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ISMP 2006 and Rio de Janeiro

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ISMP 2006 and Rio de Janeiro

Rio de Janiero welcomed the participants to the 19th International Symposium on Mathematical Programming with its world famous features: beautiful landscape, friendliness and hospitality of the people, excellent food and great caipirinhas. The weather was sunny and beautiful before and after, but was less inviting during the exact days of the conference, which we believed was skillfully arranged by the organizers to ensure exceptionally high attendance of the sessions of the Symposium. And indeed it further improved an exceptional scientific program, which included outstanding plenary and semi-plenary talks, great contributed and invited sessions. Thanks to the infectious Brazilian style and at the same time, the exciting (due to being in Rio against all odds, namely Varig's ill-timed bankruptcy, after perilous journeys around the world) atmosphere made communication and interaction among all of us extremely pleasant. It is difficult to capture this atmosphere on paper in a short time and we resisted trying. However, we are reproducing for the OPTIMA readers one of the peaks of the conference which was, as is traditional for the Mathematical Programming Symposia, the awarding of the triennial prizes. We have asked the chairmen of the committees to provide detailed descriptions of each prize which are given below. These interesting outcome is presented below showing once again the achievements and exciting challenges of Mathematical Programming in the past and in the next few years.

As the new editorial board we hope to earn a warm welcome from the readers of OPTIMA by starting our first issue with the reminder of the good times in Rio.

Alberto Caprara, Andrea Lodi,
Katya Scheinberg

The A. W. Tucker Prize

Description and Committee

The A. W. Tucker Prize is awarded for an outstanding paper or Ph.D. thesis by a student in the area of mathematical programming. This year on the Tucker Prize committee were Monique Laurent, Jong-Shi Pang, Ruediger Schultz and Tom McCormick (chair).

Finalists and Winner

The 2006 prize attracted 21 nominations (a record number) of amazingly high quality, most of which were Ph.D. theses. There was a lot of geographical diversity in the nominations: 11 from North America, 7 from Europe, and 3 from elsewhere; also diversity in area of study: doing some double-counting, 11 were on continuous optimization, 12 on discrete, 9 on stochastic problems, and 14 included some computation. The committee was especially pleased to see so much computation, and that such strong work is being done in the stochastic area.

The three finalists were **José Rafael Correa**, **Dion Gijswijt** and **Uday V. Shanbhag** and overall, **Dr. Shanbhag**'s impressive dissertation, by its breath and depth, qualified the work as the winner of the 2006 A.W. Tucker Prize.

José Rafael Correa graduated as a Mathematical Engineer from Universidad de Chile in 1999. He completed his Ph.D. in Operations Research at MIT under the supervision of Michel Goemans and Andreas Schulz in June 2004. Currently Dr. Correa is an Assistant Professor at the School of Business at Universidad Adolfo Ibáñez.

Dr. Correa was named as a Tucker Prize finalist for his Ph.D. thesis titled "Approximation Algorithms for Packing and Covering Problems". This thesis develops approximation algorithms for applied problems in three quite different areas. The first comes from scheduling packets in an interconnection network, which gets abstracted into a problem of coloring edges in bipartite graphs, and then further into a bin-packing problem. The second considers the natural problem of packing rectangles (or higher-dimensional cubes) into boxes. It extends previous results about 2-dimensional bin-packing to find an asymptotic polynomial time approximation scheme for packing d-dimensional cubes into unit cubes, and it gets new results for packing rectangles into square bins. In both cases new tools are developed to make the arguments work. The third considers the classic problem of scheduling precedence-constrained jobs on a single machine to minimize the average weighted completion time. It significantly extends known results by using linear programming relaxations to show that essentially all known 2-approximation algorithms comply with the "Sidney decomposition", and then shows that the sequencing problem can be seen as a special case of vertex cover.

Dion Gijswijt completed his curriculum in Mathematics at the University of Amsterdam, where he graduated in 2001, and received his Ph.D. degree under the supervision of Lex Schrijver in September 2005. He is currently a researcher at the Eötvös University in Budapest, and he will join the University of Amsterdam in September 2006.

Dr. Gijswijt has been selected as a Tucker Prize finalist for his Ph.D. thesis entitled "Matrix Algebras and Semidefinite Programming Techniques for Codes". This thesis presents a novel method for bounding the maximum cardinality of a nonbinary code with given minimum Hamming distance, which is one of the most basic problems in coding theory and an instance of the stable set problem in Hamming graphs. The method also applies to the dual problem of bounding the minimum size of covering codes. The new bound he proposes improves the classical linear programming bound of Delsarte, and gives sharper estimates than the state-of-the art methods on many instances of codes up to length 12 on alphabets of sizes 3, 4, and 5.

The method of Dr. Gijswijt relies on deep insight from noncommutative algebra allied with the use of semidefinite optimization. While the algebraic ingredient in the Delsarte method is the commutative Bose-Mesner algebra of the Hamming scheme, the noncommutative Terwilliger algebra plays a crucial role in Gijswijt's method. Extending earlier work of Schrijver for the binary case, Gijswijt finds the explicit block-diagonalization of the Terwilliger algebra, which enables him to apply symmetry reduction and reformulate his new bound via a compact semidefinite program of size O(n^3) for a code with word length n. Gijswijt also studies the link to the matrix-cut method of Lovász and Schrijver.

This work shows how sophisticated algebraic techniques can be successfully used for exploiting symmetries and formulate compact semidefinite relaxations for hard combinatorial optimization problems, thus adding a new algebraic technique to the mathematical programming toolbox.

