# OPTIMA 93

Mathematical Optimization Society Newsletter

## MOS Chair's Column

November 15, 2013. Thanks to everyone for your support. It was a honor to be elected to chair the society and I am looking forward to working with our Council, our editors, and all supporters of mathematical optimization. I want to especially thank former chair Philippe Toint and his dynamic predecessor Steve Wright for making it easy to step into such a well-run society.

Our membership is at an all-time high, thanks to the wildly successful ISMP 2012 in Berlin, followed by the fantastic turnout for ICCOPT 2013 in Lisbon. Going along with the surge in membership, the society is in a strong financial position and quite able to support new initiatives.

I'm certain my three-year term as chair will pass by in a flash. So if you have suggestions for possible new MOS activities, now is the time to contact me!

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## Note from the Editor

Dear MOS community, After seven years and twenty one issues it is time for me to step down from the editorial board of *Optima*. I will be taking on other editorial duties within MOS and Volker Kaibel will be the new *Optima* editor, beginning his new duties in 2014. Sam Burer will also remain on the *Optima* board.

I have joined *Optima* in 2007 as co-editor, with Alberto Caprara, while Andrea Lodi became the new editor-in-chief. Together we redesigned *Optima*'s format, introducing two articles per issue - one main article and one related discussion column. We also increased the number of issues per year to three. Alberto and Andrea were the driving force behind these changes and *Optima* has maintained this new format for seven years now. In 2011, Alberto and Andrea stepped down and I took over the role of the main editor, while Sam Burer and Volker Kaibel joined as co-editors and they have been contributing tremendously to the newsletter ever since.

The seven years flew by very quickly and I find it astonishing to look back at all the articles we published. This would not have been possible without our extremely efficient and creative designer Christoph Eyrich. I am very grateful to him as well as to Sam and Volker for making my job easy and enjoyable. Finally, I would like to thank all of the authors who contributed articles to *Optima* over the years. I am very grateful and somewhat baffled at the fact that nearly none of our invitations to write an article for *Optima* have been turned down. Moreover, the articles were typically submitted, if not strictly by the requested due dates, then soon after (which allowed us to stick to the three issues per year goal). In short it has been a great privilege to be in the middle of this vibrant medium for the MOS community and I am looking forward to enjoying *Optima* as a reader from now on.

Katya Scheinberg
*Optima editor*

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Daniel Bienstock

## Progress on solving power flow problems

### 1 Introduction

Power flow problems are a mathematical representation of the physics of electrical networks, in particular transmission systems used to convey power from generators to consumers over large geographical areas. Recent developments have spurred a new emphasis on the accurate and efficient solution of such problems. Partly, the motivation stems from pressing practical issues such as the incorporation of renewable generation, a continuing growth in demand, increasing operational costs, and a growing risk of cascading failures leading to large-scale blackouts. Moreover, after a long period of relatively few methodological advances, optimization researchers have recently produced a series of interesting new developments that will likely result in significant practical changes in the near future. In this article we will provide a brief introduction to power flow problems, review the prior history of algorithms, and describe some of the recent research developments.

Power flow problems can surprise optimization experts, both by their difficulty and by the choice of algorithms that have traditionally proved successful. However, recent developments relying on semidefinite programming hold promise that a new, modern outlook may soon take hold. In the rest of this section we will present a mathematical description of problems of interest; in Section 2 we describe traditional methods still heavily in use to address the problems, as well as some of the challenges. Section 3 outlines the use of modern convex optimization methods. Section 4 describes the semidefinite programming formulations mentioned above.

To begin we will provide a very brief description of the underlying physics. Readers are referred to [1], [5] or [31] for more complete background. A transmission system (or "grid") is a network whose edges are power lines, and some of whose nodes ("buses") correspond to generators and some to demand nodes ("loads"). The purpose of a transmission system is to convey power from generators to loads. Strictly speaking, generators produce electrical current at a given voltage. "Current" describes the quantity of charge moving across a unit area per unit time, whereas "voltage" is (potential) energy per unit charge. The product of voltage and current has the units of energy per unit time, or power. Modern power grids use AC (alternating current) power; this means that both voltage and current produced at a generator are time-dependent, sinusoidal quantities with a common frequency $\omega$. Thus, for example, the voltage at a bus $k$ at time $t$ has the form

$$v_k(t) = V_k^{max}\cos(\omega t + \theta_k), \tag{1}$$

and likewise with currents. The frequency, $\omega$, is large (50 or 60 Hz) and, in steady state operation, is strictly held to a common value across all generators in a given grid. These facts make possible a simplification of the representation of physical quantities, using steady-state averages. First, the (steady-state) voltage at a bus $k$ will be represented using a complex number $V_k$ which has the form $V_k^{max}/\sqrt{2}e^{j\theta_k}$. The quantity $\theta_k$ is the (steady-state) *phase angle* at $k$ (here, $j = \sqrt{-1}$) .

The interdependence between voltages, current and power flows is then obtained through relationships that incorporate power line parameters and the laws of physics. In particular, a line $km$ (joining buses $k$ and $m$), will have a (typically small) *resistance* $r_{km}$ and a *reactance* $x_{km}$, both positive (the order of the buses $k$ and $m$ is irrelevant in this context). Denoting

$$z_{km} \doteq r_{km} + jx_{km}, \qquad y_{km} \doteq z_{km}^{-1} \tag{2}$$

the (complex) *current* on $km$ is given by

$$I_{km} = y_{km}(V_k - V_m). \tag{3}$$

This is Ohm's law for AC current. Further, the *power injected by $k$ into $km$* is

$$P_{km} = \mathcal{R}e(V_k I_{km}^*) \tag{4}$$

where $\mathcal{R}e$ indicates real part and $*$ is the conjugate operator. Equations (3) and (4) can be obtained from (1) by averaging over one period. The quantity $P_{km}$ is referred to as active (or real) power; whereas the imaginary part of the quantity $V_k I_{km}^*$, denoted $Q_{km}$, is called *reactive* power (and does have a concrete physical interpretation).

