of papers to be processed.

We would appreciate it if you would send a message to optim@impa.br announcing your intention to submit, preferably including a tentative title of the paper. If you do so, you will receive specific directions and deadlines for submission.

Additional Information For further information about the workshop, please consult the Optimization home page at IMPA (http://www.impa.br/~optim/).

Advanced Design Problems in Aerospace Engineering

July 11-18, 1999, Erice, Italy

The meeting will be coordinated by Prof. Aldo Frediani (Pisa University, Italy) and Prof. Angelo Miele (Rice University, Houston, USA). There will be a set of lectures on the design of new generation aircraft (subsonic, transonic, supersonic and hypersonic) and space vehicles (orbital, interplanetar) with a particular emphasis on the interactions of mathematical methods and numerical applications, including optimization, on the design of aerospace vehicles. A limited number of people can attend the meeting. Details can be found on the web page of Ettore Majorana Center of Erice (http://www.ccsem.infn.it).

International Conference on Optimization and Numerical Algebra

September 27-30, 1999, Nanjing Normal University, Nanjing, China

First Announcement

Objectives The conference aims to review and discuss recent advances and promising research trends in some areas of Optimization and Numerical Algebra.

Topics Include *Linear Programming and Nonlinear Programming, Convex Programming and Nonconvex Programming, Nonsmooth Optimization, Global Optimization, Stochastic Programming, Multiobjective Optimization, Network Programming, Variational Inequalities, Linear and Nonlinear Systems of Equations, Least-Squares Problems, Computation of Eigenvalue Problems, Matrix Computation and Generalized Inverses, Applications of Optimization and Numerical Algebra*

Conference Organizers Z. Bai, bzz@lsec.cc.ac.cn (Chinese Academy of Sciences, Beijing); Q. Ni, nifqs@dns.nuaa.edu.cn (Nanjing University of Aero- and Astronautics, Nanjing); L. Qi, l.qi@unsw.edu.au (University of New South Wales, Sydney, Australia); Y. Song, yzsong@pine.njnu.edu.cn (Nanjing Normal University, Nanjing); W. Sun, wysun@pine.njnu.edu.cn (Nanjing Normal University, Nanjing); Y. Yuan, yyx@indy1.cc.ac.cn (Chinese Academy of Sciences, Beijing).

Sponsorship National Natural Science Foundation of China, Chinese Academy of Sciences, Chinese Mathematical Society, Chinese Computational Mathematics Society, Chinese Society of Mathematical Programming, Institute of Computational Mathematics and Sci-Eng Computing, Nanjing Normal University, Nanjing University of Aeronautics and Astronautics

Scientific Program Committee X. Chang (McGill University, Canada), Z. Cao (Fudan University, Shanghai), C. Dang (City University of Hong Kong, Hong Kong), J. Ding (University of Southern Mississippi, USA), M. Fukushima (Kyoto University, Japan), B. He (Nanjing University, Nanjing), C. Kanzow (University of Hamburg, Germany), A. Rubinov (University of Ballarat, Australia), J. Shi (Science University of Tokyo, Japan), T. Tanaka (Hirosaki University, Japan), Ph.L. Toint (University of Namur, Belgium), C. Xu (Xian Jiaotong University, Xian), X. Yang (University of Western Australia, Australia), J. Zhang (City University of Hong Kong, Hong Kong), S. Zhang (Erasmus University, The Netherlands), S. Wu (National Cheng Kung University, Taiwan), J. Yuan (Federal University of Parana, Brazil)

