OPTIMA 82
Mathematical Programming Society Newsletter

Steve Wright

MPS Chair's Column

March 16, 2010. You should recently have received a letter concerning a possible change of name for MPS, to "Mathematical Optimization Society". This issue has been discussed in earnest since ISMP 2009, where it was raised at the Council and Business meetings. Some of you were kind enough to send me your views following the mention in my column in Optima 80. Many (including me) believe that the term "optimization" is more widely recognized and better understood as an appellation for our field than the current name, both among those working in the area and our colleagues in other disciplines. Others believe that the current name should be retained, as it has the important benefits of tradition, branding, and name recognition. To ensure archival continuity in the literature, the main titles of our journals Mathematical Programming, Series A and B and Mathematical Programming Computation would not be affected by the proposed change; only their subtitles would change.

As the letter explains, there will be a final period for comment on the proposed change, during which you are encouraged to write to me, to incoming chair Philippe Toint, or to other Council or Executive Committee members to express your views. Following discussion of membership feedback, Council will vote on a motion to put the name change (which involves a constitutional change) to a vote of the full membership. If Council approves, you will then be asked to vote on the proposal in the same manner as in the last election for officers – that is, by online ballot, with an option for a paper ballot if you prefer. The proposal will pass if votes in favor exceed votes against on the membership ballot. We anticipate that the matter will be settled by May. Results will be announced by mail and on the web site www.mathprog.org.

We have been greatly saddened by the loss of life in earthquakes around the world in these early months of 2010. Fortunately, our colleagues in Chile were not affected severely by the earthquake of February 27 in that country. The MPS-organized conference ICCOPT 2010 will take place as planned in Santiago, Chile during July 26–29, preceded by a School for graduate students and young researchers on July 24–25. Conference facilities were left unscathed by the earthquake, and the airport and local transportation networks are operating normally. The local organizing committee (headed by Alejandro Jofre) is making a great effort to ensure a successful conference. I urge you to attend and contribute, and help add another chapter to the short but illustrious history of ICCOPT.

Many of us will be making plans for other mid-year conferences, and there are many to choose from in 2010. Our web site lists upcoming meetings at http://www.mathprog.org/?nav=meetings, including the MPS-organized IPCO 2010 (Lausanne, June 9–11) and other MPS-sponsored meetings: the International Conference on Stochastic Programming (ICSP) XII (Halifax, August 14–20), the International Conference on Engineering Optimization (Lisbon, September 6–9), and the IMA Conference on Numerical Linear Algebra and Optimization (Birmingham, September 13–15).

Note from the Editors

The topic of this issue of Optima is *Mechanism Design* – a Nobel prize winning theoretical field of economics.

We present the main article by Jay Sethuraman, which introduces the main concepts and existence results for some of the models arising in mechanism design theory. The discussion column by Garud Iyengar and Anuj Kumar address a specific example of such a model which can be solved by the means of optimization.

Optima 82 publishes the obituary of Paul Tseng by his friends and colleagues Dimitri Bertsekas and Tom Luo, we are honored to remember the contribution of this distinguished colleague.

The issue is also filled with announcements and advertisements. Among them, we like to point out the announcement of the recently published book about the first 50 years of Integer Programming based on the commemoration of the seminal work of Gomory which was held in Aussois as part of the 12th Combinatorial Optimization Workshop in 2008. Announcing such a book, we get the chance of correcting a mistake in the printed version of Optima 76 in which an article by Jon Lee about that workshop was published without Jon's name. We apologize for that.

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Jay Sethuraman

Mechanism Design for House Allocation Problems: A Short Introduction

1 Introduction

How can a group of agents make a collective choice when not all of the relevant information is commonly known to them? Hurwicz [16, 17] provided the first mathematical formulation of this problem. In this formulation, participants exchange *messages*, and the mechanism itself is thought of as a black-box that determines an outcome as a function of all the messages exchanged. Each agent is, of course, strategic in the sense that he may choose not to communicate certain information if he believes that doing so may result in an outcome that he prefers less. In this environment, what sorts of outcomes can be implemented as equilibria of these message games? This is the central question that the area of *mechanism design* is concerned with. There are a number of recent surveys that offer an accessible overview of the area; especially recommended are the ones by Jackson [19], and the wealth of material available on the Nobel Prize website [26] that discusses the contributions of Hurwicz, Maskin, and Myerson, who were awarded the 2007 Economics Prize for "having laid the foundations of mechanism design theory." (The recent book by Nisan et al. [25] covers a lot of ground, and, in particular, has two chapters on mechanism design theory.)

In this short article, we study the mechanism design question for a particular class of problems that have come to be known as house allocation problems. These form a special class of mechanism design problems that are relatively well-understood. We start, however, with the Gibbard-Satterthwaite theorem, which shows that the only mechanisms that can be implemented truthfully in dominant strategies are dictatorships.

2 The Gibbard-Satterthwaite Theorem

Suppose we are given a set of alternatives A with $|A| \geq 3$ and a set of agents N with $|N| = n \geq 2$. Let Σ be the set of all permutations of A, and let $\sigma_i \in \Sigma$ be agent i's preference over the alternatives. The vector $p = (\sigma_1, \sigma_2, \ldots, \sigma_n)$ is called a preference profile. A social choice function f selects an alternative for each preference profile. Formally, it is a function $f : \Sigma^n \to A$. While this description is reminiscent of a voting problem – the alternatives are candidates in an election, the agents are the voters, and the chosen alternative is the winner – it is general enough to model a wide variety of situations. Nevertheless, the voting problem is a useful example to keep in mind.

There are, of course, many social choice functions, but for practical purposes we are interested in social choice functions satisfying some reasonable properties. Motivated by the voting example, we can demand the following two properties of social choice functions:

- **Unanimity:** If $a \in A$ is the top choice of each agent in a profile p, then $f(p) = a$.
- **Monotonicity**: Suppose $f(\sigma_1, \sigma_2, \ldots, \sigma_n) = a$. Suppose for each agent i, σ'_i is such that $x\sigma'_i a$ only if $x\sigma_i a$. Then $f(\sigma'_1, \sigma'_2, \ldots, \sigma'_n) = a$.

We say that a social choice function f is *dictatorial* if there is an agent i such that f selects a whenever i ranks a as his top choice. The following theorem is now easy to prove.

Theorem 1. *Suppose there are exactly two agents. Then the only social choice functions that satisfy unanimity and monotonicity are dictatorial.*

Proof. Consider the profile p in which agent 1's preference ordering is $(a, b, \ldots)$ and agent 2's preference ordering is $(b, a, \ldots)$. We first argue that f must select a or b for profile p. Suppose otherwise, and that $f(p) = c \neq a, b$ (such an alternative exists because $|A| \geq 3$). Consider what f chooses when agent 1 changes his preference ordering to $(b, a, \ldots)$: Unanimity dictates that f choose b, whereas monotonicity dictates that f continue to choose c (as c's relative ordering with respect to the other alternatives has not been altered). So $f(p) = a$ or $f(p) = b$. In fact this argument shows that on any profile, the outcome chosen should be *Pareto optimal*: it should be the first choice of agent 1 or of agent 2. Without loss of generality, assume that f chooses a. We argue next that agent 1 is a dictator.

As before, pick an alternative $c \neq a, b$. Consider the profile q in which agent 1 has the preference ordering $(c, a, b, \ldots)$ and agent 2 has the ordering $(b, c, a, \ldots)$. We prove that $f(q) = c$. By the argument in the previous paragraph, $f(q)$ is either b or c. But if $f(q) = b$, then monotonicity implies that $f(p) = b$ as well, contrary to our assumption that $f(p) = a$. So $f(q) = c$. Thus, by monotonicity, $f(r) = c$ for any preference profile r in which agent 1 has c as his top-choice and agent 2 has a preference ordering $(b, c, \ldots)$. Now, consider the profile s in which agent 1 has a preference ordering $(c, b, a, \ldots)$ and agent 2 has an ordering $(b, a, \ldots, c)$. In particular, agent 2 ranks c as his last choice. By Pareto optimality, $f(s) = c$ or $f(s) = b$. However, if $f(s) = b$, then monotonicity implies that $f(r) = b$, a contradiction. Thus $f(s) = c$. But by monotonicity, $f(s) = c$ for any profile in which agent 1 ranks c first. But c was an arbitrarily chosen alternative, so our argument shows that f must choose the alternative that appears as agent 1's top choice. □

Theorem 1 can be extended to the case of more than two agents, the main idea being the following: given any non-dictatorial social choice function f^n for an n agent problem we can construct a social choice function f^{n-1}, for $(n-1)$ agents as follows: choose a $\sigma \in \Sigma$, and let $f^{n-1}_\sigma(p) = f^n(p, \sigma)$, for all $p \in \Sigma^{n-1}$. It is not hard to show that for any $n > 2$, if f^n is non-dictatorial, unanimous, and monotonic, then so is f^{n-1}.

To summarize, we have shown that as long as there are at least 2 agents and at least 3 alternatives, the only social choice functions that satisfy unanimity and monotonicity are dictatorships. One can now ask: what do these properties mean? And why are the relevant to mechanism design? Consider a situation in which f is known to the agents, so that any agent i can evaluate the outcome with the knowledge of the preferences of the other agents. Let $\sigma_{-i} = (\sigma_1, \sigma_2, \ldots, \sigma_{i-1}, \sigma_{i+1}, \ldots \sigma_n)$ denote the preference profiles of all the agents other than i. If the social choice function f is such that agent i (weakly) prefers the outcome $f(\sigma_i, \sigma_{-i})$ to $f(\sigma'_i, \sigma_{-i})$, for every $\sigma'_i \in \Sigma, \sigma_{-i} \in \Sigma^{n-1}$, then f is said to be *strategyproof*. Thus, if f is a strategyproof social choice function, then the participating agents do not have an incentive to lie about their *true* preferences to the mechanism designer, *regardless of the preferences of the other agents*. The following result relates strategyproofness to monotonicity and unanimity.