Using (4), and writing

$$y_{km} = g_{km} + jb_{km} \tag{5}$$

one obtains

$$\begin{aligned} P_{km} &= |V_k|^2 g_{km} - |V_k||V_m| g_{km}\cos\theta_{km} \\ &\qquad - |V_k||V_m| b_{km}\sin\theta_{km}, \\ Q_{km} &= -|V_k|^2 b_{km} + |V_k||V_m| b_{km}\cos\theta_{km} \\ &\qquad - |V_k||V_m| g_{km}\sin\theta_{km}, \end{aligned}$$

where $\theta_{km} \doteq \theta_k - \theta_m$. These equations are simplifications[1] however they capture the essential properties of power flows. Using these equations one can represent network-wide requirements. Given a bus $k$, consider the expressions

$$P_k = \sum_{km} (|V_k|^2 g_{km} - |V_k||V_m| g_{km}\cos\theta_{km} - |V_k||V_m| b_{km}\sin\theta_{km}), \tag{6}$$

and

$$Q_k = \sum_{km} ( - |V_k|^2 b_{km} + |V_k||V_m| b_{km}\cos\theta_{km} - |V_k||V_m| g_{km}\sin\theta_{km}) \tag{7}$$

(where the sums are over all lines of the form $km$) we see that if $k$ is a generator bus then $P_k$ is the net *power injection* of the generator into the grid. And if $k$ is a load bus, then $P_k$ is the *negative* of the net power consumed at $k$. For all other buses $k$ the sum will be zero. Similar considerations apply to the $Q_k$ quantities.

Armed with these definitions, we can now present the two central problems in power flow analysis. First, we have the *power flow problem* (**PF** for short), also known as the *load flow* problem. Here buses are partitioned into two classes:

- "PV" buses (typically generators). At a PV-bus $k$, the voltage magnitude $|V_k|$ and the active power injection $P_k$ are specified. The phase angle $\theta_k$ and the reactive injection $Q_k$ are unknown.
- "PQ" buses. At a PQ-bus $k$ both $P_k$ and $Q_k$ are known, but $|V_k|$ and $\theta_k$ are unknown.

If the total number of buses is $n$, we therefore have a system of $2n$ equations (6) and (7) on $2n$ unknowns. The objective of the problem is to solve this system of equations; from an engineering perspective the goal is to determine the pattern of power flows that arise as a result of a given choice of generator set-points (output and voltage magnitude) and demands. A critical point here is that, unlike other human networks (such as data networks) it is the underlying *physics* that determines the flows – they cannot be directly routed. Hence an outcome of the computation might be to determine that e.g. the power flow on a given line is too large. This is a risky outcome, because such a line will likely, eventually, "trip", i.e., be taken out of service, possibly causing congestion. The utility of a power flow computation is that it (hopefully) verifies the safety of a network-wide operating mode.

A second problem of interest is the (AC) *optimal power flow problem*, or *OPF* for short (we will drop the "AC" prefix, below). As the name suggests, this is an optimization problem. Its goal is to determine generator outputs so as to meet demand at minimum cost. In typical power engineering practice this problem is run with some frequency (e.g., every fifteen minutes) using demand estimates. This has proved feasible because, typically, demands (aggregated at the network level) change slowly over a fifteen-minute time span. In this problem the left-hand side quantities $P_k$ and $Q_k$ in (6) and (7) (resp.) are not given in advance, instead they are variables constrained by two-sided inequalities

$$P_k^L \le P_k \le P_k^U \quad \text{and} \quad Q_k^L \le Q_k \le Q_k^U \tag{8}$$

for each $k$. In other words, the (net) output of each bus must lie in a given range. For a load bus, the range indicates the possibility of reducing ("shedding") load so as to attain feasibility; the range will probably be tight. For generators, (8) indicates the feasible output envelope, which could be broad (when a load is co-located with a generator the range parameters are adjusted). Additionally, voltage magnitudes are similarly constrained:

$$V_k^L \le |V_k| \le V_k^U, \quad \text{for all } k. \tag{9}$$

This constraint models a stability and efficiency requirement – grids are built to operate at high voltage, and even small deviations (typically, lower than desired voltages) carry a financial cost and a risk burden. Typically voltages magnitudes are allowed to deviate by at most a few percentage points from their target values. In fact, voltage magnitudes should be near-constant across the entire grid (this observation relies on the so-called per-unit system; see [1], [5], [31]).

The objective function for OPF problems takes the form

$$\min \sum_{i \in \mathcal{G}} F_i(P_i) \tag{10}$$

where $\mathcal{G}$ is the set of generator buses, and each function $F_i$ is a convex quadratic function in $P_i$ (or a piecewise-linear approximation). This function approximates the cost of producing a certain power output. Thus, in summary, the OPF problem is

$$\min \sum_{i \in \mathcal{G}} F_i(P_i), \quad \text{subject to: (6), (7), (8) and (9).}$$

Additionally, one may impose other constraints. A typical example concerns line constraints. For example, the absolute value of the phase angle difference on a line $km$ may be upper bounded:

$$|\theta_k - \theta_m| \leq \theta_{km}^{max}. \tag{11}$$

Constraints (11) arise, typically, in the case of long lines (longer than $200$ miles, say) and they model frequency (synchronism) requirements – large phase angle differences increase the likelihood that generators will fall out of sync, possibly the single most serious risk faced by a transmission system (see, e.g., [5], page 114). A maximum phase-angle difference of $45^o$ (or near) is common; and in normal practice most phase differences will be much smaller.

Likewise, we may have constraints of the form

$$|I_{km}| \leq I_{km}^{max} \text{ or } |P_{km}| \leq P_{km}^{max}. \tag{12}$$

These are thermal constraints. When the amount of power of current that a line carries is too high, that line will overheat, and potentially sag, increasing the chance of a contact (or an electrical arc) with a foreign object; this would normally cause the line to trip, as discussed above. In normal operation, line overheating is a condition to be monitored and avoided. The modeling needed to understand the behavior of line temperature in response to power flows and ambient conditions is quite complex, see [6, 21].

In addition to these, many other types of constraints can (and do) arise in a practical setting. For example, the above discussion omits the modeling of transformers (which can be incorporated by a modification of the line equations defining $P_{km}$ and $Q_{km}$, above). We have also not described the modeling of capacitance, or shunt, effects. Further, digital controls are widely found, and a new generation of digital devices is now finding its way into operation. Also, sometimes the objective function will be modified so as to represent a discrepancy, or error, in meeting some operational goal. There is some disagreement in the power engineering community as to how to precisely characterize which modeling features are needed in a formulation to guarantee that a solution is actually implementable, or even how to model particular engineering details. Some authors have argued that is in fact undesirable to model OPF as an optimization problem, because of the difficulty in precisely modeling many engineering details.