Invited Speakers O. Axelsson (Nijmegen University, The Netherlands), J.R. Birge (University of Michigan, USA), T.F. Coleman (Cornell University, USA), D.Y. Cai (Tsinghua University, Beijing), S.C. Fang (North Carolina State University, USA), M. Ferris (University of Wisconsin, USA), M. Fukushima (Kyoto University, Japan), W. Gander (ETH, Switzerland), J.Y. Han (Chinese Academy of Sciences, Beijing), E.X. Jiang (Fudan University, Shanghai, Shanghai), P. Kall (University of Zurich, Switzerland), W.W. Lin (National Tsinghua University, Taiwan), W. Niethammer (Karlsruhe University, Germany), L. Qi (University of New South Wales, Australia), D. Ralph (University of Melbourne, Australia), Z. Shen (Nanjing University, Nanjing), E. Spedicato (University of Bergamo, Italy), J. Stoer (Wuerzburg University, Germany), J. Sun (National University of Singapore, Singapore), J.G. Sun (University of Umea, Sweden), K. Tanabe (The Institute of Statistical Mathematics, Japan), Ph.L. Toint (University of Namur, Belgium), K.L. Teo (The Hong Kong Polytechnic University, Hong Kong), J.-Ph. Vial (University of Geneva, Switzerland), A.J. Wathen (Oxford University, UK), T. Yamamoto (Ehime University, Japan), Y. Yuan (Chinese Academy of Sciences, Beijing), J. Zhang (City University of Hong Kong, Hong Kong), X. Zhang (Chinese Academy of Sciences, Beijing), Y. Zhang (Rice University, USA)

Call for Papers Titles and abstracts of invited and contributed papers must be received by July 10, 1999. The abstracts should be typed in LaTeX, not exceed one page, and be sent by e-mail (niqfs@dns.nuaa.edu.cn or wysun@pine.njnu.edu.cn).

MAPSP '99

Fourth Workshop on Models and Algorithms for Planning and Scheduling Problems

Second Announcement

June 14-18, 1999, Renesse, The Netherlands

Conference Approach The workshop aims to provide a forum for scientific exchange and cooperation in the field of planning, scheduling, and related areas. To maintain the informality of the previous workshops and to encourage discussion and cooperation, there will be a limit of 100 participants and a single stream of presentations.

Invited Speakers Michel Goemans, CORE, Louvain-la-Neuve, Belgium; Martin Gröetschel, ZIB, Berlin, Germany; Michael Pinedo, New York University, New York, USA; Lex Schrijver, CWI, Amsterdam, The Netherlands; Eric Taillard, IDSIA, Lugano, Switzerland; Richard Weber, University of Cambridge, England; Joel Wein, Polytechnic University, Brooklyn, USA; Gerhard Woeginger, Technische Universitaet Graz, Austria

The invited speakers will present a one-hour lecture. Abstracts of these talks can be found on the web http://www.win.tue.nl/~mapsp99.

Contact Address Cor Hurkens, Department of Mathematics and Computing Science, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands; Fax: (0031) 402465995; E-mail: wscor@win.tue.nl

Organizers/Program Committee Emile Aarts, Eindhoven University of Technology, Philips Research; Han Hoogeveen, Eindhoven University of Technology; Cor Hurkens, Eindhoven University of Technology; Jan Karel Lenstra, Eindhoven University of Technology; Leen Stougie, Eindhoven University of Technology; Steef van de Velde, Erasmus University, Rotterdam

Instructions for Participants Persons interested in participating are encouraged to send an e-mail to: mapsp99@win.tue.nl. Details about dates, accommodation, fees, travel, etc., can be obtained at the web site address listed above.

Important Dates May 1, 1999 - Last date for early registration; June 9-11, 1999 - IPCO '99, Graz; June 14-18, 1999 - MAPSP '99

Conference Sponsors This conference is supported by: Eindhoven University of Technology; Dutch Technology Foundation STW; EIDMA Euler Institute in Discrete Mathematics and its Applications; BETA Research Institute for Business Engineering and Technology Application; IPA Institute for Programming Research and Algorithmics; Baan Company, Ede, The Netherlands; TNO Physics and Electronics Laboratory, The Hague, The Netherlands; Centre for Quantitative Methods CQM B.V., Eindhoven, The Netherlands; OM Partners, Capelle a/d IJssel, The Netherlands; Numetrix, Brussels, Belgium; Philips Research Laboratories, Eindhoven, The Netherlands; and ORTEC Consultants BV, Gouda, The Netherlands.