Proposition 1. *Any strategyproof social choice function $f : \Sigma^n \to A$ that is onto satisfies unanimity and monotonicity.*

An immediate implication of Theorem 1 and Proposition 1 is the following result, due to Gibbard [14] and Satterthwaite [34].

Theorem 2. *Any strategyproof social choice function $f : \Sigma^n \to A$ that is onto is a dictatorship.*

The Gibbard–Satterthwaite theorem identifies a set of properties of social choice functions that are mutually incompatible. On the face of it, this is a negative result that imposes severe restrictions on which kinds of social choice functions can be implemented. A closer examination of the result, however, suggests that the impossibility result can be overcome in a number of ways, by weakening or getting rid of some of the assumptions on which the theorem rests. In particular, the Gibbard-Satterthwaite result assumes that the preference domain Σ from which the agents draw their preferences is *universal*, meaning that all orderings of the alternatives are permitted. There are many settings, however, in which the natural preference domain is much smaller. In the rest of this article we examine one such class.

3 House Allocation: Deterministic Mechanisms

3.1 Housing Markets

There is a large (and growing) literature on *house allocation* problems, originating from the early work of Shapley and Scarf [35], who considered the following model. Suppose there are n agents, each of whom owns a distinct house. Each agent has a strict preference ordering of all the houses (including his own). How should the agents reallocate the houses amongst themselves?

To relate this question to our earlier discussion, we should specify N and A, the set of agents and alternatives, respectively. Clearly, N is simply the set of agents, but A, the set of alternatives, includes all possible matchings of agents to houses. Thus, $|A| = n!$, and the universal preference domain would include all possible (strict and weak) orderings of these alternatives. However, there are many applications in which the preference of agent i is sensitive only to i's own assignment, and is insensitive to how the other houses are assigned to the other agents. In particular, it is reasonable to look at the model in which the agent preferences over individual houses is extended to agent preferences over assignments as follows: i (weakly) prefers the matching μ to the matching μ' if i (weakly) prefers his assignment under μ to his assignment under μ'. As it turns out, under this definition of agent preferences, one can overcome the Gibbard-Satterthwaite Theorem:

Theorem 3. *For the house allocation problem with strict preferences, there is a non-dictatorial, strategyproof social choice function f onto A, the set of all matchings of houses to agents.*

Proof. The proof is constructive and builds a matching for any given profile of strict preferences (of the agents over individual houses). Construct a graph in which there is a node for each remaining agent; there is an arc from node i to node j if agent i's top choice among the remaining houses is the one owned by agent j. As there are finitely many agents, and each node has an out-degree of 1, there must be a cycle. Agents in the cycle exchange their houses in the obvious way: if $(i, j, k, \ldots, \ell)$ is the cycle, then i is assigned the house that j owns, j is assigned the house that k owns, etc. Note that each of the agents in the cycle gets their top choice. The agents involved in the cycle (and hence the houses they collectively own) are removed from the problem, and the same algorithm is applied to the reduced problem. Observe that the reduced problem has the same structure as the original problem: each house in the reduced problem is owned by a distinct agent in the reduced problem. □

Let $f(\cdot)$ be the outcome of this algorithm on any reported profile of agent preferences. Observe that $f(\cdot)$ is individually rational: each agent's assignment is at least as good as the house he owned. We show that $f(\cdot)$ is strategyproof. Suppose for some profile of preferences $(\sigma_1, \sigma_2, \ldots, \sigma_n)$, agent i drops out of the problem in the kth iteration of the algorithm, assuming σ_i is his true preference. Notice that the only objects that i strictly prefers to the one that he's assigned to are those that belong to agents who depart sooner. But none of these agents can be made to point to i by any manipulation on the part of agent i, as i's preference report only affects the agents he points to, not the agents who point to him. Thus, $f(\cdot)$ is strategyproof. Furthermore $f(\cdot)$ is onto: given any matching μ of the houses to agents, construct a preference profile for the agents in which $\mu(i)$ is i's top choice; on this preference profile, clearly, the algorithm will return μ as the final allocation. Finally, it is clear that $f(\cdot)$ is not dictatorial: one can easily construct preference profiles in which any given agent is not assigned his top choice under $f(\cdot)$. □

The algorithm just described is attributed to Gale and is called the top-trading cycles (TTC) algorithm. Note that the house allocation problem just described is an instance of an exchange economy with indivisible objects, and there are two standard solution concepts – the competitive equilibrium and the core – one can associate with such a problem. Given a matching μ of the houses to the agents, and a price vector $p = (p_1, p_2, \ldots, p_n)$, with p_i indicating the price of the house owned by agent i, we say that (μ, p) is a *competitive equilibrium* if (i) $p_{\mu(i)} \leq p_i$ and (ii) $p_j > p_i$, if i prefers the house owned by j to the house $\mu(i)$. The first condition ensures that each agent can afford the house he is assigned to, and the second condition ensures that no better house is affordable. A matching μ is in the *core* if no coalition S of agents can do better by reallocating the houses they own amongst themselves (where, by doing better, we mean that each agent in S gets a house that he likes at least as much, and at least one agent in S gets a house that he strictly prefers).

The description of the TTC algorithm suggests a very simple construction of the price vector: all the houses that are eliminated in iteration k of the algorithm have a price of $n - k$. It is a simple matter to verify that this vector of prices supports the TTC outcome as a competitive equilibrium. Furthermore, a simple inductive argument shows that the core contains a unique matching, which is the one found by the TTC algorithm! Shapley and Scarf [35] originally proved the existence of a core matching (and that it can be supported as a competitive equilibrium) using more complicated machinery. Later in the paper they describe Gale's TTC algorithm and show how it, too, finds a core matching that can be supported as a competitive equilibrium. The uniqueness of the core and the strategyproofness of the core mechanism were proved by Roth & Postlewaite [31] and Roth [30] respectively. Later, Ma [22] proved that the TTC mechanism is characterized by the requirements of *strategyproofness*, *Pareto-efficiency*, and *individual rationality*: Pareto-efficiency is simply the core condition for the grand coalition of all agents, whereas individual rationality is the core condition applied to singleton agents.

3.2 House Allocation

A closely related class of models, first studied by Hylland and Zeckhauser [18], is called the house allocation problem. In this model there are n agents, each with strict preferences over n indivisible objects. The objective is to find a matching of the agents to the objects. As before, we focus on direct mechanisms in which the agents report their preference orderings to the mechanism, which finds an assignment for each agent. In contrast to the housing markets, where the initial property rights play a prominent role, it is not immediately clear what properties to demand of the mechanism: for example, if the n objects are owned by the n agents collectively, so that each agent has an equal claim to each object, how should the final allocation be done? Of course, much depends on the preferences of the agents – if the top choices of the agents are all distinct objects, efficiency would imply that each agent be given their best object. Difficulties arise when multiple agents have the same top choice. In particular, what should the assignment be when everyone ranks the same object as their top choice?

Suppose only deterministic mechanisms are permitted, so that any given preference profile must be mapped to a particular allocation of agents to profiles. Suppose also that preferences are strict. The class of *priority mechanisms* are prominent: start with a given ordering of the agents (the ordering does not depend on reported preferences), and let the agents choose their best available objects according to this ordering. Clearly, the resulting allocation is Pareto efficient, and the mechanism is coalitionally strategyproof (immune to joint manipulation by an arbitrary coalition of agents). It also satisfies a property called *reallocation-proofness*, which simply means that no pair of agents have an incentive to misreport their preferences, even if they are allowed to exchange the objects that they are finally assigned to. However, the priority mechanism is essentially *dictatorial*: the allocation chosen is always among the top-choices of the first agent in this ordering. However, the class of mechanisms satisfying all of these properties is far more general and includes mechanisms other than priority mechanisms. As an example, associate with each object a priority ranking of the agents (which is also fixed exogenously). The TTC algorithm can be generalized to this "two-sided" model, assuming that each object is owned by the agent at the top of its priority list – one difference here is that in the standard TTC algorithm each agent owns exactly one object, whereas here some agents may own more objects and some none. Moreover, once some objects and agents are removed from the problem, those agents are removed from the priority lists of the remaining objects as well. This class of mechanisms was introduced by Abdulkadiroglu and Sonmez [4], and it is not difficult to show that these mechanisms are reallocation-proof, coalitionally strategyproof, and Pareto efficient. In fact, one can generalize this class of mechanisms even further: rather than fix a priority ordering for each object exogenously, one can fix an *inheritance function* that, at each stage, determines the ownership of each object as a function of the partial allocation of the objects to the agents at that stage. The only requirement of an inheritance function is that if an agent i owns an object a at a certain stage, then a is continued to be owned by i as long as i remains in the problem. The key result of Papai [27] is that every coalitionally strategyproof, reallocation-proof, Pareto efficient mechanism is a TTC mechanism with an inheritance function, and vice-versa. In recent work, Pycia and Unver [29] characterize all coalitionally strategyproof, Pareto efficient mechanisms and observe that this is a superset of the TTC mechanisms with inheritance functions.

We end our brief discussion of deterministic mechanisms with a couple of axiomatic characterizations. A mechanism is said to be *neutral* if it is insensitive to the relabeling of the houses: if π is any permutation of the houses, and every object a is replaced by $\pi(a)$ in all preference lists, then the mechanism assigns an object a to agent i originally if and only if it assigns $\pi(a)$ to agent i in the relabeled problem. Another useful property is *consistency*, which relates the allocation of the mechanism in the original problem to its allocation in a reduced problem. Suppose for a given problem the mechanism determines an allocation in which agent i receives the object $\mu(i)$. Suppose agent j and object $\mu(j)$ are no longer in the problem, so that the mechanism is applied to the reduced preference lists in which $\mu(j)$ does not appear in any preference list and agent j is absent. The mechanism is *consistent* if it allocates $\mu(i)$ to each of the remaining agents i. Svensson [38] showed that every coalitionally strategyproof and neutral mechanism is a *priority* mechanism; Ergin [11] showed that every Pareto efficient mechanism that is consistent and neutral is a priority mechanism.