An example is provided by the left (lower bounding) voltage constraints (9). It is important that individual buses maintain "high" voltages, however, having many load buses at low voltages indicates a more serious condition than having one bus at a comparatively lower voltage; further, drooping voltages are correlated with decreased reactive power injections (the $Q_k$) values and both are associated with decreased efficiency in the transmission of active power (with increased thermal losses and heating of power lines). Modeling these interrelated conditions in a way that is comprehensive without being overly conservative is a challenge; the choice of the parameters $V_k^L$ seems to be somewhat heuristic.

For a wider discussion see [42]. Also see [41] for a review of the history of power flow problems, solution approaches, and a discussion of practical challenges. From the perspective of applied optimization, it is not uncommon to have to deal with a situation with a myriad of constraints that are difficult to model, some of which may be important and some not. What is perhaps unique about the power flow setting is the nontrivial complexity of the many engineering details, the nonlinearity of many relationships (across different scales of data) and the inflexibility resulting from having to deal with laws of physics that cannot be bypassed. We would thus argue that it is therefore even more important to *efficiently* address the basic version of the OPF problem, as described above, so that multiple runs become practicable.

A final ingredient in the PF and OPF problems is their size. Modern transmission systems are large, and even regional systems may encompass tens of thousands of buses and lines. While some aggregation and simplification is possible, problem instances involving thousands of buses are common.

There is a vast literature on OPF problems going back more than fifty years; Carpentier [11] formulated the problem and proposed a number of techniques based on convex optimization. A broad review of the history of the problem and proposed solutions appears in [9].

## 2 Traditional solution approaches

In power engineering practice, typical PF and OPF problems are handled routinely, and with ease, using traditional methods that we will survey below. This may come to a surprise to readers, given the obvious complexity of the problems (which in fact are NP-hard) and their size. Thus: are the problems easy, or not? Is this yet another case of theoretically hard problems that in practice prove manageable? As we will see, despite their success, the traditional methods can fall short, and on very critical problem instances where accuracy is of paramount practical importance.

To understand this apparent contradiction, consider the PF problem. In routine practice an instance under consideration will be feasible, and furthermore a "near" solution will be known, for example, the solution to a different PF instance with slightly different data. And even when an approximate solution is not available, if one assumes that the grid is in a stable configuration all voltage magnitudes should be approximately equal (to $1$, after an appropriate scaling step) and all phase angle differences should be very small. Hence one can use as a starting point for Newton-Raphson, the vector where all voltage magnitudes are $1$ and all phase angle differences $\theta_{km}$ are zero.

In either case one can employ an iterative algorithm, and start from the known point. By far, the most popular algorithm in power engineering practice is the classical Newton-Raphson method [30][2]. Frequently, a remarkably small number of iterations are needed for convergence, especially when dealing with a grid not under stress.

On the negative side, Newton-Raphson can fail to converge. This will happen in particular if the Jacobian of the system becomes singular at some iteration. This condition can sometimes be bypassed (e.g., by reducing the step size) but there is no theoretical guarantee of convergence. Experimentation shows that non-convergence is especially common (in fact: almost the default) when modeling a grid under stress. In such cases, the 'flat' starting point will be far from feasible. When convergence to a feasible solution proves problematic, some popular commercial software packages used in actual operation (such as Powerworld [34]) may alter problem data on the fly in order to attain convergence. See, for example, [32], [33]. Frequently, it is argued that the number or $L^2$-norm of the changes to problem data is minimal, however it is not clear how the claim can be validated. The difficulty in diagnosing infeasibility of a PF instance is a significant challenge, for example because PF problems may be run as 'what-if' studies to determine the impact of a control action (possibly in a situation of emergency).

Similar considerations apply to the OPF problem. Many commercial software systems are available to address the OPF problem(s); additionally these systems attempt to accommodate a number of engineering features that are difficult to model directly. Frequently such systems iteratively solve a sequence of problems, relying on linearizations of the power flow equations. Note that the linearization of a non-convex constraint will result in a linear inequality that may not be globally valid (see [10]). Of course, methods of this type may only converge to a local optimum, and convergence itself is not guaranteed. Nevertheless, it must be stressed that all the software systems described here benefit from extensive industry experience supporting the underlying models, from the modeling of many complex engineering details, and from (usually) very good software implementations and often, clever mathematics.

Here, we see the same paradigm as described above: on routine problems one can (probably) compute the optimum quickly, especially if a good starting point is known. Yet on the other hand, when dealing with a grid under stress, the algorithms can (and do) break down.

The lack of a categorical proof of infeasibility of a PF instance or optimality of an OPF instance can be problematic. In the PF case, non-convergence of the Newton-Raphson method *could* be due to genuine infeasibility (as is often claimed) – or it could be due to algorithmic shortcomings. This is not just a theoretical assertion; nonconvexities are observed in both the PF and OPF problems, both in realistic problem instances and also in extremely simple, but realistically parameterized instances. See, e.g., [18] for very simple 3-bus examples that nevertheless exhibit a remarkable degree of nonconvexity.

Likewise, the lack of proof of optimality (or a bound for the optimality gap) for an OPF instance can be nontrivial especially (once again) when dealing with a grid under distress, where, potentially, the conditions that cause such stress (such as low voltages) can make transmission less efficient (i.e., more costly). Further, we note that (relying on a common optimization "trick") as a stand-in for evaluating feasibility of a PF problem, one can instead solve an OPF-like problem with an objective of the form

$$\min \sum_k \left( a_k (P_k - \tilde{P}_k)^2 + b_k (Q_k - \tilde{Q}_k)^2 \right)$$

(with constraints (8), (9)) where the quantities $\tilde{P}_k$ and $\tilde{Q}_k$ are desired target values. The resulting optimization problem may be easier to solve than the original PF instance. However, non-convergence, or convergence to a significantly suboptimal solution, again becomes problematic. Once again, the setting under consideration is that of a grid under stress, where the goal of the computation is to determine how the system will behave, for example in response to a control action.

In summary, thus, traditional methods that lack a theoretical guarantee of convergence to a guaranteed solution (i.e., a solution feasible for a PF instance, or optimal to an OPF instance) prove fast and reliable on routine problem instances. This is of course important. When considering systems under stress the traditional solution methods become much less reliable. This is also quite important, and has become more so under growing congestion, increasing costs (which push grid operators toward riskier operating modes) and the incorporation of new technologies such as renewable generation.