Uday V. Shanbhag obtained his undergraduate degree in engineering at the Indian Institute of Technology, Bombay (Mumbai) in 1993, and his Ph.D. in the department of Management Science and Engineering at Stanford University in 2006 under the direction of Walter Murray. Currently, he is an Assistant Professor in the Department of Mechanical and Industrial Engineering at the University of Illinois at Urbana-Champaign.

Uday Shanbhag's doctoral dissertation, titled "Decomposition and Sampling Methods for Stochastic Equilibrium Problems", deals with a novel class of extremely difficult yet practically very important optimization problems constrained by equilibrium conditions and subject to uncertainty. Successive chapters deal with stochastic quadratic programs with recourse (extending previous works on importance-sampling-based L-shape decomposition methods), mathematical programs with complementarity constraints (MPCCs) (proposing an interior-point based method that calculates stationary solutions satisfying an MPCC second-order condition, compared to prior methods that found only first-order solutions), two-stage MPCCs with uncertainty (solving via a primal-dual method that relies on sampling and a scenario-based decomposition), and a two-period spot-forward market under uncertainty formulated as a stochastic Nash-Stackelberg game (motivated by applications to electric power markets with oligopolies). The research utilizes and advances the state-of-the-art nonlinear programming and Monte Carlo sampling methods for tackling such problems. Several new ideas and formulations are introduced; great care is placed on the highly technical convergence analysis; and the results from the implementation of the proposed methods on realistic applications are interpreted with interesting insights. The end product is an outstanding piece of work that combines theory, algorithms, applications, and implementations, bringing together and significantly advancing several areas in continuous optimization, and enabling the application of new optimization paradigms.

The Beale-Orchard-Hays Prize

Description and Committee

The Beale-Orchard-Hays committee composed by Bill Cook, Michael Juenger, Franz Rendl, and Steve Wright (chair) received 8 nominations in various areas of computational mathematical programming. According to its character, the prize is awarded to a paper appearing within the past three years that describes development of software, the computational evaluation and testing of new algorithms, and the development of new methods for empirical testing of algorithms, all in the area of mathematical programming. The nominations spanned a broad spectrum of the discipline and included both well established and new software packages, along with testing of new algorithmic approaches.

Winner

The prize was awarded to **Nick Sahinidis** and **Mohit Tawarmalani** for their paper "A polyhedral branch-and-cut approach to global optimization", Mathematical Programming, Series B 103 (2005), pp. 225-249. The approaches described in this paper are implemented in the BARON system which represents a powerful approach for the global optimization of nonlinear optimization problems, including problems with integer variables. The nominated paper develops techniques that enhance previous versions of BARON. In particular, it uses factorable decompositions of nonlinear functions into subexpressions to construct polyhedral outer approximations that exploit convexity more thoroughly and yield tighter underestimators. BARON also incorporates techniques from automatic differentiation, interval arithmetic, and other areas to yield an automatic, modular, and relatively efficient solver for the very difficult area of global optimization.

The George B. Dantzig Prize

Description and Committee

The George B. Dantzig prize is awarded jointly by the Mathematical Programming Society and the Society for Industrial and Applied Mathematics. The prize is awarded for original research, which by its originality, breadth and depth, is having a major impact on the field of mathematical programming. The contribution(s) for which the award is made must be publicly available and may belong to any aspect of mathematical programming in its broadest sense. Strong preference is given to candidates that have not reached their 50th birthday in the year of the award. Committee Members this year were Arkadi Nemirovskii, Lex Schrijver, Jong-Shi Pang and Bob Bixby (chair).

Winner

The committee has selected **Eva Tardos** as the lone recipient of the Dantzig Prize this year. She is warded the prize for deep and wide-ranging contributions to mathematical programming. In the 1980s, she solved a long-standing open problem by finding the first strongly polynomial-time algorithm for minimum-cost flows, a truly ground-breaking result. Most subsequent work on minimum-cost flows, including several of the currently fastest algorithms has roots in her revolutionary method. This first result in network flow is just one of numerous results by Tardos in the area. More generally, a wide range of network flow models can be viewed as a special case of linear programming, and hence is solvable in polynomial time. Tardos has made significant advances in the design of more efficient polynomial-time algorithms for these problems by exploiting their network structure. Among these is the first polynomial-time combinatorial algorithm for generalized network flows. She also obtained polynomial-time algorithms for certain multi-commodity flow problems; among them the maximum concurrent flow problem. These results have led to further generalizations for a wide array of combinatorially-defined linear programs, generally known as fractional packing and covering problems.

Tardos has been a leader in the use of sophisticated mathematical programming techniques in the design of approximation algorithms for NP-hard discrete optimization problems. For example, for the generalized assignment problem, Tardos showed that any feasible solution to a natural linear programming relaxation can be rounded to be integer, where the resulting solution cost not greater than twice that of the fractional solution.

Throughout the years, Eva Tardos has renewed herself scientifically by changing the focus of her work, most recently by laying foundations for new, important directions in algorithmic game theory. Her work showing performance bounds for "selfish routing" has gained attention for its crystalization of the notion of the price of anarchy. One statement of her breakthrough result is that the negative effect of allowing users to selfishly route their traffic is completely offset by building a network of double the capacity.

Tardos is a stimulating lecturer and collaborator, always willing to exchange ideas, and often with clever and surprisingly ingenious approaches that turn out to be basis for subsequent research. She has trained an impressive line of students, written many valuable surveys, and has played a particularly important role in linking Mathematical Programming and Computer Science. The depth, breadth, originality, and impact of her work make her a very deserving winner of the Dantzig Prize.