4 House Allocation: Probabilistic Mechanisms

There are significant disadvantages to relying only on deterministic mechanisms for house allocation. For example, if all the agents have the same preference ordering of the objects, every deterministic mechanism must favor some agents over the others, and this can be viewed as *unfair*. One way to restore fairness is to allow for money, but this may not be appropriate in all situations: concrete examples include organ exchanges and public school assignments, but there are several others. In these markets, it is important to find *fair* allocations, but without using money, necessitating the use of randomization in the mechanism. Randomized mechanisms have been extensively studied for house allocation problems, and somewhat less so for the Shapley-Scarf housing market (as here, the TTC outcome is compelling).

Consider again the problem of *fair* allocation of a number of indivisible objects to a number of agents, each desiring at most one object. If there is only one object, there is really only one *fair* and *efficient* solution: assign the object to an agent chosen uniformly at random among the n agents. When there are many objects the problem becomes substantially more interesting for a number of reasons: (i) many definitions of fairness and efficiency are possible; (ii) richer mechanisms emerge; and (iii) since preferences of the agents over the objects have to be solicited from the agents, *truthfulness* of the allocation mechanism becomes important. Under a probabilistic mechanism each agent's allocation is described by a *vector*, with the jth component indicating the probability that this agent is assigned object j. As not every pair of vectors can be compared, strategyproofness (and other properties) can be defined in many ways, of which we consider two. A mechanism is said to be strategyproof if the allocation under truthful reporting of preferences dominates the allocation obtained by any other preference report, where the domination is in the sense of first-order stochastic dominance. In other words, a mechanism is strategyproof if the probability of receiving one of the k best objects is maximized when the agent reports his preference ordering truthfully, for every k. A mechanism is said to be *weakly* strategyproof if it is not possible to obtain an allocation that dominates the allocation under truthful reporting.

A natural idea is to use a priority mechanism, with the priority ordering chosen uniformly at random, i.e., every ordering of the agents is equally likely. This is the *random priority* (RP) mechanism, formally first studied by Abdulkadiroglu and Sonmez [2], and has a number of attractive features: it is efficient (every assignment chosen with a positive probability is Pareto efficent), strategyproof, and *treats equals equally*: agents with identical preferences are treated in an identical manner, a priori). This may seem like the last word on the problem, but it turns out that there are other compelling randomized mechanisms.

To illustrate RP, consider the following example, due to Bogomolnaia and Moulin [7]. Suppose there are 4 agents $1, 2, 3, 4$ and 4 objects a, b, c, d. Consider the preference profile in which agents 1 and 2 rank the objects $a > b > c > d$ and agents 3 and 4 rank the objects $b > a > d > c$. The probabilistic assignment computed by the RP mechanism is

	a	b	c	d
1	5/12	1/12	5/12	1/12
2	5/12	1/12	5/12	1/12
3	1/12	5/12	1/12	5/12
4	1/12	5/12	1/12	5/12

For example, agent 1 gets assigned object a whenever 1 is ranked first, or whenever 1 is ranked second, and 2 is not first. The former event occurs with probability 1/4 and the latter with probability 1/6, which explains the first entry of the matrix.

A second natural mechanism, proposed by Hylland and Zeckhauser [18], is to adapt the competitive equilibrium with equal incomes (CEEI): endow each agent with a dollar, and find a price for each house so that the market clears: given the price vector, each agent consumes a bundle (fractions of each house) that maximizes his total utility, while staying within his budget. A standard fixed-point argument shows the existence of market-clearing prices. Moreover the (probabilistic) allocation so found is envy-free (a stronger property than equal treatment of equals) and efficient (in a stronger sense than Pareto efficiency). Efficiency follows by definition, and envy-freeness follows from the fact that every agent has the same budget, and so can afford every other agent's allocation at the current price. But there are two potential objections to this solution: it can be shown that this mechanism is not strategyproof; and furthermore, the computational and informational requirements of implementing this mechanism are prohibitive: it requires the solution of a fixed-point problem, and requires a complete knowledge of the utility functions of the agents. In contrast, the random priority mechanism is very simple to implement, and only requires the agents to rank the objects.

A third class of mechanisms, due to Bogomolnaia and Moulin [7], is the *probabilistic serial* (PS) mechanism, which combines the attractive features of RP and CEEI: it requires the agents to report their preferences over objects, not the complete utility functions, and yet computes a random assignment that is *envy-free* and *ordinally efficient*. Ordinal efficiency is stronger than the Pareto efficiency satisfied by RP, but weaker than *ex ante* efficiency of CEEI, but given the "ordinal" nature of the input to the mechanism (only preference rankings are used, not complete utility functions), this is perhaps the most meaningful notion of efficiency for an ordinal mechanism. The PS mechanism can be motivated by the example we discussed earlier. In that example, agents 1 and 2 both prefer a to b, whereas agents 3 and 4 prefer b to a. Yet, the RP mechanism assigns a with positive probability to agents 3 and 4, and assigns b with positive probability to agents 1 and 2, creating a potential inefficiency. As an alternative, consider a mechanism that assigns a with probability 1/2 to agents 1 and 2, and assigns b with probability 1/2 to agents 3 and 4; likewise for objects c and d. The resulting allocation matrix

	a	b	c	d
1	$1/2$	0	$1/2$	0
2	$1/2$	0	$1/2$	0
3	0	$1/2$	0	$1/2$
4	0	$1/2$	0	$1/2$

is one in which every agent's allocation stochastically dominates (in the first-order sense) his allocation under RP! It is this sense in which RP is inefficent. The PS mechanism corrects for this inefficiency, and can be described as follows: imagine that each agent eats his best available object at each point in time at unit speed. Once one unit of an object is consumed, it is removed from the problem. On this example, initially, agents 1 and 2 eat object a, whereas agents 3 and 4 eat object b; at $t = 1/2$, unit amounts of a and b are consumed, so a and b are no longer available; from this point on, agents 1 and 2 eat object c (their best available object) and agents 3 and 4 eat object d. At $t = 1$, all of the objects are consumed, and each agent has consumed a unit amount. The resulting random assignment is exactly the one shown earlier. The PS mechanism always finds an assignment that is ordinally efficient, which means that there is no other dominating (in the sense of first-order stochastic dominance) assignment matrix. Morever, it determines an envy-free allocation for precisely the same reason that the CEEI solution does: here, the eating speeds are identical across agents. All this is achieved, unfortunately, at the expense of strategyproofness: the PS mechanism is not strategyproof, but is weakly strategyproof in the sense that by reporting false preferences no agent can find an allocation that dominates his allocation under true preferences. Furthermore, Bogomolnaia and Moulin [7] show that no mechanism that is ordinally efficient and strategyproof can treat equals equally, which is a very strong and somewhat disappointing impossibility result. On the positive side, however, a number of recent papers have examined "large markets" (markets with many copies of each object, say): the general flavor of these results is that, in appropriately chosen large markets, RP becomes ordinally efficient or that PS becomes strategyproof [10, 21].

In contrast to the literature on deterministic mechanisms for house allocation, there are very few axiomatic characterizations of these mechanisms. In particular, it is believed that RP is the only mechanism satisfying equal treatment of equals, Pareto efficiency, and strategyproofness.

5 A Unified Model: Probabilistic Mechanisms

In recent work, Athanassoglou and Sethuraman [6] discuss the following model that subsumes all the house allocation models discussed so far. There are n agents and n objects, and agent i is endowed with e_{ij} units of house j, with each $e_{ij} \in [0, 1]$. To keep things simple, assume that each agent owns at most the equivalent of a full object, and that at most one unit of any object is owned by the agents, so that the endowment matrix is a doubly sub-stochastic matrix. Each agent i has (ordinal) preferences over the set of houses expressed by the complete and transitive relation $\succeq_i$. While this preference relation allows for indifferences, again for simplicity, assume a strict preference ordering for each agent. What we wish to find is an allocation, which, as before, is described by an assignment matrix, with the rows indexing the agents and columns indexing the objects; like the endowment matrix, the assignment matrix will be a doubly sub-stochastic matrix, as we assume that each agent is interested in at most the equivalent of one object. Observe that this model generalizes the most prominent models studied in the house allocation literature. In particular, if the endowment matrix is a permutation matrix, we recover the Shapley-Scarf [35] housing market model; if the endowment matrix is identically zero, we get the house allocation model; and if the endowment matrix is a sub-stochastic matrix with entries in $\{0, 1\}$, we get the the house allocation problem with existing tenants, considered by Abdulkadiroglu and Sonmez [3] and Yilmaz [39]. As before, the objective is to find compelling allocation mechanisms and explore their properties such as efficiency, strategyproofness, fairness. Furthermore, as some of the agents enter the market with an endowment *individual rationality* – an agent's final allocation should (weakly) dominate his endowment – becomes important. Thus, the input to the mechanism is a strict preference profile and an endowment matrix, where the preference orderings are assumed to be private information, but the endowments are not. A motivating example to keep in mind is the following situation: suppose the final assignment of objects to agents will be made based on a given fractional assignment matrix E, so that agent i will receive object j with probability e_{ij}. Interpreting this fractional assignment matrix as the endowment of the agents, the mechanism computes an alternative assignment matrix in which each agent's random allocation stochastically dominates her endowment, yielding a "superior" lottery for each agent (this is the individual rationality requirement). Ordinal efficiency of the proposed mechanism implies that this new lottery cannot be improved upon for all the agents simultaneously.