## 3 Convex optimization algorithms and formulations

MATPOWER [45, 46], a freely available package of PF and OPF solvers, includes a primal-dual, logarithmic barrier method especially designed for the OPF problem. MATPOWER can also use, as "plugins", a number of modern solver systems for convex optimization problems, such as MINOS [36], SNOPT [39], IPOPT [44], CONOPT [13] and KNITRO [28]. Clearly, these solvers, though modern and extremely well implemented, can only guarantee convergence to a local optimum, for the simple reason that PF and OPF problems are extremely non-convex. Recent work [8, 10, 35] performed an experimental evaluation of the solvers mentioned in the above paragraph, using as testbed a subset of the popular IEEE test family [20].

[As a parenthetical remark, it is to be noted that extremely few realistic problem instances of transmission systems are publicly available, possibly because of the proprietary nature of the data. As a result, many of the instances in the IEEE family are old, and small. Relatively recent snapshots of the Polish national grid have now become publicly available; at (roughly) 3,000 buses they constitute an interesting family of (probably) realistic small- to medium-size examples.]

The experiments in [8] used, as the largest example, the "300-bus" case. The experiments reveal a wide disparity in performance profiles among the solvers, with occasional non-convergence. On the positive side, often the solvers converged to a global optimum (but without proving so, of course). On the negative side, convergence typically required on the order of twenty seconds (for the fastest solver) and performance significantly worsened when running instances with increased low levels. This is a form of stress test – when loads increase in a grid, resources obviously become scarcer. We should add that twenty seconds is (as far as we can tell) much too slow a performance when only 300 buses are concerned – practical applications would require at least one order of magnitude more buses.

It is worth pointing out a widely used convex (in fact: linear) approximation to the PF and OPF problems. This is the DC "approximation," so-called because it is reminiscent of Ohm's law for DC (direct current) circuits. The approximation rests on a number of observations:

- First, a typical line $km$ in a high-voltage transmission system will have $r_{km} \ll x_{km}$ (small resistance compared to reactance). In the DC approximation, all lines are modeled as having zero resistance. Recalling (2), (5), a calculation shows that for any line $km$, $g_{km} = 0$, and $b_{km} = -x_{km}^{-1}$.
- As mentioned before, in a transmission system under stable operation, after scaling, $|V_k| \approx 1$ for all buses $k$. In the DC approximation we assume $|V_k| = 1$ for all $k$.
- Another characteristic of stability is that $|\theta_{km}|$ should be small for all $km$. In the DC-approximation we replace $\cos\theta_{km}$ with $1$ and $\sin\theta_{km}$ with $\theta_{km}$, for all lines $km$.

Using these simplifications (6) becomes linear:

$$P_k = \sum_{km} x_{km}^{-1}(\theta_k - \theta_m), \tag{13}$$

and reactive power is ignored. We obtain a linearly constrained optimization problem. In practice, the DC approximation is very heavily used, even when the conditions that make the approximation accurate do not hold. It is attractive when, in particular, a large volume of computations is needed. It is also useful because of its modest need for accurate data (relative to the full OPF formulation). A discussion of the pros and cons of the DC approximation is given in [43].

## 4 Conic formulations

Suppose we represent voltage at bus $k$ using rectangular coordinates, i.e. $V_k = e_k + jf_k$. With this representation, the current $I_{km} = y_{km}(V_k - V_m)$ is linear in $e_k, f_k, e_m, f_m$, and the power injection $P_{km} = V_k I^*_{km}$ is quadratic. For example, for any line $km$ one obtains

$$P_{km} = g_{\{k,m\}}e_k^2 + e_k\left(-g_{\{k,m\}}e_m + b_{\{k,m\}}f_m\right) - f_k\left(b_{\{k,m\}}e_m + g_{\{k,m\}}f_m\right) + g_{\{k,m\}}f_k^2. \quad (14)$$

(and a similarly expression for $Q_{km}$). Thus, writing

$$w = (e_1, \ldots, e_n, f_1, \ldots, f_n)^T \in \mathbb{R}^{2n},$$

equation (8) becomes,

$$P_k^L \le w^T M_k w \le P_k^U \quad \text{and} \quad Q_k^L \le w^T N_k w \le Q_k^U \quad (15)$$

for appropriate matrices $M_k, N_k \in \mathbb{R}^{2n \times 2n}$. The detailed structure of the $M_k$ and $N_k$ matrices is unimportant here. However, say that a $2n \times 2n$ matrix $A$ is *block skew-symmetric* if there exist $n \times n$ matrices $B$ and $C$ with $B$ symmetric such that

$$A = \begin{pmatrix} B & C \\ -C & B \end{pmatrix}. \quad (16)$$

Then one can show that for each bus $k$, both $M_k$ and $N_k$ are block skew-symmetric – this is a direct consequence of the structure of equations such as (14).

Similarly, (9) becomes

$$(V_k^L)^2 \le e_k^2 + f_k^2 \le (V_k^U)^2, \quad \text{for all } k. \quad (17)$$

Note that the quadratic in (17) is diagonalized, and hence its corresponding matrix (which we will call $D_k$) is block skew-symmetric. In summary, the OPF problem with objective as in (10), and injection and voltage bounds can be written as

$$\begin{aligned} &\min \sum_{i \in \mathcal{G}} F_i(w^T M_i w) \\ &\text{s.t.} \\ \text{for } 1 \le k \le n: \quad & P_k^L \le w^T M_k w \le P_k^U \\ & Q_k^L \le w^T N_k w \le Q_k^U \\ & (V_k^L)^2 \le w^T D_k w \le (V_k^U)^2, \end{aligned}$$

where all $M_k, N_k, D_k$ are block skew-symmetric. The rectangular voltage representation is not new (for example it appears in the work of Overbye cited above). Recently, however, Lavaei and Low [26] used the above formulation to obtain an SDP relaxation of the OPF problem. Some earlier work on conic and SDP formulations can be found in [2], [22], [23]. To obtain the Lavaei-Low formulation, suppose that for each generator $i$, we have cost function

$$F_i(p) = a_i p^2 + b_i p, \quad \text{where } a_i \ge 0 \text{ and } b_i \ge 0. \quad (18)$$