Call for Applications

1999 Nanjing Award in Optimization and Numerical Algebra for Young Researcher

The applicant should be either a current student, or a researcher who obtained his/her last degree on or after January 1, 1994, or any person born on or after January 1, 1968. Only one paper can be submitted for consideration and it must also be submitted for presentation at the conference. The paper can be co-authored but the applicant must be the major contributor (significantly more than 50%).

The award selection committee consists of: T.F. Coleman (USA), coleman@tc.cornell.edu; P. Kall (Switzerland), kall@ior.unizh.ch; and J. Zhang (Hong Kong), Committee Chair, mazhang@cityu.edu.hk. Applicants should send their application to all three of the award selection committee members by e-mail on or before July 10. The application (an ASCII file) should include a short curriculum vitae and a detailed abstract of the paper (no more than three pages). The technical part of the application should be in plain TeX, LaTeX or should be a separate postscript file, as should be the full paper which may be requested if the selection committee feels it to be necessary in making its judgment.

The committee will shortlist about three papers for final competition and notify the candidates around September 10. The three selected papers will be presented in a special session in the conference. The award will be announced at the conference banquet.

Special Arrangements Conference proceedings, special issues of some journals, tours and accommodations arrangements will be indicated in the Second Announcement.

Further information can be obtained online (http://www.cc.ac.cn, http://www.njnu.edu.cn, http://www.nuaa.edu.cn, http://www.usm.edu) or by contacting the conference organizers or any members of the scientific program committee.

International Conference on Complementarity Problems

June 9-12, 1999, Madison, Wisconsin, USA

The contemporary applications and algorithms that will be emphasized at the meeting will reflect the 35 years that have passed since complementarity was formally introduced and employed as a powerful mathematical model for a wide spectrum of problems in diverse fields. The conference is intended to bring together engineers, economists, industrialists, and academics from the U.S. and abroad who are involved in pure, applied, and/or computational research of complementarity and related problems.

The conference will consist of invited presentations, and is limited to 100 participants (including the speakers). A refereed volume of proceedings of the conference will be published. There are three major themes of the conference: engineering and machine learning applications, economic and financial applications, and computational methods. Each theme will be represented by several experts in the area.

Further details on the meeting, including registration deadlines, hotel and travel information can be found online (http://www.cs.wisc.edu/cpnet/iccp99).

–MICHAEL FERRIS, OLVI MANGASARIAN, JONG-SHI PANG (CO-ORGANIZERS)

Workshop on the Theory and Practice of Integer Programming

in Honor of Ralph E. Gomory on the Occasion of his 70th Birthday

We are pleased to announce a workshop in celebration of Ralph Gomory's 70th birthday. The focus of the workshop will be on integer linear programming. The workshop is sponsored by DIMACS, as part of the 1998-99 Special Year on Large-Scale Discrete Optimization, and by IBM. The workshop will be held August 2-4, 1999, at the IBM Watson Research Center in Yorktown Heights, New York. The workshop will include lectures by leading international experts covering all aspects of integer programming. We hope that the lecture program will be of particular interest to young researchers in the field, including Ph.D. students and postdoctoral fellows.

A conference banquet will be held with Alan Hoffman (IBM) as the Master of Ceremonies. The banquet speakers will include Paul Gilmore (University of British Columbia), Ellis Johnson (Georgia Tech), and Herb Scarf (Yale).

For more details, please see http://dimacs.rutgers.edu/Workshops/Gomory/.