The Fulkerson Prize

Description and Committee

The Fulkerson Prize is for outstanding papers in the area of discrete mathematics. The prize is sponsored jointly by the Mathematical Programming Society and the American Mathematical Society. Up to three awards are presented at each (triennial) International Symposium of the Mathematical Programming Society. To be eligible for this year's award, a paper had to be published in a journal between January 2000 and December 2005. The committee consisted of Noga Alon, Bill Cunningham and Michel Goemans (chair).

Winners

Three prizes were given this year and the three winning papers are:

Manindra Agrawal, Neeraj Kayal and Nitin Saxena, "PRIMES is in P", Annals of Mathematics, 160 (2004), pp. 781--793.

Testing whether an integer is a prime number is one of the most fundamental computational and mathematical problems. The existence of short certificates for both compositeness and primality was known since the 70's and suggested that primality testing might be in P. Yet, despite numerous efforts and a flurry of algorithms, it was not until 2002 that Agrawal, Kayal and Saxena devised the first deterministic polynomial-time algorithm for primality testing. Earlier algorithms had either assumed the generalized Riemann hypothesis, or were randomized or were only subexponential. This is a stunning development. This result is a true masterpiece, combining algebraic and number theoretic results in a seemingly simple way.

Mark Jerrum, Alistair Sinclair and Eric Vigoda, "A polynomial-time approximation algorithm for the permanent of a matrix with nonnegative entries", J. of ACM, 51 (2004), pp. 671--697.

The permanent of a matrix has been studied for over two centuries, and is of particular importance to statistical physicists as it is central to the dimer and Ising models. For a 0-1 matrix, it represents the number of perfect matchings in the corresponding bipartite graph. Although polynomial-time computable for planar graphs, the computation of the permanent is #P-complete for general graphs as shown by Valiant in 1979. This opened the search for approximation schemes. In this paper, Jerrum, Sinclair and Vigoda give the first Fully Polynomial Randomized Approximation Scheme for computing the permanent of any 0-1 matrix or any nonnegative matrix. This is a remarkable result. Their algorithm is based on updating a Markov chain in a way that quickly converges to a rapidly mixing non-uniform Markov chain on perfect matchings and near-perfect matchings. Their work builds upon the earlier pioneering work of Jerrum and Sinclair who initiated the use of rapidly mixing Markov chains for combinatorial problems.

Neil Robertson and Paul D. Seymour, "Graph Minors. XX. Wagner's Conjecture", Journal of Combinatorial Theory, Series B, 92(2004), pp. 325--357.

Kuratowski's theorem says that a graph is planar if and only if it does not contain K_5 or $K_{\{3,3\}}$ as a minor. Several other excluded minor characterizations are known, and Wagner conjectured that any minor-closed graph property can be characterized by a finite list of excluded minors. Restated, this says that for any infinite family of finite graphs, one of its members is a minor of another one. In a remarkable tour de force, Robertson and Seymour proved Wagner's conjecture, and this paper appeared as part 20 of their monumental work on the theory of graph minors. Their proof of the Graph Minor Theorem required the development of many graph theoretic concepts, such as linkages and tree-width. This is a spectacular achievement in graph theory with far reaching consequences. It shows, for example, that embeddability in any fixed surface can be characterized by a finite list of excluded minors, or that the disjoint paths problem can be solved in polynomial time for a fixed number of terminals.

The Lagrange Prize

Description and Committee

The Lagrange Prize in Continuous Optimization, given jointly by Mathematical Programming Society and Society for Industrial and Applied Mathematics, is awarded for outstanding works in the area of continuous optimization. The committee for the 2006 prize consisted of John Dennis, Nick Gould, Adrian Lewis, and Mike Todd (chair).

Winners

A number of strong nominations were received, and the committee had a lively discussion concerning their merits. The committee unanimously chose Roger Fletcher, Sven Leyffer and Philippe Toint to win the prize for their papers:

Roger Fletcher and Sven Leyffer, "Nonlinear programming without a penalty function", Mathematical Programming, 91 (2002), pp.239-269

Roger Fletcher, Sven Leyffer, and Philippe L. Toint, "On the global convergence of a filter-SQP algorithm", SIAM J. Optimization, 13 (2002), pp. 44-59.

The citation reads: In the development of nonlinear programming over the last decade, an outstanding new idea has been the introduction of the filter. This new approach to balancing feasibility and optimality has been quickly picked up by other researchers, spurring the analysis and development of a number of optimization algorithms in such diverse contexts as constrained and unconstrained nonlinear optimization, solving systems of nonlinear equations, and derivative-free optimization. The generality of the filter idea allows its use, for example, in trust region and line search methods, as well as in active set and interior point frameworks. Currently, some of the most effective nonlinear optimization codes are based on filter methods. The importance of the work cited here will continue to grow as more algorithms and codes are developed.

The filter sequential quadratic programming (SQP) method is proposed in the first of the two cited papers. Many of the key ideas that form the bases of later non-SQP implementations and analyses are motivated and developed. The paper includes extensive numerical results, which attest to the potential of the algorithm.

The second paper complements the first, using novel techniques to provide a satisfying proof of correctness for the filter approach in its original SQP context. The earlier algorithm is simplified, and in so doing the analysis plays its natural role with respect to algorithmic design.