Setting $a_i = b_i = 0$ for $i \notin \mathcal{G}$, we can then rewrite the above optimization problem as

$$\begin{aligned} \textbf{O1:} \quad & \min \sum_k \alpha_k \\ & \text{s.t. for } 1 \le k \le n: \\ & P_k^L \le w^T M_k w \le P_k^U \\ & Q_k^L \le w^T N_k w \le Q_k^U \\ & (V_k^L)^2 \le w^T D_k w \le (V_k^U)^2 \\ & \begin{bmatrix} b_k w^T M_k w - \alpha_k & \sqrt{a_k}\, w^T M_k w \\ \sqrt{a_k}\, w^T M_k w & -1 \end{bmatrix} \preceq 0. \end{aligned}$$

(Remark: This is not a QCQP [quadratically constrained quadratic program] but we can easily reduce it to one, by replacing the semidefinite constraint for each $i \in \mathcal{G}$ with the system

$$|w^T M_i w| - p_i \le 0, \quad a_i p_i^2 + b_i w^T M_i w - \alpha_i \le 0,$$

where $p_i$ is a new variable.) We can next construct the semidefinite relaxation of problem **O1**:

$$\begin{aligned} \textbf{R1:} \quad & \min \sum_k \alpha_k \\ & \text{s.t.} W \succeq 0, \text{ and for } 1 \le k \le n: \\ & P_k^L \le M_k \bullet W \le P_k^U \\ & Q_k^L \le N_k \bullet W \le Q_k^U \\ & (V_k^L)^2 \le D_k \bullet W \le (V_k^U)^2 \\ & \begin{bmatrix} b_k M_k \bullet W - \alpha_k & \sqrt{a_k}\, M_k \bullet W \\ \sqrt{a_k}\, M_k \bullet W & -1 \end{bmatrix} \preceq 0. \end{aligned}$$

Clearly, if **R1** has a rank-1 optimal solution, it constitutes an optimal solution to **O1**. To investigate this condition, [26] constructs a dual problem to **O1**. For $1 \le k \le n$, and given reals $\lambda^U, \lambda^L, \mu^U, \mu^L, \gamma^U, \gamma^L$, define

$$h_k(\lambda^U, \lambda^L, \mu^U, \mu^L, \gamma^U, \gamma^L) \doteq \lambda_k^L P_k^L - \lambda_k^U P_k^U + \mu_k^L Q_k^L - \mu_k^U P_k^U + \gamma_k^L (V_k^L)^2 - \gamma_k^U (V_k^U)^2,$$

and consider the optimization problem with variables, for each $k$,

$$\lambda_k^U, \lambda_k^L, \mu_k^U, \mu_k^L, \gamma_k^U, \gamma_k^L, g_k, r_k$$

given by:

$$\begin{aligned} \textbf{L1:} \quad & \max \sum_k h_k(\lambda_k^U, \lambda_k^L, \mu_k^U, \mu_k^L, \gamma_k^U, \gamma_k^L) - \sum_k r_k \\ & \text{s.t.} \quad \lambda_k^U, \lambda_k^L, \mu_k^U, \mu_k^L, \gamma_k^U, \gamma_k^L \ge 0; \; g_k, r_k \in \mathbb{R}, \text{ all } k, \\ & \sum_k [\lambda_k M_k + \mu_k N_k + \gamma_k D_k] \succeq 0 \quad (19) \\ \text{and } \forall k: \quad & \lambda_k = \lambda_k^U - \lambda_k^L + 2\sqrt{a_k} g_k + b_k \\ & \mu_k = \mu_k^U - \mu_k^L \\ & \gamma_k = \gamma_k^U - \gamma_k^L \\ & \begin{bmatrix} 1 & g_k \\ g_k & r_k \end{bmatrix} \succeq 0. \end{aligned}$$

Then we have [26]:

**Theorem 4.1.** *Assume* **O1** *is feasible.* **(1)** *Problem* **L1** *is the dual of* **O1**. **(2)** *Problem* **R1** *is the Lagrangian dual of* **L1** *with* $W$ *acting as the dual variable for constraint (19). Further, strong duality holds.*

A key ingredient in the analysis in [26] is the following:

**Assumption C.** At optimality for problem **L1**, the matrix in the left-hand side of (19) has null space of dimension 2.

Now a key observation is that the matrix in (19) is block skew-symmetric. As a result, suppose that $\lambda^*, \mu^*, \gamma^*$ are the optimal values for the variables $\lambda, \mu, \gamma$ in **L1**; and likewise that $W^*$ is an optimal matrix for **R1**. Denote

$$A^* \doteq \sum_k \left[\lambda_k^* M_k + \mu_k^* N_k + \gamma_k^* D_k\right].$$

Then by complementary slackness,

$$A^* \bullet W^* = 0. \tag{20}$$

Let the spectral decomposition of $W^*$ be $W^* = \sum_{i=1}^{2n} \theta_i v_i v_i^T$, where $\theta_i \geq 0$ for all $i$. Then by (20),

$$\sum_{i=1}^{2n} \theta_i v_i^T A^* v_i = 0.$$

But by (19), $A^* \succeq 0$, and so

$$v_i^T A^* v_i = 0, \quad \text{for all } i \text{ with } \theta_i > 0. \tag{21}$$

Assumption C now implies that *at most* two eigenvalues $\theta_i$ are strictly positive. Assuming e.g. $V_k^L > 0$ for at least one $k$ we have $W^* \neq 0$, and so at least one the $\theta_i$ is positive.

We will now show that, without loss of generality, problem **R1** has a rank-1 solution. Obviously this holds if $W^*$ has rank one. Assuming otherwise, we have

$$W^* = \theta_1 v_1 v_1^T + \theta_2 v_2 v_2^T$$

where $v_1, v_2$ form an orthonormal pair. Write $v_1 = (s_1^T, t_1^T)^T$ where $s_1, t_1 \in \mathbb{R}^n$. Consider the vector

$$\hat{v}_1 \doteq (-t_1^T, s_1^T)^T.$$

Clearly $\hat{v}_1^T v_1 = 0$ and $\hat{v}_1 \neq v_1$. Moreover, it is straightforward to verify that for any block skew-symmetric $2n \times 2n$ matrix $R$,

$$\hat{v}_1^T R \hat{v}_1 = v_1^T R v_1 \tag{22}$$

and so in particular, $A^* \hat{v}_1 = 0$. By Assumption C we must therefore have

$$\hat{v}_1 = \pm v_2.$$

Thus, using (22) we have that for any block skew-symmetric $R$,

$$R \bullet W^* = (\theta_1 + \theta_2) R \bullet v_1 v_1^T.$$

It follows that $(\theta_1 + \theta_2) v_1 v_1^T$ is feasible for problem **R1** and with equal objective value as $W^*$, and therefore, optimal, as desired. For additional details see [26].