Invited Lecturers include: Karen I. Aardal, Utrecht University; Egon Balas, Carnegie Mellon University; Francisco Barahona, IBM Watson Research Center; Imre Barany, Hungarian Academy of Sciences; Daniel Bienstock, Columbia University; Robert Bixby, Rice University; Charles E. Blair, University of Illinois; Vasek Chvatal, Rutgers University; Sebastian Ceria, Columbia University; Gerard Cornuéjols, Carnegie Mellon University; William H. Cunningham, University of Waterloo; John J. Forrest, IBM Watson Research Center; Michel X. Goemans, Universite Catholique de Louvain; Ralph Gomory, Sloan Foundation; Peter Hammer, Rutgers University; T.C. Hu, University of California at San Diego; Ellis Johnson, Georgia Tech; Mike Juenger, Universitat zu Koeln; Berhard Korte, University of Bonn; Thomas L. Magnanti, Massachusetts Institute of Technology; George L. Nemhauser, Georgia Institute of Technology; Gerd Reinelt, Universität Heidelberg; Martin W.P. Savelsbergh, Georgia Institute of Technology; Herbert E. Scarf, Yale University; Andras Sebö, University of Grenoble; Bruce Shepherd, Lucent Bell Laboratories; Bernd Sturmfels, University of California at Berkeley; Mike Trick, Carnegie Mellon University; Leslie Earl Trotter, Jr., Cornell University; Robert Weismantel, University of Magdeburg; David P. Williamson, IBM Watson Research Laboratory; Laurence Alexander Wolsey, Université Catholique de Louvain; and Günter Ziegler, Technische Universität Berlin.

Conference Organizers: William Cook, Rice University; and William Pulleyblank, IBM Watson Research Center.

Symposium on Operations Research 1999, SOR '99

During September 1-3, 1999, an International Symposium, SOR '99, organized by the German Operations Research Society (GOR) will take place in Magdeburg, Germany. All areas of Operations Research will be covered at this conference. For more information, contact: G. Schwödiauer (general chair), University of Magdeburg, Faculty of Economics and Management, P.O. Box 41 20, D-39016 Magdeburg, Germany; phone +49 391 6718739; fax +49 391 6711136; E-mail schwoediauer@wiwi.uni-magdeburg.de. Additional information about the conference can be found online (http://www.uni-magdeburg.de/SOR99/).

CALL FOR NOMINATIONS

Optimization Prize for Young Researchers

PRINCIPAL GUIDELINE: The Optimization Prize for Young Researchers, established in 1998 and administered by the Optimization Section (OS) within the Institute for Operations Research and Management Science (INFORMS), is awarded annually at the INFORMS Fall National Meeting to one (or more) young researchers for the most outstanding paper in optimization that is submitted to or published in a refereed professional journal. The prize serves as an esteemed recognition of promising colleagues who are at the beginning of their academic or industrial career.

DESCRIPTION OF THE AWARD: The Optimization award includes a cash amount of US$1,000 and a citation certificate. The award winners will be invited to give a one-hour lecture of the winning paper at the INFORMS Fall National Meeting in the year the award is made. It is expected that the winners will be responsible for the travel expenses to present the paper at the INFORMS meeting.

ELIGIBILITY: The authors and paper must satisfy the following three conditions to be eligible for the prize:

(a) the paper must either be published in a refereed professional journal no more than three years before the closing date of nomination, or be submitted to and received by a refereed professional journal no more than three years before the closing date of nomination;

(b) all authors must have been awarded their terminal degree within five years of the closing date of nomination;

(c) the topic of the paper must belong to the field of optimization in its broadest sense.

THE PRIZE COMMITTEE: The prize committee for 1999 consists of John Birge, Gerard Cornuéjols, Michel Goemans, Jong-Shi Pang and Michael Todd.

NOMINATION: Nominations should be sent before July 15, 1999 to Gerard Cornuéjols Graduate School of Industrial Administration Carnegie Mellon University Pittsburgh, PA 15213, or to any other member of the prize committee. Nominations should be accompanied by a supporting letter.

Combinatorial Optimization

W.J. Cook, W.H. Cunningham, W.R. Pulleyblank
and A. Schrijver

Wiley, 1998

ISBN 0-471-55894-X

Combinatorial optimization is by now a mature field, yet few textbooks covering the area are available. This new book (sometimes referred to as the "4 Bill's book," because of the first names of the authors) is highly welcome, and will certainly become a classic in the area.