International conference on Algorithmic Operations Research

The 2nd International conference on Algorithmic Operations Research (AlgOR 2007) will be held at Simon Fraser University-Surrey, Canada during January 21-23, 2007. To submit a paper for presentation or to register for the conference please use the online registration form at the conference web page. An international Operations Research Case Competition for students is also planned. For further details please visit the conference web page at:

http://math-optima1.surrey.sfu.ca/algor2007/orc.htm

Selected papers from conference will be published as a special issue of the journal Algorithmic Operations Research (http://journals.hil.unb.ca/index.php/AOR)

CIME School on Nonlinear Optimization

preliminary announcement

Organizes a school on
Nonlinear Optimization
The school will take place in Cetraro,Cosenza, Italy - Grand Hotel S. Michele - from July 1 to July 7, 2007.

Lectures:

Prof. Immanuel Bomze
Univ. of Vienna, Austria
Global Optimization

Prof. Vladimir Demianov
St. Petersbourg State Univ., Russia
Non-smooth optimization

Prof. Roger Fletcher
Univ. of Dundee, UK
Sequential Quadratic Programming

Prof. Tamas Terlaky
Mc Master Univ. Canada
Interior Point Methods

Course directors are

Prof. Gianni Di Pillo
Univ. La Sapienza, Rome, Italy
dipillo@dis.uniroma1.it

Prof. Fabio Schoen
Univ. di Firenze, Italy
fabio.schoen@unifi.it

Registration begins January 2007. It is free, but is required in advance. Visit CIME at www.cime.unifi.it for registration.

Match, match, match, and match again

Gerhard J. Woeginger*

Abstract

We discuss four discrete problems, one problem for each occurrence of the word 'match' in the title of this paper. The solutions of all four problems are based on underlying matching problems.

Introduction

Matching is one of the most fundamental, most popular, and most studied problems in Math-ematical Programming. Every OPTIMA reader knows that the goal of a matching problem is – speaking somewhat sloppily – to pair up objects so that the sum of the weights of these pairs becomes maximum (or minimum). Every OPTIMA reader knows everything that can possibly be known about perfect matching, maximum cardinality matching, bipartite matching, aug-menting paths, the Hungarian method, and the blossom algorithm of Edmonds. (And if thereshould exist some reader who does not know everything, he might want consult the survey [7]by Gerards or the books [9] by Schrijver.)

In this paper we will discuss four discrete problems that can be solved quite elegantlythrough the classical matching machinery. At first sight, however, none of these four problems appears to be a 'clean' matching problem:

- Problem #1 is a scheduling problem. The matched objects are jobs, and feasible match ings must agree with an underlying partial order.
- Problem #2 is a geometric problem. The matched objects are points, and in feasible matchings certain pairs of edges must not show up together. (In general, matching problems with such forbidden pairs are NP-complete.)
- Problem #3 again is a geometric problem. The matched objects are points and wedges, and feasible matchings must satisfy a global covering condition. (In general, matching problems with an additional global condition are NP-complete.)
- Problem #4 concerns a two-player game. The game is centered around paths, and not around matchings.

All four problems can be rewritten and reformulated, so that they turn into standard matching problems. All four problems are easy to grasp, and they all possess nice and short solutions. Hence, these problems might be appropriate for spicing up homework assignments and class- room exercises.

1 Scheduling under precedence constraints

Fujii, Kasami & Ninomiya [3] consider a processing system with two machines and n jobs. Each of the jobs must be processed for one time unit without interruption on one of the two machines. The jobs are partially ordered by a transitive, anti-symmetric, irreflexive precedence relation: If $i \prec j$ holds for two jobs i and j, then the processing of job i must be completed before the processing of job j can start. Two jobs i and j are called *compatible*, if neither $i \prec j$ nor $j \prec i$ holds, that is, if i and j can be processed simultaneously on the two machines without violating the precedence constraints. A job without predecessors is called a *minimal* job. The problem is to decide whether all jobs can be processed within t time units. Without loss of generality, we will assume that there are exactly $n = 2t$ jobs. (If $n > 2t$, the problem trivially has no solution. If $n < 2t$, then we may add $2t - n$ dummy jobs to the instance that do not interact with the other jobs through the precedence constraints.)

Here is a solution for this problem. We construct an undirected auxiliary graph G with the jobs as vertices, and an edge between job i and job j if and only if i and j are compatible. If there exists a feasible schedule, then the pairs of jobs that are processed simultaneously yield a perfect matching in the auxiliary graph G. The surprise is that also the reverse statement is true: If G contains a perfect matching, then there exists a feasible schedule.

Lemma 1 *If the auxiliary graph G has some perfect matching, then it also has a perfect matching that matches two minimal jobs.*

Proof. Put a cost of 2 on every edge that connects two minimal jobs, and put a cost of 1 on all the other edges in G. Compute a perfect matching $\mathcal{M}$ of maximum cost. Suppose for the sake of contradiction that $\mathcal{M}$ does not contain any edges that connect two minimal jobs.

Consider a minimal job a and its partner b in $\mathcal{M}$; since b is non-minimal, it is preceded by some other minimal job a'. Repeated application of this observation yields that there exists a cyclic sequence $a_1, \ldots, a_s, a_{s+1} = a_1$ of minimal jobs with non-minimal partners $b_1, \ldots, b_s, b_{s+1} = b_1$ in $\mathcal{M}$, such that $a_{i+1} \prec b_i$ holds for all $i = 1, \ldots$, s. Now consider the four jobs a_i, b_i, a_{i+1}, and b_{i+1} for some fixed i.