Assumption C is the key for obtaining rank-1 solutions. Does it always hold? Clearly not – the OPF problem is NP-hard. However, in [26] the authors present arguments that would suggest that it does hold in realistic instances, possibly after some small perturbations to the data. Indeed, they show that that is the case for the IEEE test instances. Following the publication of [26] other researchers have taken up this question. A number of small, simple, realistic examples where Assumption C does not hold appear in D. Molzahn's PhD thesis [37]; also see [27]. As of this writing, the preponderance of opinion is that Assumption C may not always hold on realistic instances – but that it will *nearly* hold, that is to say problem **R1** will usually have a low rank solution.

In any case, it is clear that the work in [26] has drastically redrawn the map of the OPF problem and almost certainly additional positive results are forthcoming. Further investigations are presented in [14, 15, 29] and references therein.

## 5 Approximate solution algorithms

Can we efficiently solve the SDP relaxation of the OPF problem discussed above? We point out that problems **R1** and **L1** are very sparse. In fact, their nonzero pattern is identical to the (graph-theoretic) set of neighbors of bus $k$; furthermore each $D_k$ has just two nonzeros. Overall, the nonzero pattern of **R1** and **L1** is essentially identical to the adjacency matrix of the underlying network (plus diagonal entries). Real-world grids are quite sparse, with average degree approximately $3$, say.

Reference [37] describes a number of experiments using the SDP solver SeDuMi [38], including an algorithm that takes advantage of the sparsity so as to preprocess formulation **L1** before presenting it to the solver (using matrix completion techniqes, see [24]). On the IEEE 300-bus case, they report running times on the order of $5$ seconds (using a current computer); this is comparable or better than the running times reported in [8] for local-optimality convex solvers (see Section 3). On various versions of the Polish grid (roughly 3000 buses) [37] reports running times on the order of $1500$ seconds. The sparsity-aware preprocessing proved invaluable; without its use the Polish grid examples could not be solved.

In both cases, these running times exceed the times reported by commercial vendors (used to obtain possibly local optima) by two orders of magnitude or more; we have observed that the DC approximation for the Polish grid solves, typically, in a fraction of a second. Of course, the SDP formulation (when the rank condition applies) has the supreme advantage of being correct. This is key when analyzing a grid under distress. Hence we are in a situation already discussed above: in routine situations, heuristic and approximate solvers are very fast and practicable, and, empirically, provide correct solutions. What is new is that now there is a theoretically sound, albeit slow, method for handling non-routine situations.

Future research will possibly produce significant speedups for the SDP relaxation, as well as a way to bypass the rank condition. As an intermediate step, researchers have produced a number of alternatives to the (linear) DC approximation, with some guarantees of accuracy under various degrees of grid "stress", while retaining computational practicability. These approximations linearize the power flow equations around the "flat start" point (e.g. $|V_k| = 1$, $\theta_{km} = 0$ for all $k$ and $km$) but with more flexibility than the DC approximation. See [3, 12, 17] (convex quadratic approximation). These approximations are less accurate than the full OPF formulation; however they remain highly practicable and have some theoretical guarantees. Quite likely, algorithms of this type will see increased usage in practical settings, in particular in volume computation settings.

A different approach is discussed in [7]. Consider equation (6), which describes the (active) power balance at bus $k$. If we assume $g_{km} \approx 0$ for all $km$ and all voltage magnitudes approximately equal to $1$, then (6) becomes

$$P_k \approx \sum_{km} (-b_{km} \sin \theta_{km}). \tag{23}$$

Reference [7] describes a way to solve the system of equations (23) for given $P_k$ while constraining the solution in terms of line flow limits. To describe this development we will choose an orientation for each line; for a line $i = uv$ we write $a_{ui} = 1$, $a_{vi} = -1$. For any bus $k \neq u, v$ we write $a_{ki} = 0$. For a line $i = km$ we write $\beta_i = b_{km}$, and we denote by $u_i$ the (active power) flow limit on $i$. In [7] the following formulation is discussed, where $\mathcal{L}$ indicates the set of (directed) lines:

$$\min_{\rho} \sum_{i \in \mathcal{L}} \beta_i \psi(\rho_i) \tag{24}$$

$$\text{s.t.} \sum_{i \in \mathcal{L}} \beta_i a_{ik} \rho_k = P_k \quad \forall k \tag{25}$$

$$|\rho_i| < \min\{1, u_i\}. \quad \forall_i \in \mathcal{L} \tag{26}$$

Here for $|x| < 1$,

$$\psi(x) \doteq \int_{-1}^{x} \arcsin(y)\, dy,$$

a convex function of $x$ since $\arcsin(x)$ is increasing for $x \in [-1, 1]$. Note that this formulation may be infeasible, and if the optimal solution of Eq. (24) occurs on the boundary of Eq. (26), then the solution is likely unstable. Suppose optimization problem (24-26) is well-defined, that is to say, it has an optimal solution which strictly satisfies (26). Then denoting by $\theta_i$ the optimal Lagrange multiplier for constraint (25) we obtain that for every line $i = km$,

$$\arcsin(\rho_i) = \theta_k - \theta_m, \tag{27}$$

in other words, we obtain an *exact* solution to (23) (by (25)). However, the objective function that we used to obtain this result is *not* what is desired in the OPF setting. In [4], the authors present an interior point (logarithmic barrier function) algorithm that solves the OPF problem under the assumptions described above ($g_{km} \approx 0$ for all $km$ and all voltage magnitudes approximately equal to 1) to guaranteed tolerance, in polynomial time.