The primary use of this book is as a textbook. It is equally suitable for undergraduate and graduate courses. The coverage is indeed extensive, starting from elementary topics such as the shortest path and minimum spanning tree problems to more advanced topics more appropriate for (even advanced) graduate courses (or self-study) such as total dual integrality, weighted matroid intersection or the maximum cut problem on planar graphs. The book is self-contained with a brief appendix with the important concepts/results in linear programming and a chapter on NP-completeness. There are also many exercises (without solutions) throughout the book, spanning a wide spectrum of difficulty.

The book covers a variety of very recent advances, such as novel approaches for the minimum cut problem (the deterministic and random contraction algorithms of Nagamochi and Ibaraki, and Karger, respectively). Elegance and simplicity were probably the criteria used by the authors in selecting which developments to present. When appropriate, simple applications are discussed in the book. For example, the authors show how the search of a rectilinear planar layout can be formulated as a minimum cost flow problem, or how the max flow/min cut theorem can be used to decide if sports teams will be eliminated.

Being primarily a textbook does not mean that researchers cannot benefit from its reading. Because of the wide array of results covered in this book, most, or maybe even all of us except maybe the authors themselves, will discover a few gems (both in terms of results and proofs) while reading it.

The exposition is clear and mathematically rigorous. Also, the authors give intuitive (or informal) explanations whenever possible and sometimes mention why alternative approaches fail. The proofs are written with great care.

One aspect that I particularly liked about the book is the fact that several problems are looked at from different perspectives. Combinatorial optimization is at the intersection between combinatorics, linear programming and algorithms, and we are often reminded of this multi-faceted aspect of the field in this book. As an illustration, for the non-bipartite matching problem, the Tutte-Berge formula is given and proved in Section 5.1, Edmonds' blossom algorithm is presented in Section 5.2, and polyhedral results are discussed and proved in several ways in Chapter 6 (including solving matching problems using a cutting plane algorithm based on a minimum odd cut separation routine).

In summary, this book should definitely be considered by instructors of combinatorial optimization courses and can also be invaluable to any researcher in the field.

—MICHEL GOEMANS, LOUVAIN LA NEUVE

Handbook of Discrete and Computational Geometry

edited by Jacob E. Goodman and Joseph O'Rourke

CRC Press, 1997

ISBN 0-8493-8524-5

This extensive handbook sums up the knowledge in discrete geometry and the newer field of computational geometry, two fields that have flourished in the last decade due to the collaboration between them. The book contains 52 chapters written by different leading researchers in their respective areas. The chapters are arranged in six major sections, ranging from more theoretical aspects of *combinatorial and discrete geometry* and the theory of *polytopes and polyhedra*, over *algorithms and complexity of fundamental geometric objects* and a summary of the most important *geometric data structures and searching* and *computational techniques*, to *applications of discrete and computational geometry*.

Although some chapters are probably of interest mainly for researchers in specialized areas (like a chapter on polyominoes), much of the covered material is relevant for mathematical programmers: on the one hand, geometric notions are often prevalent in optimization with several variables, like the obvious relation between linear programming theory and convex polytopes, to which six chapters of the book are devoted; but as spatial and geometric computation is becoming more and more important in applications, it is also indispensable to have some knowledge of the basic techniques and results of computational geometry, as they are given in the later parts of the handbook.

The book is very clearly structured. After a short introduction, each chapter (or subsection) starts with a glossary, giving concise definitions of the main concepts and technical terms. The main text contains explanations of the concepts and a summary of the main results; sometimes the main ideas of proofs are sketched, and open problems are mentioned at the end. Very often, comparisons between different algorithms or results are summarized in tables. Each chapter concludes with references to other sources like monographs, textbooks, or survey articles, and to related chapters in the book. The cited literature is listed with each chapter.

There is only little overlap between different chapters, like recurring definitions of the affine span or what a subdivision is. Each chapter can be read independently of the other chapters, and the material is presented in such a way that a novice can quickly grasp the main concepts and results of a subject. The layout and visual appearance are very appealing and help to emphasize the structure. This, as well as the extensive index of terms, makes the book also well accessible as a reference work.