As the agents come to the market with different endowments, interpreting the "fairness" requirement is challenging. Consider the following instance with three agents $\{1,2,3\}$ and three objects $\{a,b,c\}$. Agent 1 prefers a to b and b to c; agents 2 and 3 prefer b to a and a to c. The initial endowments are specified in braces, next to the preference ordering. Here, agent 1 is endowed with b, agent 2 with a, and agent 3 with c.

$$\begin{array}{lll} 1: & a \succ b \succ c & \{b\} \\ 2: & b \succ a \succ c & \{a\} \\ 3: & b \succ a \succ c & \{c\} \end{array}$$

It is clear that the only individually rational and efficient assignment is one in which 1 gets a, 2 gets b and 3 gets c. Clearly agent 3 will envy both agents 1 and 2. However, this envy is *not justified* because it is not possible for agents 1 and 2 to give up any portion of their endowments to agent 3, receive a positive share of house c and still maintain individual rationality. In contrast, in the following example,

$$\begin{array}{lll} 1: & a \succ c \succ b & \{b\} \\ 2: & b \succ c \succ a & \{a\} \\ 3: & b \succ a \succ c & \{c\} \end{array}$$

the assignment discussed earlier – giving a to 1, b to 2, and c to 3 – is still individually rational and efficient. However there are other individually rational and efficient allocations because agents 1 and 2 are willing to give up some of b and a respectively for any object in the sets $\{a,c\}$ and $\{b,c\}$ respectively. In this context, if all of c is allocated to agent 3, then this agent could justifiably envy agents 1 and 2. This is because instead of giving agents 1 and 2 their best objects, the mechanism could have found a different allocation in which agents 1 and 2 do a little worse, still maintain individual rationality, and agent 3 does a little better. In particular, the assignment

	a	b	c
1	$\frac{1}{2}$	0	$\frac{1}{2}$
2	0	$\frac{1}{2}$	$\frac{1}{2}$
3	$\frac{1}{2}$	$\frac{1}{2}$	0

is individually rational, efficient, and is envy-free.

Notice that in the TTC algorithm agents 1 and 2 would swap their endowments, leaving agent 3 with object c. Thus the key difference between the TTC method and the probablistic solution just obtained is in the interpretation of the endowments: TTC allows the agents to trade their objects directly, but this may not be possible always. (As a concrete example in school assignment in the US: residing in a certain neighborhood confers a right to attend a particular public school, but this right is not tradeable.) In such environments the only role of the initial endowment is that of a guarantee: each agent is assured of a final assignment that is at least as good as his initial endowment, but owning a "superior" object does not necessarily imply a "superior" allocation.

For the house allocation problem with fractional endowments, Athanassoglou and Sethuraman [6] design an algorithm to find an assignment that is individually rational, ordinally efficient, and eliminates justified envy of the sort discussed in the example above. (The formal definition appears in Yilmaz [39].) This algorithm falls under the general class of simultaneous eating algorithms (of which the PS mechanism is a special case). In particular, it allows each agent to "eat" her most preferred available object at rate 1, *as long as there is some way to complete the assignment so that the individual rationality constraints are not violated*; this continues until some object is completely consumed, or some individual rationality constraint is in danger of being violated. In the latter case the agents, whose continued consumption of their best available houses would violate some individual rationality constraint, are forbidden from consuming their most preferred houses even if they are available, and they move on to their next best house. It is interesting that these events can be tracked by solving a parametric flow problem on an appropriately defined network, see [6] for a formal description. The implication is that for this very general model, there is a mechanism that always finds an ordinally efficient, individually rational allocation that avoids justified envy. The price for this generality, however, is that strategyproofness in any form is not possible. In fact, it is known that even the bare basic requirements of efficiency and individual rationality are already incompatible with strategyproofness in this general model. These and other impossibility results are also discussed in [6].

6 Applications

The models discussed so far have a number of applications, the most prominent ones being the assignment of schools to students [4] and the organization of kidney exchanges [32]. There are a wealth of papers and articles that describe these applications in greater detail. We focus on two issues, one specific to each of these applications.

6.1 Kidney Exchange

Suppose there is a patient who needs a kidney, has a willing donor, but the donor's kidney is not a medical match for the patient. Suppose there is a second such patient-donor pair, and suppose that the first donor can give a kidney to the second patient, and the first patient can receive a kidney from the second donor. If such pairs can be identified, then the two transplants can be performed, and neither patient enters the waiting list for kidneys. This motivates the need for an organized kidney exchange in which such patient-donor pairs can make their presence known. The connections to the housing market problem is very clear: the patients are the agents, the donors are the objects, and the quality of the match will serve as the preference ranking of the agents. This simple connection already raises a lot of questions: for example, what if one of the patients has a willing donor who makes a donation, and in return, the patient does not get a kidney but gets a preferred position in the wait-list. Similarly, what if there is an altruistic donor who is not tied to any patient? These considerations already call for a model in which some agents may own objects, but others come to the market with no object, and some objects are in the market unattached to any owner. This is called the house allocation model with existing tenants and was explored even before the kidney exchange application was formally studied.

An important constraint in the kidney exchange problem is that all the transplants involved in an exchange must be performed simultaneously. This, along with other practical considerations, makes *pairwise* kidney exchanges more attractive. Furthermore, as a first approximation to the actual problem, we may simply check whether a particular kidney is a medical match for a particular patient or not. This naturally leads to a matching problem in a graph, but with a non-standard objective. In terms of the house allocation model, this is an extreme special case in which each agent classifies each object as either *acceptable* or *unacceptable*; any acceptable object is just as good as every other acceptable object.

Matching models with this extreme form of indifference have been studied by Bogomolnaia and Moulin [8] and Katta & Sethuraman [20] for the case of a bipartite graph, in which agents represent one side, and the objects the other. The work of Katta & Sethuraman [20] provides a complete solution to the house allocation problem in which agent preferences have indifferences that are significantly more general. In this article, we discuss only the special case of extreme indifference, also called the case of dichotomous preferences. Each agent indicates only the subset of objects that she finds acceptable, each of which gives her unit utility; the other objects are unacceptable, yielding zero utility. Any solution is simply evaluated by the expected utility it gives to each agent, which is simply the probability that she is assigned an acceptable object. Suppose the preference profile in a given instance is such that some set of three agents have only two acceptable objects among them. Then, it is obvious that the combined utilities of these three agents cannot exceed 2. The general algorithm for solving the problem builds on this trivial observation: it consists of locating such a *bottleneck* subset of agents, and allocating their acceptable objects amongst them in a *fair* way, eliminating these agents and their acceptable objects, and recursively applying the idea on the remaining set of agents and objects. It is an elementary exercise to show that these problems can be solved as the problem of finding a maximum flow in the following network: there is a node for each agent and for each object, and there is an infinite-capacity arc from agent node i to object node j if i finds j acceptable. Augment the network by adding a source node s, a sink node t; arcs with capacity $\lambda \geq 0$ going from s to each agent node, and arcs with unit capacity going from each object node to the sink t. We view $\lambda \geq 0$ as a *parameter*, and study the minimum-capacity $s-t$ cuts (or simply minimum cuts) in the network as λ is varied. Let λ^* be the (smallest) *breakpoint* of the min-cut capacity function of the parametric network. Clearly, $\lambda^* \leq 1$. If $\lambda^* = 1$ then every agent can be assigned an acceptable object; otherwise, the agents on the source-side of the min-cut form a bottleneck set, and each of them can only be matched to an acceptable object with probability λ^*. The objects they get matched to with these probabilities is given by any flow with value $n\lambda^*$ in this network. Eliminating these agents and they objects they find acceptable (every one of those objects will necessarily be removed from the problem), we get a reduced problem on which we apply the same algorithm. The breakpoints of the min-cut capacity function of a parametric network are well-understood [13]. Efficient algorithms to compute these breakpoints have been discovered by several researchers, see for example Ahuja, Magnanti, and Orlin [5] and the original paper of Gallo, Grigoriadis and Tarjan [13].

This mechanism just described is coalitionally strategyproof, envy-free, and efficient in a very strong sense: this mechanism finds a utility vector for the agents that Lorenz dominates the utility vector found by any other mechanism. (A vector x Lorenz dominates a vector y if the sum of the k smallest components of x is (weakly) larger than the sum of the k smallest components of y, for every $k \geq 1$.) In fact, the house allocation problem with dichotomous preferences is closely related to the sharing problem, first introduced by Brown [9]. Brown's work was partly motivated by a coal-strike problem: during a coal-strike, some "non-union" mines could still be producing. In this case, how should the limited supply of coal be distributed equitably among the power companies that need it? Since power companies vary in size, it would not be desirable to give each power company the same amount of coal. Moreover, even if such an equal sharing was desirable, the distribution system may not allow for a perfect distribution because of capacity constraints. Brown [9] modeled the distribution system by a network in which there are multiple sources (representing the coal-producers), multiple sinks (representing power companies), and multiple transshipment nodes; each edge in the network has a capacity which is an upper bound on the amount of coal that can traverse that edge. Also, each sink node has a positive "weight" reflecting its relative importance, and the utility of any sink node is the amount of coal it receives divided by its weight. The goal is to distribute the supply of coal so as to maximize the utility of the sink that is worst-off. Megiddo [23, 24] considered the *lexicographic sharing* problem, where the objective is to lexicographically maximize the utility vector of all the sinks, where the kth component of the utility vector is the kth smallest utility. The algorithm described earlier is precisely the one that finds a lex-optimal flow for this sharing problem!