- If b_i and b_{i+1} are compatible, then we could replace the edges $[a_i, b_i]$ and $[a_{i+1}, b_{i+1}]$ in $\mathcal{M}$ by $[a_i, a_{i+1}]$ and $[b_i, b_{i+1}]$. This would increase the cost of $\mathcal{M}$; a contradiction.
- If $b_i \prec b_{i+1}$ holds, then $a_{i+1} \prec b_i$ together with the transitivity of the precedence constraints implies ai+1 $a_{i+1} \prec b_{i+1}$. Then a_{i+1} and b_{i+1} are not compatible; a contradiction.

Therefore, bi+1 Á bi must hold for i = 1; : : : ; n. But this means that the jobs b1; : : : ; bs; b1 form a closed cycle in the precedence relation! This contradiction completes the argument. ■

Now it is obvious how to get a polynomial time algorithm for the scheduling problem: We apply Lemma 1 to ¯nd a perfect matching that matches two minimal jobs i and j. We schedule i and j into the earliest empty time slot, remove them from G, and we repeat this procedure until G becomes empty.

Of course there are faster (and more complex) algorithms for solving this two-machine scheduling problem. The fastest known algorithm has a running time linear in the number of jobs and the number of precedences; it is due to Gabow & Tarjan [4]. The corresponding problem for *three* machines is an outstanding open question in complexity theory; see Garey & Johnson [6]. All natural generalizations of the above approach to three machines turn out to be incorrect or do not work in polynomial time.

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2 Crossing-free matchings

The following problem was one of the twelve problems posed at the 1979 *William Lowell Putnam Mathematical Competition* (that's the earliest reference that I could find, although I suspect the problem to be much much older than that): Consider a set R of n red points and a set B of n blue points in the Euclidean plane, such that no three points of $R \cup B$ lie on a common line. Show: There exist n pairwise disjoint, straight line segments that match the blue points to the red points.

And here is the solution: Consider a matching $\mathcal{M}$ between the point sets R and B that minimizes the total Euclidean length of the corresponding n straight line segments. Suppose that $\mathcal{M}$ contains two line segments R_1B_1 and R_2B_2 that intersect in a common point X. The triangle inequality yields $|R_1X| + |XB_2| > |R_1B_2|$ and $|R_2X| + |XB_1| > |R_2B_1|$, which leads to

$$
\begin{aligned}
|R_1B_1| + |R_2B_2| &= (|R_1X| + |XB_1|) + (|R_2X| + |XB_2|) \\
&= (|R_1X| + |XB_2|) + (|R_2X| + |XB_1|) \\
&> |R_1B_2| + |R_1B_1|.
\end{aligned}
$$

Thus, by switching the partners of R_1 and R_2 in $\mathcal{M}$ one could decrease the total length of $\mathcal{M}$. Therefore, $\mathcal{M}$ is crossing-free.

Hershberger & Suri [8] construct crossing-free matchings in $O(n \log n)$ time. Their approach is purely based on geometric observations, and does not use graph theory. Their time complexity $O(n \log n)$ is best possible, since the problem contains sorting as a special case.

3 Illuminating the ocean

In the following problem, the reader should think of the *plane* as an ocean, of the *points* P_k as lighthouses in this ocean, and of the *wedges* W_k as regions that are illuminated by floodlights: There are n points $P_1, \ldots, P_n$ in the Euclidean plane. Furthermore, there are n rays that emanate from the origin and cut the Euclidean plane into n wedges $W_1, \ldots, W_n$. These wedges span angles $\alpha_1, \ldots, \alpha_n$ with $\sum_{k=1}^{n} \alpha k = 2\pi$. Show: The wedges $W_1, \ldots, W_n$ can be translated from the origin to the points $P_1, \ldots, Pn$ (one wedge per point), such that their translates again cover the entire plane.

The special case of this problem with $n = 4$ points and with angles $\alpha_k \equiv \pi/2$ was posed at the 1967 *All Soviet Union Mathematical Competition*. We will describe a solution of stunning beauty for the general problem. It is due to Galperin & Galperin [5], and it is based on an auxiliary matching problem. Let $\vec{v}_k$ denote the vector of length $1/\cos(\pi_k/2)$ in the direction of the angle-bisector of W_k. We write $\langle \vec{a}, \vec{b} \rangle$ to denote the inner product of the two vectors $\vec{a}$ and $\vec{b}$, and we write $\|\vec{a}\|$ to denote the length of vector $\vec{a}\ \vec{b}$. Translating wedge W_k from the origin to some other anchor point P yields the region $W_k[P]$.

Lemma 2 *Consider two points P and Q, and two integers k, l with $1 \le k; l \le n$. If Q lies inside $W_k[P]$ but outside $W_l[P]$, then $\langle \vec{PQ}, \vec{v}_k \rangle > \langle \vec{PQ}, \vec{v}_l \rangle$.*

Proof. Let β denote the angle between $\vec{PQ}$ and $\vec{v}_k$. Then $Q \in W_k[P]$ yields $|\beta| \le \alpha_k/2$, which implies $\cos(\beta) \ge \cos(\alpha_k/2)$. Whence

$$
\begin{aligned}
&\langle \vec{PQ}, \vec{v}_k \rangle = \cos(\beta) \cdot \|\vec{PQ}\| \cdot \|\vec{v}_k\| \\
&= \cos(\beta) \cdot \|\vec{PQ}\| / \cos(\alpha_k/2) \ge \|\vec{PQ}\|:
\end{aligned}
$$