I will list only those chapters that are probably most interesting from the point of view of mathematical programming. The chapters about *polytopes* has already been mentioned. The algorithmic problems that are treated in the section about *algorithms* include *convex hull computations*, *Voronoi diagrams and Delaunay triangulations*, various other sorts of optimal *triangulations* and mesh generation, *geometric reconstruction problems* of objects about which only partial or indirect information (like projections or cross-sections) is available, and *shortest paths and networks* in geometric settings like a surface or room with obstacles.

The fundamental geometric problems that are dealt with in the section about *data structures* are *point location* (locating a point among a previously given set of regions); *range searching* (the problem of preprocessing a set of objects so that one can quickly report or count the objects contained in a query region; for example, in a rectangle or half-space); *ray shooting and lines in space*, and *geometric intersection* problems.

The section about *computational techniques* consists of four chapters covering the geometric aspects and applications of techniques which are relevant for algorithm design in general: *randomized algorithms*, *robust geometric computation*, dealing with problems of numerics and degeneracy, *parallel algorithms in geometry*, and the technique of *parametric search*.

The section about *applications* starts with a chapter on *linear programming in low dimensions*. Then there is even a chapter on *mathematical programming* (by Mike Todd). The other practical chapters *algorithmic motion planning*, *robotics*, *computer graphics*, *pattern recognition*, *graph drawing*, *splines and geometric modeling*, geometric problems in automated *design and manufacturing* like molding, milling, and inspection of parts, *solid modeling*, *geometric applications of the Grassmann-Cayley algebra* to mechanical problems of bar frameworks in robotics, *rigidity and scene analysis, sphere packing and coding theory*, and *crystals and quasicrystals*. The final chapter on *computational geometry software* (by Nina Amenta) provides a valuable starting point for people that are looking for ready-made computer programs for their geometric problems, and gives the sources for many publicly available codes, complete with internet addresses.

–GÜNTER ROTE, GRAZ

gallimaufry

Professor ***Sigfried Schaible***, University of California, Riverside, was elected AAAS Fellow in September 1998 by the American Association for the Advancement of Science "for pioneering studies in optimization and operations research, particularly in fractional programming and generalizations of convexity and monotonicity for mathematical programming and its extensions."