In the case of kidney exchanges, the associated graph is really non-bipartite, and this has been formally worked out by Roth, Sonmez and Unver [33] using classical combinatorial tools. The ideas described for the bipartite case, however, essentially carry over, yielding a simpler proof of many of those results.

6.2 School Choice

Abdulkadiroglu and Sonmez [4] considered the problem of assigning students to schools and formulated it as a matching problem, and advocated two broad types of solutions for it. One is the familar and well-studied stable matching model, introduced earlier by Gale and Shapley [12]. The other is the TTC algorithm with object priorities reflected by an inheritance table. In both cases, the students were the agents whose preferences had to be elicited. The schools were viewed as passive objects whose priority lists were exogenous. After an extensive evaluation of both mechanisms, the public school system in NYC (and in some other American cities) has been using the Gale-Shapley mechanism for assigning students to high schools [1]. There is, however, a supplementary round that is meant to assign the applicants who are unassigned after the main round. The assignment process in the main round takes into account several factors including the student priorities at each school based on standardized test scores, and is therefore a two-sided matching problem. In contrast, the assignment process in the supplementary round is quite simple: all unassigned applicants are invited to rank order high schools with vacant capacity; all students at this stage have the *same* priority to attend any school. Thus we are led naturally to a setting in which agents rank heterogeneous "objects," for which they have equal claims. Motivated by these considerations, Pathak [28] recently studied two "lottery" mechanisms: first is the *single* lottery mechanism, in which a single random ordering of the agents is drawn; any ties (at *any* school) are broken in favor of the student whose lottery number is lower. A natural alternative – which *seems* fairer to the students at first glance – is the *multiple* lottery mechanism, which allows each school to conduct its own lottery. The actual assignment is made by the TTC mechanism applied to the preference profiles of the students and the priority profiles of the schools [4]. Pathak [28, pp. 3] notes that during the course of the design of the new assignment mechanism, policymakers from the Department of Education believed that the single lottery mechanism is less equitable than the multiple lottery mechanism. Remarkably, Pathak shows that, for the special case of the problem in which each school has exactly one vacant spot, the distribution of assignments is exactly the same under both mechanisms! In a recent paper, Sethuraman [36] proves the equivalence of the single and multiple lottery mechanisms in full generality. This result is proved by introducing a new class of mechanisms called *Partitioned Random Priority* (PRP). Under the PRP mechanism, we are given an arbitrary partition $S_1, S_2, \ldots, S_k$ of the "schools;" the schools within each S_i use a *common* lottery, and distinct S_i's use an independent lottery. (Note that if each school is in a partition by itself we recover the multiple lottery mechanism; if all the schools belong to a single partition, we recover the single lottery mechanism.) The key result is that the distribution of assignments under the PRP mechanism is the same, *regardless of the partition of the schools*. The analog of Pathak's result when schools have multiple seats follows: if there are k schools and school i can admit q_i students, then make q_i copies of school i, and let S_i consists of these q_i copies.

7 Conclusion

The literature on house allocation problems is extensive, nevertheless a number of challenging open questions remain. One important direction for future research is the study of dynamic versions of these problems. For example, the kidney exchange problem is really a dynamic problem, in which donor-patient pairs arrive or leave, and patients need to make a trade-off between accepting a less-preferred option now versus waiting for a potentially better match in the future. While there are a few recent papers that take up such problems [40], much remains to be done. Another intriguing question is to obtain an effective characterization of the domains for which the Gibbard-Satterthwaite impossibility result can be avoided. The house allocation and related problems represent one such general class of problems, but there are many others. The work on integer programming approaches to social choice [37] is a first step in this direction, but we are still far from a complete understanding of this question.

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Discussion Column

Garud Iyengar and Anuj Kumar

Parametric Network Flows in Adword Auctions

1 Introduction

Sponsored search advertising is a major source of revenue for internet search engines. Close to 98 % of Google's total revenue of $6 billion for the year 2005 came from sponsored search ads. It is believed that more than 50 % of Yahoo!'s revenue of $5.26 billion was from sponsored search advertisement. Sponsored search ads work as follows. A user queries a certain *adword*, i.e., a keyword relevant for advertisement, on an online search engine. The search engine returns the links to the most "relevant" webpages and, in addition, displays certain number of relevant sponsored links in certain fixed "slots" on the result page. Every time the user clicks on any of these sponsored links, she is taken to the website of the advertiser sponsoring the link and the search engine receives certain price per click from the advertiser. The likelihood that a user clicks an ad is a function of the slot; therefore, advertisers have a preference over which slot carries their link. The click likelihood is also a function of the exogenous brand values of the advertisers; therefore, search engines prefer allocating more desirable slots to advertisers with higher exogenous brand value. Thus, search engines need a mechanism for allocating slots to advertisers. Since auctions are very effective mechanisms for revenue generation and efficient allocation, they have become the mechanism of choice for assigning sponsored links to advertising slots.

Adwords auctions are dynamic in nature – the advertisers are allowed to change their bids quite frequently. In this note, we design and analyze static models for adword auctions. We use the dominant strategy solution concept in order to ensure that the static model adequately approximates dynamic adword auctions. We consider the case where the private known valuation for a click is independent of the ad slot, i.e., the advertiser values clicks but is indifferent about where that click originated. The search engine wants to assign slots to advertisers using an auction that induces the advertisers to be truthful, i.e., the auction is incentive compatible, and leaves them noworse off than if they had not participated in the adword auction, i.e., it is rational for advertiser to participate in the auction. Among all these auctions, the auctioneer wants to choose one that maximizes revenue. In this note we show that the revenue maximizing incentive compatible, individually rational, auction can be implemented in a computationally tractable manner using parametric network flows.

2 Revenue Maximizing Adword Auction

Suppose there are n advertisers bidding for $m(\leq n)$ slots on a specific adword. Let c_{ij} denote the click-through-rate when advertiser i is assigned to slot j. For convenience, we will set $c_{i,m+1} = 0$ for all $i = 1, \ldots, n$. We assume that for all bidders i, the rate c_{ij} is strictly positive and non-increasing in j, i.e., *all* bidders rank the slots in the same order. The rates c_{ij}, for *all* (i, j) pairs $i = 1, \ldots, n$, $j = 1, \ldots, m$, are known to the auctioneer, and *only* the rates $(c_{i1}, c_{i2}, \ldots, c_{im})$ are known to bidder i, i.e., each bidder only knows her click-through-rates.

We assume that the true per-click-value v_i of advertiser i is private information and is independent of the slot j. We assume an *independent private values* (IPV) setting with a commonly known prior distribution function that is continuously differentiable with density $f(v_1, \ldots, v_n) = \prod_{i=1}^n f_i(v_i) : \mathbb{R}^n_+ \mapsto \mathbb{R}_{++}$. Even through we use dominant strategy as the solution concept, we still need the prior distribution in order to select the optimal revenue maximizing mechanism.

We restrict attention to direct mechanisms – the revelation principle guarantees that this does not introduce any loss of generality. In this setting, advertiser i submits a bid b_i which is the amount she is willing to pay for a ad-slot. Let $\mathbf{b} \in \mathbb{R}^n_+$ denote the bids of the n bidders. An auction mechanism for this problem consists of the following two components:

1. An allocation rule $\mathbf{X} : \mathbb{R}^n_+ \mapsto \{0,1\}^{n\times m}$ such that $\sum_{i=1}^n X_{ij}(\mathbf{b}) = 1$, for all $j = 1, \ldots, m$, and $\sum_{j=1}^m X_{ij}(\mathbf{b}) \leq 1$, $i = 1, \ldots, n$. Thus, $\mathbf{X}(\mathbf{b})$ is a matching that matches bidders to slots as a function of the bid $\mathbf{b}$. We denote the set of all possible matchings of n advertisers to m slots by $\mathcal{M}_{nm}$.
2. A payment function $\mathbf{T} : \mathbb{R}^n_+ \mapsto \mathbb{R}^n$ that specifies the amount each of the n bidders pays the auctioneer. Note that this payment function depends on the bids of *all* the advertisers.

In Lemma 1 we show that the payment of a bidder who is not allocated any slot can be set to zero without any loss of generality. Thus, we can define the per click payment t_i of advertiser i as $t_i(\mathbf{b}) = \frac{T_i(\mathbf{b})}{\sum_{j=1}^m c_{ij} X_{ij}(\mathbf{b})}$. For all $i = 1, \ldots, n$, $v_i \in \mathbb{R}_+$, and $\mathbf{b}_{-i} \in \mathbb{R}^{n-1}_+$

$$u_i(b, v; (\mathbf{X}, \mathbf{T}), \mathbf{b}_{-i}) = \sum_{j=1}^m (c_{ij} v - t_i(b, \mathbf{b}_{-i})) X_{ij}(\mathbf{b}, \mathbf{b}_{-i}) \qquad (1)$$

denote the utility the advertiser i of type v who bids b and all the other advertisers bid $\mathbf{b}_{-i}$. We restrict attention to mechanisms $(\mathbf{X}, \mathbf{T})$ that satisfy the following two properties:

1. Incentive compatibility (IC): For all $i = 1, \ldots, n$, $v_i \in \mathbb{R}_+$, and $\mathbf{b}_{-i} \in \mathbb{R}^{n-1}_+$, $v_i \in \operatorname{argmax}_{b \in \mathbb{R}_+} \{u_i(b, v; (\mathbf{X}, \mathbf{T}), \mathbf{b}_{-i})\}$, i.e., truth telling is ex-post dominant.
2. Individual rationality (IR): For all $i = 1, \ldots, n$, $v_i \in \mathbb{R}_+$, and $\mathbf{b}_{-i} \in \mathbb{R}^{n-1}_+$ $\max_{b \in \mathbb{R}_+} \{u_i(b, v; (\mathbf{X}, \mathbf{T}), \mathbf{b}_{-i})\} \geq 0$, i.e., we implicitly assume that the outside alternative is worth zero.