A symmetric argument centered around the angle between $\vec{PQ}$ and vector $\vec{v}_l$ yields that $\langle \vec{PQ}, \vec{v}_l \rangle < \|\vec{PQ}\|$. ■

We now fix some arbitrary point Q in the plane, and we define the Q-cost of assigning wedge W_k to point P_i as $\langle \vec{P_iQ}, \vec{v}_k \rangle$. We compute a matching $\mathcal{M}$ between wedges and points that maximizes the total Q-cost. By renumbering the points, we may assume that for $k = 1, \ldots, n$ matching $\mathcal{M}$ assigns wedge W_k to point P_k, and that hence the translated wedges are $W_k[P_k]$. Here is the first beautiful observation:

Lemma 3 *If matching $\mathcal{M}$ maximizes the Q-cost, then point Q is covered by one of the translated wedges.*

Proof. Suppose not. Observe that for every point P_i, there exists some wedge W_k with $Q \in W_k[P_i]$; we denote this situation by $i \to k$. Clearly, the relation $\to$ contains some directed cycle $c_1 \to c_2 \to \cdots \to c_s \to c_s+1 = c_1$ for some $s \ge 2$. But then the following cyclic switch would increase the Q-cost of $\mathcal{M}$: For $k = 1, \ldots, s$ re-assign wedge $W_{c_{k+1}}$ from point $P_{c_{k+1}}$ to point P_{c_k}. By Lemma 2, this contradicts the maximality of $\mathcal{M}$. ■

Here is the second beautiful observation: The Q-costs do depend on the choice of Q, but the optimal matching $\mathcal{M}$ *does not*. Indeed, let π be some assignment of wedges to points and consider two points Q_1 and Q_2. Then the difference between the two objective values of π under $Q1$-costs and under $Q2$-costs equals nX

Since this difference is independent of the

$$
\begin{aligned}
&\sum_{k=1}^{n} \langle \vec{P_kQ_1}, \vec{v}_{\pi(k)} \rangle - \sum_{k=1}^{n} \langle \vec{P_kQ_2}, \vec{v}_{\pi(k)} \rangle = \\
&= \sum_{k=1}^{n} \langle \vec{P_kQ_1} - \vec{P_kQ_2}, \vec{v}_{\pi(k)} \rangle = \\
&= \sum_{k=1}^{n} \langle \vec{Q_2Q_1}, \vec{v}_{\pi(k)} \rangle = \langle \vec{Q_2Q_1}, \sum_{k=1}^{n} \vec{v}_k \rangle.
\end{aligned}
$$

assignment π, matching $\mathcal{M}$ maximizes the Q-cost for every possible point Q. Hence, by Lemma 3 every possible point Q is covered by one of the translated wedges.

4 A path-forming game

Player 1 and Player 2 play the following game on an undirected graph G: They alternately select the next vertex of a *simple* path $\mathcal{P}$ in G. Player 1 is free to select the starting vertex of $\mathcal{P}$ in his first move. The first player unable to move loses the game. Who wins this game, if both players play optimally?

We will show that Player 1 has a winning strategy if and only if G has no perfect matching. It is easy to see that Player 2 has a winning strategy, if G does contain a perfect matching $\mathcal{M}$: Whenever Player 1 moves to some vertex v, then Player 2 reacts by simply moving to the partner vertex of v in $\mathcal{M}$. And Player 1 wins the game, if G does not contain a perfect matching: Player 1 fixes an arbitrary maximum cardinality matching $\mathcal{M}$ (that by assumption is non-perfect), and then starts the path in a vertex u' that is not covered by $\mathcal{M}$. Whenever Player 2 moves to some vertex v, Player 1 reacts by moving to the partner vertex of v in $\mathcal{M}$. Note that Player 2 must always move to a vertex u'' that is covered by $\mathcal{M}$; otherwise, the augmenting path $\mathcal{P}$ from u' to u'' could be used to increase the cardinality of $\mathcal{M}$.

This game shows up (for instance) as an exercise in Bondy & Murty [2]. Interestingly, Bodlaender [1] has proved that the corresponding path-forming game in *directed* graphs is PSPACE-complete, and hence intractable.

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Symposium in honor of George Nemhauser

A two-day symposium will be held in honor of George's 70th birthday. George has had a profound impact on many people in the operations research and mathematical programming community. He was president of MPS from 1989-1992. This event will provide the participants with an opportunity to show their appreciation. The symposium will include a small number of scientific presentations on general topics and also special presentations that cover George's work and his contributions through various stages of his career. Speakers at the symposium include:

Cindy Barnhart
Bob Bixby
Bill Cook
Gerard Cornuejols
Marshall Fisher
Rob Garfinkel
Martin Groetschel
John Jarvis
Tom Magnanti
Bill Pulleyblank
Don Ratliff
David Ryan
Mike Todd
Mike Trick
Laurence Wolsey

The Symposium will be held at Georgia Tech, start at noon on Thursday, July 26, 2007 and end at 1:30 on Friday, July 27 (George's birthday is the 27th). There will be a banquet on Thursday evening. The registration fee is $100. Further announcements and registration details will follow.

Mike Ball & Martin Savelsbergh

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Peter Ladislaw Hammer

December 23, 1936 – December 27, 2006

Peter Ladislaw Hammer was born in Timisoara, Romania, on December 23, 1936. He earned his Ph.D. in mathematics under Academician Grigore C. Moisil at the University of Bucharest in 1966. He defected to Israel in 1967 where he became a professor at the Technion in Haifa. After moving to Canada, he taught from 1969 to 1972 at the University of Montreal, and from 1972 to 1983 at the University of Waterloo. In 1983, he moved to the USA and became a professor at Rutgers University, where he founded RUTCOR - the Rutgers Center for Operations Research. He remained the director of RUTCOR until his untimely death in a tragic car accident, on December 27, 2006.