## Notes

1. They ignore, for example, "shunt" effects.
2. Other, slower iterative methods are also possible, see, e.g., [1]

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Larry Snyder

# Multi-period optimal power flow problems

An important new development in the research on optimal power flow (OPF) problems has been the formulation and analysis of *multi-period OPF problems*. In some cases these problems have emerged in response to changes (or anticipated changes) in electricity grids, as the U.S. and other countries develop "smart grids" that will integrate a two-way communications infrastructure into the electricity network and require many new kinds of decisions for grid operators. In other cases the problems have been a natural stop on the trajectory of OPF research and have been studied recently simply because, due to advances in the optimization technologies used to solve OPF problems, they can be.

Although single-period OPF problems are ubiquitous in practice today, multi-period problems are likely to play a significant role in the planning and operation of transmission grids in the future. Since Dan Bienstock's column discusses single-period problems, I will use this discussion column to provide a brief introduction to multi-period OPF problems (sometimes called dynamic OPF problems).

Power planning problems exhibit several requirements that cannot be modeled without inter-temporal constraints. I'll focus on *ramping constraints*, which restrict the magnitude of changes in generation quantities between consecutive time periods to reflect technical requirements of the generators. The classical (single-period) OPF model can naturally be extended to $T$ periods by adding time indices to the parameters and decision variables in (5)–(8) (referring to the equation numbering in Dan's column) and replacing the objective function (9) with:

$$\min \sum_{t=1}^{T} \sum_{k\in\mathcal{G}} F_k(P_k(t),t). \tag{1}$$

Ramping constraints take the form

$$-\Delta_k^- \le P_k(t) - P_k(t-1) \le \Delta_k^+ \tag{2}$$

for constants $\Delta_k^-, \Delta_k^+ \ge 0$. (Other inter-temporal constraints could be added, for example, to model minimum or maximum total power output over the horizon.) The multi-period OPF model with ramping is then given by

$$\min \sum_{t=1}^{T} \sum_{k\in\mathcal{G}} F_k(P_k(t),t), \quad \text{subject to: (5)–(8), (2),} \tag{3}$$

where (5)–(8) have been modified to include time indices. See, for example, [10, 11], which formulate models similar to this and propose algorithms based on interior point methods. However, the computational burden for these algorithms is large since the problem size is an order of magnitude greater than that for single-period problems.

In practice, inter-temporal features are usually captured by constraints in the unit commitment (UC) problem [6, 9], which makes binary startup and shutdown decisions about multiple generators across a multi-period time horizon, ignoring power flows and network constraints. The UC problem is typically solved once per day for the following day's operations, and then the OPF problem is solved every fifteen minutes or so, adhering to the schedule set by the UC solution and accounting for power flow constraints. Some authors have combined the two models, i.e., including OPF-type power flow constraints in the UC problem. For example, [2] and [8] propose Benders decomposition and augmented Lagrangian methods, respectively. Solution times are on the order of minutes or hours, making these joint models valuable as refinements to the UC problem but impractical as replacements for the OPF problem, which must be solved multiple times per hour.

A common explanation for the separability of OPF problems by time period is that today's power systems require demands in a short time interval to be supplied, exactly, by generation within the same interval; therefore there is no coupling of time intervals, at least with respect to supply and demand. However, the requirement that demand must match supply within a given, short time span – a requirement that has been at the heart of grid operation for more than a century – is changing, due to the development of grid-scale *energy storage systems* (ESS) such as batteries, flywheels, compressed air, etc. [1] This complicates the OPF paradigm since storage couples time periods: decisions about generation and storage charging/discharging must account for the future as well as the present. The resulting multi-period OPF models might be solved, for example, with 15-minute time periods and a 24-hour horizon, perhaps on a rolling-horizon basis.

Assume that each generator in $\mathcal{G}$ has a co-located ESS, e.g., a battery. To incorporate storage into the multi-period OPF model, we introduce new decision variables $g_k(t) \ge 0$ representing the amount of power generated at $k \in \mathcal{G}$ in time period $t$, as well as variables $r_k(t)$ representing the amount of power discharged from the ESS (or charged to the ESS, if $r_k(t) < 0$) at node $k$ in period $t$.[1] The power injected into the grid from generator $k$ is no longer given by (12), but rather by

$$P_k(t) = g_k(t) + r_k(t); \tag{4}$$

now $g_k(t)$ and $r_k(t)$ are the "direct" decision variables, whereas $P_k(t)$ is derived from it via (4) and is related back to the grid via (12). (We will use the DC (linear) approximation and thus ignore $Q_k(t)$.) The generation amounts must be nonnegative:

$$g_k(t) \ge 0. \tag{5}$$

The energy level in the ESS at node $k$ in period $t$, denoted $b_k(t)$, is governed by

$$b_k(t) = b_k(t-1) - r_k(t). \tag{6}$$

(Before proceeding, we note that (4) and (6) together are equivalent to to $b_k(t) = b_k(t-1) + g_k(t) - P_k(t)$, which is reminiscent of the equation used to describe the dynamics of periodic-review inventory systems; see, e.g., [12]. Here $b$ stands in for the inventory level, $g$ for the order quantity, and $P$ for the demand. This is not an accident, of course, since an ESS is essentially an inventory of energy.) In addition, the energy level must remain in some given range:

$$B_k^L \le b_k(t) \le B_k^U \tag{7}$$

for all $k$ and $t$. (Often $B_k^L = 0$.)

Combining these, we get the *multi-period OPF model with storage* introduced by Chandy et al. [4]:

$$\min \quad \sum_{t=1}^{T} \sum_{k\in\mathcal{G}} [F_k(g_k(t),t) + h_k(b_k(t))] + \sum_{k\in\mathcal{G}} h_k^T(b_k(T)) \tag{8}$$

subject to: (7), (11)–(12), (4)–(7)

In the objective function (8), $F_k(g_k(t),t)$ is the generation cost in period $t$, $h_k(b_k(t))$ is the ESS cost as a function of the charge level, and $h_k^T(b_k(T))$ is a terminal ESS cost. In [4], $h_k(\cdot)$ is assumed to be decreasing, an assumption that is meant to encourage high charge levels since a full ESS has more value. However, one could instead assume $h_k(\cdot)$ is increasing, in which case the function acts like a "holding cost" in the inventory sense. In (7), (11), and (12) we add time indices, and in (7) and (11) we consider the constraints related to $P$ only. The model in [4] also allows the voltage magnitudes $|V_k|$ to take values other than 1, though they are assumed to be constant. It does not include ramping constraints as in (2).