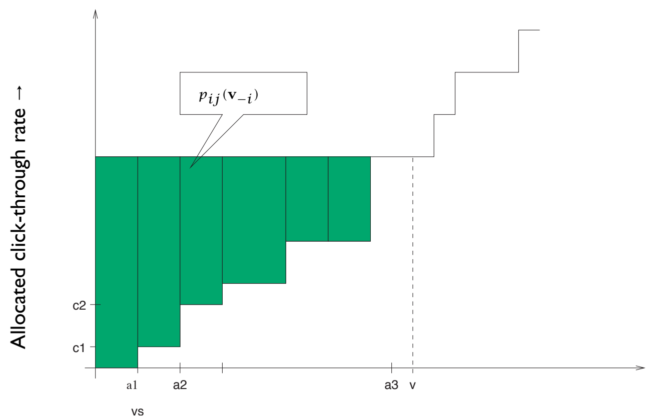

Figure 1. *Incentive Compatibility: Allocated click-though-rate as a function of valuation*

Lemma 1. *The following are equivalent characterizations of IC allocation rules.*

(a) The click-through-rate $\sum_{j=1}^m c_{ij} X_{ij}(v_i, \mathbf{v}_{-i})$ *is non-decreasing in* v_i *for all fixed* $\mathbf{v}_{-i}$.

(b) For all i *and* $\mathbf{v}_{-i}$ *there exist thresholds* $a_{i,m+1} = 0 \leq a_{im}(\mathbf{v}_{-i}) \leq a_{i,m-1}(\mathbf{v}_{-i}) \leq \cdots \leq a_{i,1}(\mathbf{v}_{-i}) \leq \infty$ *such that bidder* i *is assigned slot* j *iff* $v_i \in (a_{ij}(\mathbf{v}_{-i}), a_{i,j+1}(\mathbf{v}_{-i})]$.

(c) For each advertiser i, there exist slot prices $\{p_{ij}(\mathbf{v}_i)\}_{j=1}^m$ *of the form*

$$\begin{aligned} p_{ij}(\mathbf{v}_{-i}) &= \frac{1}{c_{ij}} \sum_{k=j}^m (a_{ik}(\mathbf{v}_{-i}) - a_{i,k+1}(\mathbf{v}_{-i}))(c_{ij} - c_{i,k+1}) \\ &= \frac{1}{c_{ij}} \sum_{k=j}^m (c_{ik} - c_{i,k+1}) a_{ij}(\mathbf{v}_{-i}) \end{aligned} \qquad (2)$$

where $0 \leq a_{im}(\mathbf{v}_{-i}) \leq a_{i,m-1}(\mathbf{v}_{-i}) \leq \cdots \leq a_{i,1}(\mathbf{v}_{-i}) \leq \infty$ *such that advertiser* i *self-selects her assigned slot.*

Note that the price $p_{ij} \equiv 0$ for $j = m+1$, i.e., the advertiser does not pay anything if she is not assigned a slot. Lemma 1 gives us a method for constructing incentive compatible auctions – first select an allocation rule that guarantees $\sum_{j=1}^m c_{ij} X_{ij}(v_i, \mathbf{v}_{i-})$ is non-decreasing in v_i for fixed $\mathbf{v}_{-i}$; next, compute the $a_{ij}((\mathbf{v}_i)$ and charge the slot prices $p_{ij}(\mathbf{v}_{-i})$. The proof of Lemma 1 follows from Holmstrom's Lemma (see, p.70 in [3]). For details see [2].

Our next task is to select a revenue maximizing auction. From [4], it follows that expected revenue of the auctioneer under any dominant strategy incentive compatible allocation rule $\mathbf{X}$ is given by

$$\mathbb{E}\left[\sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}\left(v_i - \frac{1-F_i(v_i)}{f_i(v_i)}\right) X_{ij}(\mathbf{v}) + \sum_{i=1}^{n} u_i(0,\mathbf{v}_{-i})\right]$$

where $f_i : \mathbb{R}_+ \mapsto \mathbb{R}_{++}$ is the prior density of v_i, $i = 1,\ldots,n$. Let

$$\mathbf{X}^*(\mathbf{v}) \in \underset{\mathbf{X}\in\mathcal{M}_{nm}}{\operatorname{argmax}} \left\{\sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}\left(v_i - \frac{1-F_i(v_i)}{f_i(v_i)}\right) X_{ij}\right\}. \qquad (3)$$

Suppose the virtual valuations per click $\nu_i(v_i) = v_i - \frac{1-F_i(v_i)}{f_i(v_i)}$ are non-decreasing. Then the allocation rule $\mathbf{X}^*$ results in a non-decreasing total-click-through for each of the bidders. Hence, part (a) in Lemma 1 implies that $\mathbf{X}^*$ is IC. Since the point-wise maximum is an upper bound on any expected revenue maximizing allocation, $(\mathbf{X}^*,\mathbf{T}^*)$ is expected revenue maximizing, dominant strategy incentive compatible, individually rational allocation rule with per-click prices given by (2). When the virtual valuations $\nu(v_i)$ are non-monotonic, the revenue maximizing mechanism can be constructed by first *ironing* (see [4]) the virtual valuation to obtain a non-decreasing virtual valuations $\tilde{\nu}_i(v_i)$ and then solving (3). Given $\mathbf{v}$, (3) can be solved, or equivalently $\mathbf{X}^*(\mathbf{v})$ can be computed, in $\mathcal{O}(m^2 n)$ operations as a solution to an optimal weighted matching problem.

Next, we show how to efficiently compute the thresholds $a_{ij}(\mathbf{v}_{-i})$ and the payments $p_{ij}(\mathbf{v}_{-i})$ corresponding to the rule $\mathbf{X}^*$. Recall that $a_{ij}(\mathbf{v}_{-i})$ denotes the threshold value that ensures that advertiser i will be assigned to a slot j or better. Fix an advertiser i_0. Consider the parametric assignment problem in λ:

$$\max_{\mathbf{X}\in\mathcal{M}_{nm}} \left\{\lambda \sum_{j=1}^{m} c_{i_0 j} X_{i_0 j} + \sum_{k=1,k\neq i}^{n}\sum_{j=1}^{m} \tilde{\nu}_k(v_k) c_{kj} X_{kj}\right\}. \qquad (4)$$

This is an LP and the solution $X_{i_0 j}$ is a piece-wise constant in λ. Let $\{\lambda_j\}_{j=1}^{m}$ denote values of λ such that $X_{i_0 j} = 1$ for all $\lambda \in (\lambda_j, \lambda_{j+1}]$. Since $\tilde{\nu}_{i0}$ is non-decreasing, the thresholds $a_{i_0 j} = \tilde{\nu}_{i_0}^{-1}(\lambda_j)$. However, it is not immediately obvious that the thresholds $\{\lambda_j\}$ can be efficiently computed.

The assignment problem (4) can be formulated as a parametric minimum cost network flow problem on an appropriately defined graph. Using standard results for parametric network flows, (see Exercise 11.48 in [1] and [2] for details) one can show that there exists an algorithm OptMatch that takes as input $(i_0, \mathbf{v}_{-i_0}, \mathbf{c})$ and computes the thresholds in $\mathcal{O}(m^2 n)$ operations. Once we have the thresholds, computing the slot prices is easy. Algorithm ComputePrices cycles through each of the advertisers, calls OptMatch to get the thresholds, and then uses (2) to compute the prices. Thus, Algorithm ComputePrices computes the slot prices in $\mathcal{O}(m^2 n^2)$ operations. Thus, we have established the goal that we set ourselves in this note – we have an efficient algorithm for running a revenue maximizing adword auction!

Algorithm 1 ComputePrices

$\mathbf{z} \leftarrow (\tilde{\nu}_1(v_1),\ldots,\tilde{\nu}_n(v_n))$, $c_{i,m+1} \leftarrow 0, a_{i,m+1} \leftarrow 0 \quad \forall i.$
for $i = 1$ to n **do**
  $\tilde{\mathbf{a}} \leftarrow$ OptMatch$(i, \mathbf{z}_{-i}, \mathbf{c})$.
  for $j = 1$ to m **do**
    $a_{ij} \leftarrow \psi_i^{-1}(\tilde{a}_{ij})$
    if $a_{ij} = \infty$ **then**
      $a_{ij} \leftarrow \min_{j<k\leq m} a_{ik}$
    end if
  end for
  for $j = 1$ to m **do**
    $p_{ij} \leftarrow \frac{1}{c_{ij}} \sum_{k=j}^{m} (a_{ik} - a_{i,k+1})(c_{ij} - c_{i,k+1})$
  end for
end for

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Dimitri P. Bertsekas and Zhi-Quan (Tom) Luo

Paul Tseng, 1959–2009

Paul Tseng, Professor of Mathematics at the University of Washington, Seattle, has been missing since August 13, 2009 while kayaking in the Yantze river near Lijiang, in Yunnan province of China. His friends were alerted when he did not show up for an invited talk on August 17 at an International Conference on Numeric Optimization and Numeric Linear Algebra in Lijiang, hosted by the Chinese Academy of Sciences. The sad news quickly propagated through the optimization community, where Paul is greatly loved by his many friends and widely respected for his research accomplishments. His scheduled semi-plenary lecture at the ISMP meeting in Chicago (immediately after the Lijiang conference) had to be canceled.