For more than 40 years, Peter Hammer has ranked among the most influential researchers in the fields of operations research and discrete mathematics. He has made numerous major contributions to these fields, launching several new research directions. His results have influenced hundreds of colleagues and have made a lasting impact on many areas of mathematics, computer science, and statistics.

Most of Peter Hammer's scientific production has its roots in the work of George Boole on propositional logic. More than anyone else, Peter Hammer has used and extended Boole's machina universalis to handle questions relating to decision making, analysis and synthesis as they arise in natural, economic and social sciences. Over the span of his scientific career, he has conducted eclectic forays into the interactions between Boolean methods, optimization, and combinatorial analysis, while adapting his investigations to the most recent advances of mathematical knowledge and of various fields of application. Among the main research topics which have received his attention, one finds an impressive array of methodological studies dealing with combinatorial optimization, some excursions into logistics and game theory, numerous contributions to graph theory, to the algorithmic aspects of propositional logic, to artificial intelligence and, more recently, to the development of innovative data mining techniques. His publications include 19 books and over 240 scientific papers. (See the Web site rutcor.rutgers.edu for a complete bibliography.)

At the very onset of his career, as a researcher at the Institute of Mathematics of the Academia of Romania, Peter Hammer wrote several important articles on transportation problems, jointly with Egon Balas. At the same time his advisor, Grigore Moisil, directed him to the study of Boolean algebra. In this field, a central role is played by functions depending on binary variables, and taking either binary values (i.e., Boolean functions) or real values (i.e., pseudo-Boolean functions). In a series of papers, Peter Hammer demonstrated that a large variety of relevant problems of operations research, combinatorics and computer science can be reduced to the optimization of a pseudo-Boolean function under constraints described by a system of pseudo-Boolean inequalities. A further main conceptual step in his work was the characterization of the set of feasible solutions of the above system as solutions of a single Boolean equation (or, equivalently, of a satisfiability problem). This led him, in joint work with Ivo Rosenberg and Sergiu Rudeanu, to the development of an original approach inspired from classical Boolean methods for the solution of a large variety of discrete optimization problems.

This research project culminated in 1968 with the publication of the book Boolean Methods in Operations Research and Related Areas (Springer-Verlag, 1968), co-authored by Sergiu Rudeanu. This landmark monograph, which founded the field of pseudo-Boolean optimization, has influenced several generations of students and researchers, and is now considered a "classic" in Operations Research.

In a sense, Peter Hammer's early work can be viewed as a forerunner of subsequent developments in the theory of computational complexity, since it was in effect demonstrating that a large class of combinatorial optimization problems is reducible to the solution of Boolean equations. However, this purely "reductionist" view of his work would be quite narrow. In fact, Peter Hammer has systematically used the "canonical" representation of various problems in terms of Boolean functions or Boolean equations to investigate the underlying structure, the "essence" of the problems themselves. More than often, this goal is met through a simplifying process based, once again, on the tools of Boolean algebra. This approach provides, for instance, a simple way to demonstrate that every system of linear inequalities in binary variables is equivalent to a set of inequalities involving only 0,1,-1 coefficients, as observed in a joint paper by Frieda Granot and Peter Hammer (1972). It also led Peter Hammer, Ellis Johnson and Uri Peled (1975) to early investigations into the facial structure of knapsack polyhedra.

In a related stream of research, Peter Hammer has established numerous fruitful links between graph theory and Boolean functions. In a famous joint paper with Vašek Chvátal on the aggregation of inequalities in integer programming (1977), he introduced and characterized the class of threshold graphs, inspired by threshold Boolean functions. Threshold graphs have subsequently been the subject of scores of articles and of a book by Mahadev and Uri Peled, two of Peter Hammer's former doctoral students. Other links between graphs and Boolean or pseudo-Boolean functions have been explored in joint work with Claude Benzaken, Dominique de Werra, Stephan Foldes, Toshihide Ibaraki, Alex Kelmans, Vadim Lozin, Frédéric Maffray, Bruno Simeone, etc.

Quadratic 0-1 optimization has been one of Peter Hammer's main fields of investigation. The theory of roof-duality (1984), jointly developed with Pierre Hansen and Bruno Simeone, builds on concepts from linear programming (linear relaxations), Boolean theory (quadratic Boolean equations) and networks (maximum network flow problems) to compute best linear approximations of quadratic pseudo-Boolean functions and tight bounds on the maximum value of such functions. Further research along similar lines has been conducted by Peter Hammer in collaboration with Endre Boros, Jean-Marie Bourjolly, Yves Crama, David Rader, Gabriel Tavares, X. Sun, etc.

Peter Hammer has also shown interest for the application of Boolean models in artificial intelligence and related fields, as witnessed by numerous joint papers with Gabriela and Sorin Alexe, Martin Anthony, Tiberius Bonates, Endre Boros, Yves Crama, Oya Ekin, Toshi Ibaraki, Alex Kogan, Miguel Lejeune, Irina Lozina, and other collaborators. His contributions bear on automatic theorem proving, compression of knowledge bases, algorithms for special classes of satisfiability problems, etc. About 20 years ago, he launched an innovative approach to data mining based on a blend of Boolean techniques and combinatorial optimization. The basic tenets of this approach were presented in a joint paper with Yves Crama and Toshihide Ibaraki (1988) and were subsequently developed by Peter Hammer and his coworkers into a new broad area of research, which he dubbed Logical Analysis of Data, or LAD for short. The effectiveness of the LAD methodology has been validated by many successful applications to real-life data analysis problems. In particular, some front-of-the line medical centers are increasingly using LAD in the actual practice of medical diagnosis for a variety of syndromes.