The deadline

for the next issue

of OPTIMA is

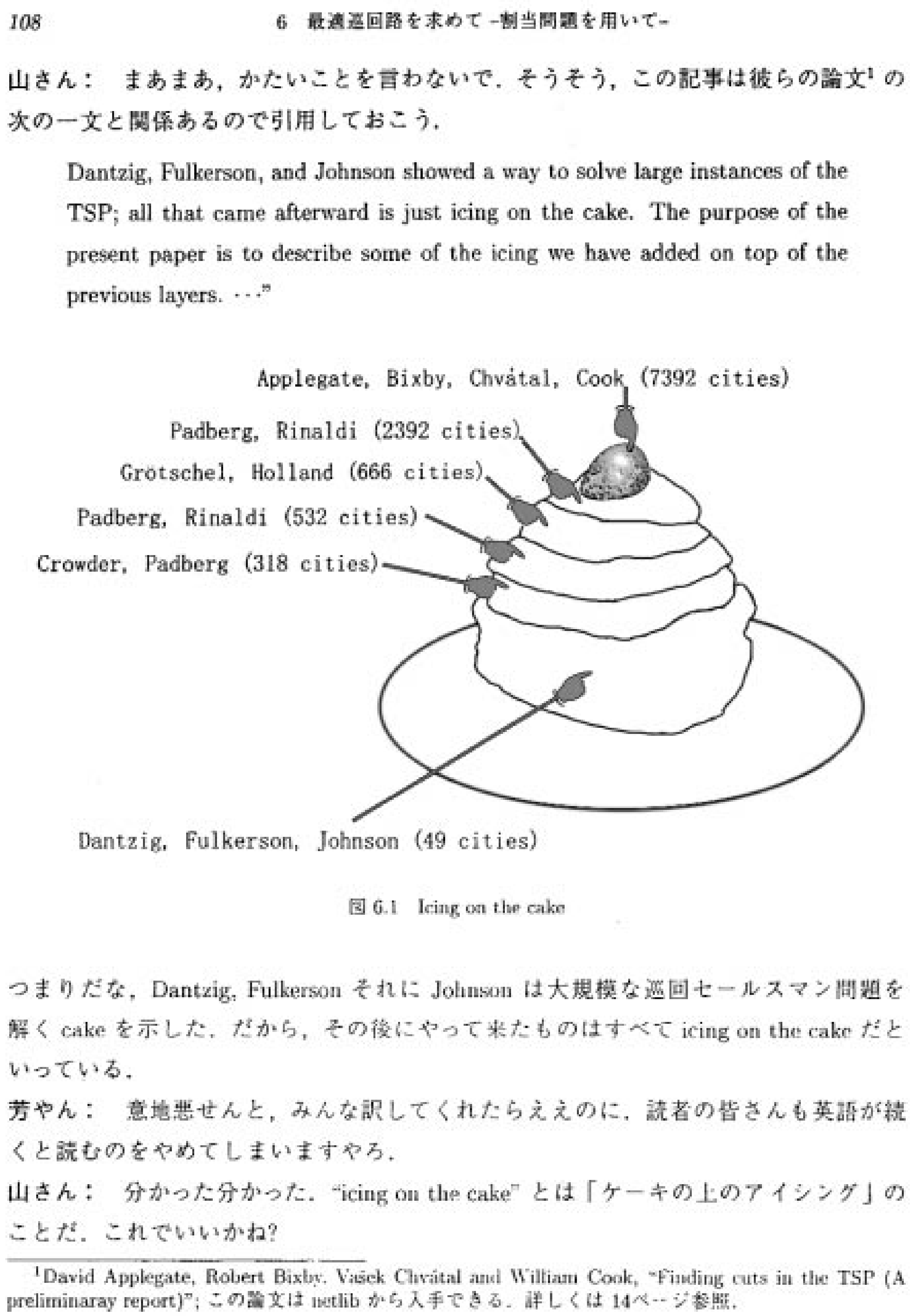
June 15, 1999.

For the electronic version of OPTIMA, please see:

http://www.ise.ufl.edu/~optima/

The following page is reprinted with permission from the book *Invitation to the Traveling Salesman Problem* by ***Yoshitsugu Yamamoto*** and ***Mikio Kubo***, published by Asakura, 1996. The authors cite a passage from the article "*7397-City Traveling Salesman Instance Solved – Another Layer of Icing on the Cake*," which appeared in OPTIMA No. 45, 1995. The Japanese illustration is clearly inspired by the subtitle!

–KAREN AARDAL



山さん：　まあまあ，かたいことを言わないで．そうそう，この記事は彼らの論文[1]の次の一文と関係あるので引用しておこう．

> Dantzig, Fulkerson, and Johnson showed a way to solve large instances of the TSP; all that came afterward is just icing on the cake. The purpose of the present paper is to describe some of the icing we have added on top of the previous layers. ···"

図 6.1　Icing on the cake

つまりだな，Dantzig, Fulkerson それに Johnson は大規模な巡回セールスマン問題を解く cake を示した．だから，その後にやって来たものはすべて icing on the cake だといっている．

芳やん：　意地悪せんと，みんな訳してくれたらええのに．読者の皆さんも英語が続くと読むのをやめてしまいますやろ．

山さん：　分かった分かった．“icing on the cake” とは「ケーキの上のアイシング」のことだ．これでいいかね?

---

[1] David Applegate, Robert Bixby, Vašek Chvátal and William Cook, “Finding cuts in the TSP (A preliminaray report)”; この論文は netlib から入手できる．詳しくは 14ページ参照．

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–SPRINGER-VERLAG, MARCH 1999
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