Following an intensive search, helped by his close friend and collaborator, Tom Luo of the University of Minnesota, the events leading to Paul's disappearance were pieced together: Paul flew from Seattle on August 11, through Shanghai and Chengdu, into Lijiang on August 13. Then from the Lijiang airport he took a taxi directly to a remote location, near the Jin'an Bridge on the Jinsha river (a tributary of the Yantze river), where he launched his kayak in rapid waters at about 4PM on August 13. He had planned to kayak for three days, or about 400 kilometers, on the Jinsha river through a beautiful but mostly uninhabited mountainous region. Paul, an avid outdoorsman and very seasoned kayaker, appeared to struggle with the water from the beginning (based on eye-witness accounts), and then disappeared from view. His kayak and backpack were found on August 30 a few kilometers downstream from where he entered. It is believed that Paul was a victim of an unfortunate accident. He is survived by his mother and his sister Nora.

Paul grew up in Taiwan and Canada, and worked primarily in the United States. His family came from China to Taiwan, where he was born on Sept 21, 1959 in Hsin-Chu. Later his family moved to Taipei, where Paul went to elementary school. Paul and his family moved to Vancouver, Canada in December 1970, where Paul graduated from high school in 1977.

Paul was well known for his adventurous and unconventional travels, often using bicycle and kayak. In the years 1986–2008, he took long bicycle trips through Europe, Central America, and Kenya, and kayaking trips in the Danube, the Mekong, the Baltic Sea, the Nile, the Red Sea, Vancouver Island, the Yellow River in China, and the Rio Madre de Dios (a headwater tributary of the Amazon River in Peru). He kayaked for long distances (as examples, from Laos to the Mekong delta in Vietnam, and from Prague to the Danube delta in Romania), often mixing with local people on the way and sharing their lifestyles. He brought back many pictures and stories, which can be found at his website http://www.math.washington.edu/~tseng/personal.html. His ambition was to kayak in all the major rivers of the world.

Paul had several other nonprofessional interests. He liked sports and he was well-known for his expert tennis game. He also had a strong interest in music, particularly in playing the piano, and he liked drawing, painting, pottery making and woodcarving. He spent a few summers drawing portraits in Stanley Park in Vancouver, and he had a summer job making wood carvings of West Coast animals. He was a minimalist in life, with a deeply held commitment to environmentalism and noble causes (he had frequently "walked for hunger"), and he tended his garden and beautiful roses with great care.

Paul received his B.Sc. from Queen's University (Kingston, Ontario) in Mathematics in 1981, and his Ph.D. from the Operations Research Center of the Massachusetts Institute of Technology (Cambridge, MA) in 1986. After working for one year at the University of British Columbia, he spent three years at the Massachusetts Institute of Technology as a postdoc in the group of Dimitri Bertsekas and John Tsitsiklis, working on optimization and distributed computation. Paul moved in 1990 to the University of Washington's Department of Mathematics, where he worked alongside Terry Rockafellar and Victor Klee.

Paul's research has been mainly in continuous optimization, with side interests in discrete optimization, distributed computation, and network and graph algorithms. He is widely recognized by his peers as one of the foremost optimization researchers of his generation, at a time of great progress in his field. He has published extensively (over 120 journal papers), and his research subjects include among others:

- Efficient algorithms for structured convex programs and network flow problems,
- Complexity analysis of interior point methods for linear programming,
- Parallel and distributed computing,
- Error bounds and convergence analysis of iterative algorithms for optimization problems and variational inequalities,
- Interior point methods and semidefinite relaxations for hard quadratic and matrix optimization problems, and
- Applications of large scale optimization techniques in signal processing and machine learning.

Paul's Ph.D. thesis was on network optimization methods and related monotropic programming problems. He coauthored with his advisor Dimitri Bertsekas, a series of papers on relaxation methods and monotropic programming, as well as a publicly available network optimization program, called RELAX, which is widely used in industry and academia for research purposes. Among his other research accomplishments, Paul, together with Tom Luo, resolved a long-standing open question on the convergence of matrix splitting algorithms for linear complementarity problems and affine variational inequalities, and was the first to establish the convergence of the affine scaling algorithm for linear programming in the presence of degeneracy. Furthermore, in a series of papers with Tom Luo, he developed a theory of error bounds and used it creatively to yield a strong convergence rate analysis for a broad class of iterative algorithms including the proximal splitting methods and the successive projection methods to convex sets, both of which find contemporary applications in compressive sensing and image processing. He was widely admired for his creative work and his productivity, and was well-liked for his cheerful and friendly manner. He has had close collaborations with several colleagues, and he served the community as a conscientious and hard-working editor in several top optimization journals for many years.

A special workshop called "Large-scale optimization: Analysis, algorithms and applications" is planned in Paul's honor for May 21, 2010, at Fudan University, in Shanghai, China, where several of his collaborators will participate and present research related to topics where Paul's work has had a major impact. See http://www.se.cuhk.edu.hk/Workshop2010/home.html where you can view and upload photos of Paul, and leave messages. You may also visit http://www-optima.amp.i.kyoto-u.ac.jp/ORB/issue34/issue34.html where you can read messages from Paul's friends and colleagues, and have the opportunity to leave your own message.

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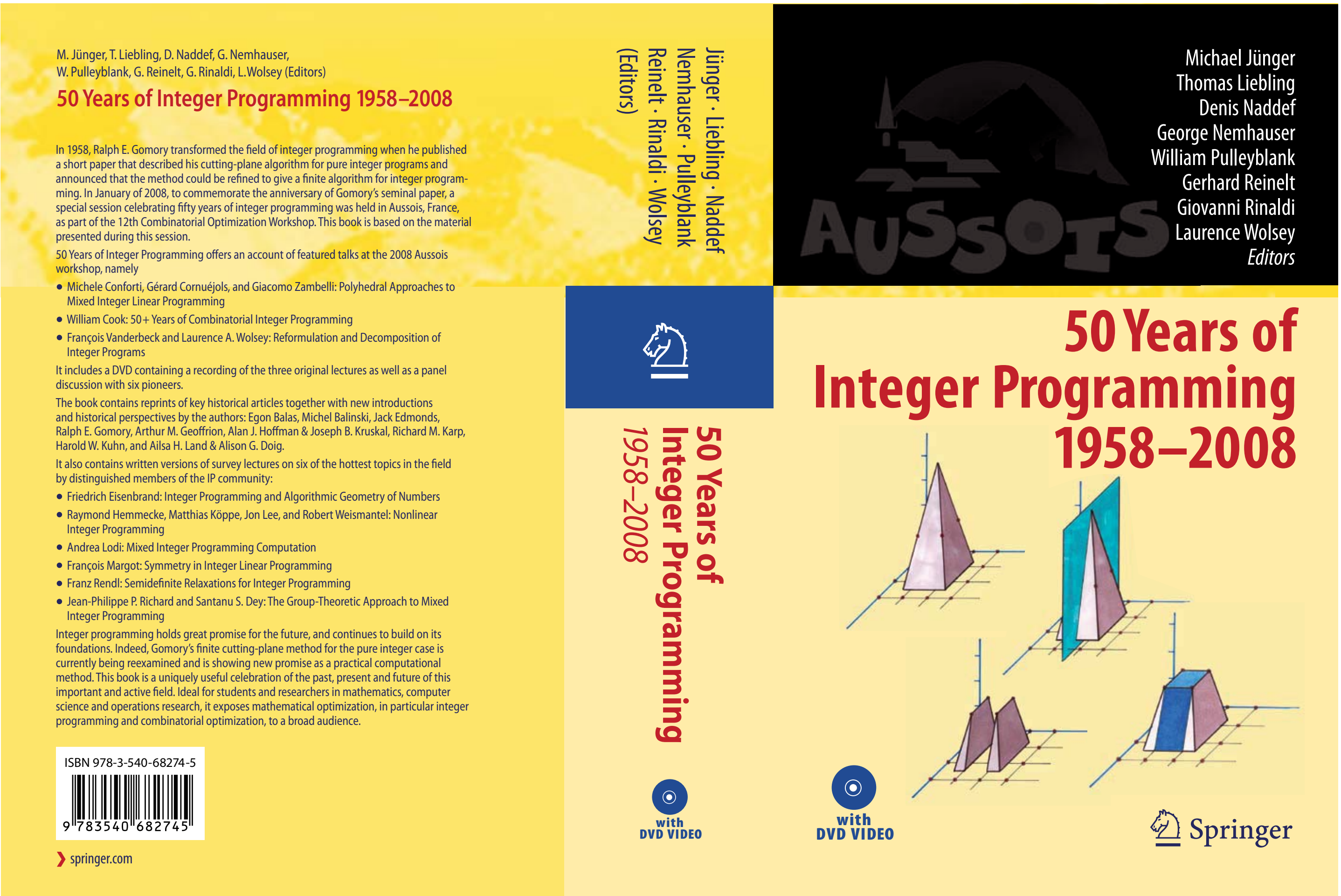

Announcements

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MIPLIB2010 – Call for contribution

Since its first release in 1992, the MIPLIB has become a standard test set used to compare the performance of mixed integer linear optimization software and to evaluate the computational performance of newly developed algorithms and solution techniques.

Seven years have passed since the last update in 2003. Again, the progress in state-of-the-art optimizers, and improvements in computing machinery have made several instances too easy to be of further interest. New challenges need to be considered!

Last year a group of interested parties including participants from ASU, COIN, FICO, Gurobi, IBM, and MOSEK met at ZIB to discuss the guidelines for the 2010 release of the MIPLIB. It will be the fifth edition of the Mixed-Integer Programming LIBrary.

Therefore, we are looking for interesting and challenging (mixed-) integer linear problems from all fields of Operations Research and Combinatorial Optimization, ideally ones which have been built to model real life problems. We would be very happy if you contributed to this library by sending us hard and/or real life instances.

We have recently opened our submission web page and are looking forward to your contributions: http://miplib.zib.de/miplib2010

24th European Conference on Operational Research (EURO XXIV)

Lisbon, Portugal, July 11–14, 2010. The 24th European Conference on Operational Research (EURO XXIV) is organized by EURO (The Association of European OR Societies) and APDIO (The Portuguese OR Society), with the support of the Faculty of Sciences of the University of Lisbon and CIO (Operational Research Centre, Portugal).