Many aspects of Peter Hammer's immense contribution to the study of Boolean functions and their combinatorial structure are to be found in a forthcoming monograph entitled Boolean Functions: Theory, Algorithms, and Applications, co-authored by Yves Crama and several other close collaborators of Peter Hammer, to be published by Cambridge University Press in 2007.

Beside his scientific production, Peter Hammer will undoubtedly be remembered for his vigorous contribution to and promotion of discrete mathematics and operations research. He was the founder and editor-in-chief of several highly-rated professional journals, including Discrete Mathematics, Discrete Applied Mathematics, Discrete Optimization, Annals of Discrete Mathematics, Annals of Operations Research and the SIAM Monographs on Discrete Mathematics and Applications. At Rutgers University, Peter Hammer was the founding Director of the operations research programme, and he was largely responsible for developing RUTCOR into an internationally recognized center of excellence and an open institute, where seminars, workshops, graduate courses, and a constant flow of visitors create a buzzing and stimulating research environment. He was also a tireless organizer of professional conferences and workshops, where he always made sure to provide opportunities for interactions between experienced scientists and younger researchers.

The importance of Peter Hammer's scientific contribution was acknowledged by the award of numerous international distinctions, including the "George Tzitzeica" prize of the Romanian Academy of Science (1966), the Euler Medal of the Institute of Combinatorics and its Applications (1999), and honorary degrees from the Swiss Federal Institute of Technology in Lausanne (1986), the University of Rome "La Sapienza" (1998), and the University of Liège (1999). He was a Fellow of the American Association for the Advancement of Science since 1974, and a Founding Fellow of the Institute of Combinatorics and its Applications. Several conferences were organized in his honor, including the First International Colloquium on Pseudo-Boolean Optimization (Chexbres, Switzerland, 1987), the Workshop and Symposia Honoring Peter L. Hammer (Caesarea Rothchild Institute, University of Haifa, 2003), and the International Conference on Graphs and Optimization (GO V, Leukerbad, Switzerland, 2006).

Peter Hammer was not only an outstanding scholar and a tireless organizer, but also a kind, generous and humorous human being. He relished the interaction with students and colleagues, and made everybody feel comfortable to work with him, be it on a mathematical question (which he was always keen to formulate) or on planning a conference. He supervised numerous graduate students with respect and fatherly understanding, considering each one of them as his "best student". He was also a true "citizen of the world": born in Romania from a Hungarian family, he subsequently took the Canadian citizenship, then the US one, wrote joint papers with co-authors of 28 different countries, fluently spoke 6 languages (or more), travelled the world extensively, spent extended periods of time in Belgium, France, Israel, Italy, Russia, Switzerland and many other countries, and developed an extended network of friends and coworkers on all continents.

Finally, last but certainly not least, Peter Hammer was a loving husband, father and grandfather. He is survived by his wife, Anca Ivanescu, whom he married in 1961 and whose family name he assumed for a few years, by his two sons Alexander and Maxim, and by four beloved grandchildren, Isabelle, Madeline, Annelise, and Oliver.

He will be missed by everyone who knew him, always and forever.

Endre Boros[1], Yves Crama[2]
and Bruno Simeone[3]

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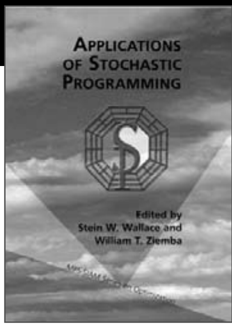

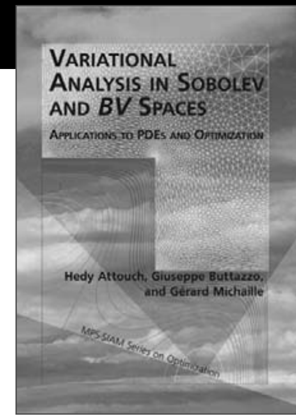

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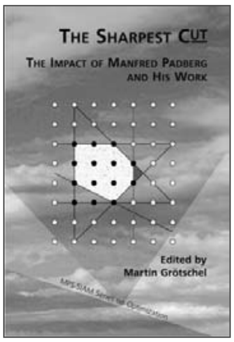

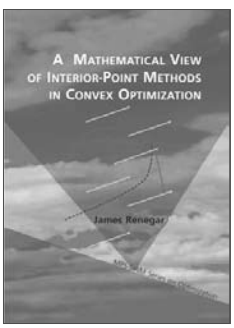

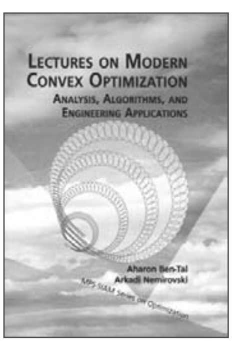

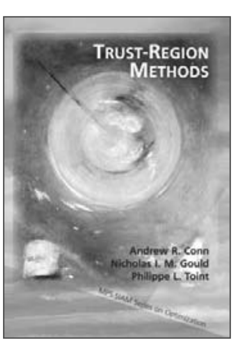

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