The Programme Committee (chaired by Silvano Martello) and the Organizing Committee (chaired by Jose Paixao), are preparing a high quality scientific programme and an exciting social programme for the Conference.

Plenary Speakers: Harold W. Kuhn and John F. Nash, Jr.

Invited Speakers: Fran Ackermann, Noga Alon, James Cochran, Elena Fernandez, Pierre Hansen, Martine Labbe, Nelson Maculan, Michel Minoux, Arkadi Nemirovski, Stefan Reichelstein, Alexander Shapiro, and Berthold Vöckin.

Contacts: prog@euro2010lisbon.org (Programme)
registration@euro2010lisbon.org
(Registration, travel and accommodation)
info@euro2010lisbon.org (General enquiries)
Website: www.euro2010lisbon.org

52th Workshop: Nonlinear Optimization, Variational Inequalities and Equilibrium Problems

July 2–10, 2010, Ettore Majorana Centre for Scientific Culture International School of Mathematics "G. Stampacchia" Erice, Italy. The Workshop aims to review and discuss recent advances in the development of analytical and computational tools for Nonlinear Optimization, Variational Inequalities and Equilibrium Problems, and to provide a forum for fruitful interactions in strictly related fields of research.

Topics include constrained and unconstrained nonlinear optimization, global optimization, derivative-free methods, nonsmooth optimization, nonlinear complementarity problems, variational inequalities, equilibrium problems, game theory, bilevel optimization, neural networks and support vector machines training, applications in engineering, economics, biology and other sciences.

The Workshop will include keynote lectures (1 hour) and contributed lectures (30 min.). Members of the international scientific

ICCOPT is a forum for researchers and practitioners interested in continuous optimization, which takes place every three years. The first version was held in 2004 at Rensselaer Polytechnic Institute (Troy, NY, USA), while the second version was organized in 2007 at McMaster University (Hamilton, Ontario, Canada).

The February 27 earthquake in Chile has not affected the conference facilities, and the international airport in Santiago and other transportation systems are operating normally. (Most damage occurred 200–600 km south of Santiago.) Hence, ICCOPT 2010 will go ahead as planned. Several deadlines have been extended (see below).

The Conference will feature a series of invited lectures, contributed talks, and streams on specific subjects.

Plenary speakers include:

- Xiaojun Chen (The Hong Kong Polytechnic University)
- Roberto Cominetti (Universidad de Chile)
- Ignacio Grossman (Carnegie Mellon)
- Rene Henrion (Weierstrass Institute for Applied Analysis and Stochastics)
- Marco Locatelli (Università di Parma)
- Zhi-Quan (Tom) Luo (University of Minnesota)
- Jorge Nocedal (Northwestern University)
- Mikhail Solodov (Instituto de Matematica Pura e Aplicada)
- Philippe Toint (Facultés Universitaires Notre Dame de la Paix)
- Stefan Ulbrich (Technische Universität Darmstadt)
- Luis Nunes Vicente (University of Coimbra)

A School on Continuous Optimization and Mathematical Modeling, addressed to PhD students and young researchers, will precede ICCOPT-2010, on July 24–25. This School will provide an introductory but up-to-date perspective in two areas, namely,

- Optimization under uncertainty (Ruszczynski, Dentcheva, Shapiro, Wets)
- Optimization in natural resources management (Alvarez, Amaya, Ramirez, Gajardo, Rapaport)

Free registration will be provided for all School participants, and they may also apply for free accommodation during the School and Conference.

Important Dates: Streams Submission: March 31, 2010
Conference: Early registration: May 28, 2010
Abstract submission: May 10, 2010
School: Applications: April 30, 2010

More Information: For further information, including abstract and stream submission, registration procedures, fees and accommodation sites, please see the conference web site at www.iccopt2010.cmm.uchile.cl, or send email to iccopt2010@dim.uchile.cl.

community are invited to contribute a lecture describing their current research and applications. Acceptance will be decided by the Advisory Committee of the School.

Invited lecturers who have confirmed the participation are: Ernesto G. Birgin, Francisco Facchinei, Christodoulos A. Floudas, David Gao, Diethard Klatte, Eva K. Lee, Marco Locatelli, Jacqueline Morgan, Evgeni A. Nurminski, Jong-Shi Pang, Mike J. D. Powell, Franz Rendl, Nikolaos V. Sahinidis, Katya Scheinberg, Marco Sciandrone, Valeria Simoncini, Henry Wolkowicz, Ya-xiang Yuan.

A special issue of Computational Optimization and Applications will be dedicated to the Workshop, including a selection of invited and contributed lectures.

The scientific and organizing commitee: Gianni Di Pillo (SAPIENZA – Universita' di Roma, Italy), Franco Giannessi (Universita' di Pisa, Italy), Massimo Roma (SAPIENZA – Universita' di Roma, Italy).

Further information: www.dis.uniroma1.it/~erice2010, erice2010@dis.uniroma1.it

Poster of the Workshop: www.dis.uniroma1.it/~erice2010/poster.pdf

Conference on Computational Management Science – CMS2010

July 28th–30th 2010, University of Vienna, Austria. The CMS conference is an annual meeting associated with the journal of Computational Management Science www.springer.com/business/operations+research/journal/10287 published by Springer.

The aim of this conference is to provide a forum for theoreticians and practitioners from academia and industry to exchange knowledge, ideas and results in a broad range of topics relevant to the theory and practice of computational methods, models and empirical analysis for decision making in economics, finance, management, and engineering.

The CMS Best Student Paper Prize will be awarded at the CMS conference. The prize is 300 EUR and the possibility of publication in the journal of Computational Management Science. Papers can be nominated by the supervisors of the students. Submission deadline is June 1st 2010. Only registered participants' papers will be considered for the prize.

Please visit www.univie.ac.at/cms2010/ for more details, abstract submission, registration and accommodation.

SIAG/Optimization Prize: Call for Nominations

The SIAM Activity Group on Optimization Prize (SIAG/OPT Prize) will be awarded at the SIAM Conference on Optimization to be held May 15–19, 2011, in Darmstadt, Germany.

The SIAG/OPT Prize, established in 1992, is awarded to the author(s) of the most outstanding paper, as determined by the prize

Mixed Integer Programming 2010

July 26–29, 2010

Georgia Institute of Technology, Atlanta, Georgia

http://www2.isye.gatech.edu/mip2010/

The 2010 Mixed Integer Programming workshop will be the 7th in a series of annual workshops held in North America designed to bring the integer programming community together to discuss very recent developments in the field. The workshop series consists of a single track of invited talks.

We encourage anyone with interests in mixed integer programming to participate. Space is limited, and thus early registration is recommended.

There will be a contributed poster session for which we invite all participants to submit an abstract; see the website for instructions. Space for posters is limited, so we may not be able to accommodate all posters. There will be ample time for discussion and interaction between the participants during the workshop.

Thanks to the generous support by our sponsors, registration is free, and travel support is available. Funding priority will be given to students and postdocs who have submitted poster abstracts.

Confirmed speakers

Karen Aardal	Quentin Louveaux
Tobias Achterberg	Jim Luedtke
Egon Balas	Andrew Miller
Pierre Bonami	Michele Monaci
Michele Conforti	James Ostrowski
Sanjeeb Dash	Marc Pfetsch
Daniel Espinoza	Jean-Philippe Richard
Matteo Fischetti	Martin Savelsbergh
Antonio Frangioni	Stefano Smriglio
Zonghao Gu	Andrea Tramontani
Ellis Johnson	Santosh Vempala
Simge Kucukyavuz	Juan-Pablo Vielma
Jon Lee	Giacomo Zambelli

Important Dates

March 31
Deadline for poster abstracts and requests for travel support

April 30
Notification on poster acceptance and travel support

July 26–29
Workshop

Organizing committee

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Ismael Regis de Farias Jr.
(Texas Tech University)

Ricardo Fukasawa
(University of Waterloo)

Matthias Köppe
(University of California, Davis)

Andrea Lodi
(University of Bologna)

committee, on a topic in optimization published in English in a peer-reviewed journal. The eligibility period is the four calendar years preceding the year of the award. For more information on the prize, including a list of previous recipients, see www.siam.org/prizes/sponsored/siagopt.php.

Eligibility: Candidate papers must be published in English in a peer-reveiwed journal bearing a publication date in the 2007–2010 calendar years (January 1, 2007–December 31, 2010). They must contain significant research contributions to the field of optimization, as commonly defined in the mathematical literature, with direct or potential applications.

Description of the award: The award will consist of a plaque and a certificate containing the citation. At least one of the prize recipients is expected to attend the award ceremony and to present the paper at the conference.

Nominations: Nominations, together with a PDF version of the paper, should be submitted via e-mail to one of the members of the SIAG/OPT Prize Committee before November 15, 2010.

Selection Committee: The members of the selection committee are: Yinyu Ye (Chair), Stanford University; Shabbir Ahmed, Georgia Institute of Technology; Philip Gill, University of California, San Diego; Etienne de Klerk, Tilburg University, The Netherlands; Jean Philippe Richard, University of Florida.

DON'T FORGET TO ATTEND IPCO 2010
The 14th Conference on Integer Programming and Combinatorial Optimization

June 9–11, 2010, Ecole Polytechnique Fédérale de Lausanne, Switzerland. The IPCO conference is under the auspices of the Mathematical Programming Society. It is held every year, except for those years in which the 'Symposium on Mathematical Programming' takes place. The conference is meant to be a forum for researchers and practitioners working on various aspects of integer programming and combinatorial optimization. The aim is to present recent developments in theory, computation, and applications in these areas.
http://ipco.epfl.ch/

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