In the special case in which the network consists of a single generator/ESS and a single load connected by a line of infinite capacity, the problem reduces to something that is very much like an inventory problem. Chandy, et al. [4] derive the KKT conditions for this problem and use them to find an explicit optimal solution. They also prove that the optimal charging pattern follows a particular shape, conditioned on a (somewhat restrictive) assumption on the cost and demand structures over the horizon. For the general (multiple-generator/load) problem, [4] again derives KKT conditions, although, assuming $F_k(\cdot)$ is convex, the problem can also be solved using a convex optimization solver such as CPLEX.

The recent results by Lavaei and Low [7] for the single-period OPF problem extend to the multi-period problem: Gayme and Topcu [5] modify the problem above to include AC power flow constraints (5) and (6); they solve an SDP relaxation of the original OPF problem and prove that, under certain assumptions, there is no duality gap.

These models can also be modified to accommodate more realistic factors such as charging/discharging capacities and efficiencies. In the latter case, one can change (6) to

$$b_k(t) = b_k(t-1) - \eta r_k(t) \tag{9}$$

if the charging and discharging efficiency are both equal to $\eta$. Typically this is not the case, so it is necessary to split the $r_k$ variables to represent charging and discharging separately:

$$b_k(t) = b_k(t-1) + \eta^{\text{in}} r_k^{\text{in}}(t) - \eta^{\text{out}} r_k^{\text{out}}(t), \tag{10}$$

where $r_k^{\text{in}}(t),\ r_k^{\text{out}}(t) \geq 0$. A minor difficulty can arise with this approach: as [3] shows, the first-order conditions may fail to satisfy the linear independence constraint qualification (LICQ), causing the Jacobian to become singular and the Newton–Raphson method to fail to converge. This happens, for example, if the ESS begins the horizon fully charged or fully empty and it is optimal for it to stay at that level during the first time period. For the multi-period OPF, the issue can be avoided using a simple modeling trick, but the results in [3] may have implications for other multi-period models that make use of (10).

In addition to storage, other features of advanced electricity grids will increase the importance of multi-period OPF problems. The increased penetration of *renewable generation resources*, and their inherent volatility and uncertainty, will mean that traditional generators will have to ramp more quickly and more often, making ramping constraints a critical feature of OPF problems. The introduction of new *demand response* (DR) programs, in which consumers are encouraged to shift their demands away from peak times via pricing or other mechanisms, will add new temporal (and stochastic) aspects to the loads themselves. Moreover, in some DR programs such as *direct load control* (in which the grid operator directly shifts the timing of consumers' HVAC or other devices), the load becomes a decision variable that functions very much like the generation variables, with additional inter-temporal constraints. In addition, new *sensing and communications* infrastructure integrated into the grid will enable better forecasts of conditions in the near future, providing opportunities for improved inter-temporal decision making.

### Note

1. Since we assume these and other quantities have units of energy per period, we use the terms *power* and *energy* interchangeably.

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Tamás Terlaky

# ICCOPT 2013

### Looking back

This year marks the 10th anniversary of the birth of ICCOPT. The idea to have a triennial continuous optimization conference was first born during ISMP 2003 on the Copenhagen-Lyngby suburban train. Jean-Philippe Vial and myself started a lively discussion on the importance of continuous optimization, and the lack of an annual forum for the continuous optimization community to come together and celebrate the most notable achievements of the field. The next day during the lunch break many members of the community came together and provided strong support to initiate the ICCOPT series as a major instrument of the Mathematical Optimization Society (MOS), MPS, at that time. As the chair of the ICCOPT Steering Committee of MOS until the summer of 2013, it is my pleasure and privilege to report on the success of ICCOPT 2013.

### ICCOPT 2013

The Fourth International Conference on Continuous Optimization took place from July 27 to August 1, 2013, in the Department of Mathematics of the Faculty of Science and Technology of the New University of Lisbon, Caparica, Portugal. The meeting was co-chaired by Katya Scheinberg (Program Committee) and by Paula Amaral and Joaquim João Júdice (Organizing Committees), and overall coordinated by Luis Nunes Vicente.

The activities started during the weekend with two 5-hour intensive Summer Courses. On Saturday, July 27, PDE-Constrained Optimization was taught by Michael Ulbrich and Christian Meyer and on Sunday, July 28, Sparse Optimization and Applications to Information Processing was taught by Mário A. T. Figueiredo and Stephen J. Wright. Both courses were of excellent quality and attended by more than one hundred participants each. ICCOPT 2013 partially supported the traveling or lodging expenses of 21 young participants, mostly PhD students, who attended the Summer School and the Conference.

The scientific program of the Conference spanned over 4 days (July 29–August 1). The featured talks were given by Paul I. Barton, Michael C. Ferris, Yurii Nesterov, and Yinyu Ye (plenaries) and by Amir Beck, Regina Burachik, Sam Burer, Coralia Cartis, Michel De Lara, Victor DeMiguel, Michael Hintermüller, and Ya-xiang Yuan (semi-plenaries). A total of 479 participants profited from these great lectures which covered nearly all topics in Continuous Optimization. The bulk of the program was organized around 13 clusters of invited sessions and sessions of contributed talks (coordinated by a crucial team of 26 co-chairs) resulting in a total of 412 talks and 13 posters. The program was enriched with a plenary session featuring talks by the three finalists of the Best Paper Prize for Young Researchers in Continuous Optimization, namely Venkat Chandrasekaran (the winner), Boris Houska, and Meisam Razaviyayn, who were selected among more than 30 applicants by a selection committee of Stefan Ulbrich (the chair), Sam Burer, and Jean-Baptiste Hiriart-Urruty.

As it is becoming the standard in all ICCOPTs, the social program included the MOS Welcome Reception on Sunday, the Reception during the Session of Poster Presentations on Monday, the Conference Banquet on Tuesday, and the Student Social on Wednesday. The winner of the Best Paper Prize was announced at the banquet followed by a live performance of Fado singing. Participants were offered lunch during every day of the meeting, outside in the interior courtyard of the Department. Most of those who stayed until the end on Thursday went on to the Conference Tour to visit a natural park and local wine cellars.

The logistics of the meeting were essentially organized by a team of 12 colleagues, namely Carmo Brás, Nelson Chibeles-Martins, Isabel Correia, Ana Luísa Custódio, Luís Merca Fernandes, Isabel Gomes, João Gouveia, Ismael Vaz, Manuel Vieira, Zaikun Zhang, led by Paula Amaral and Luis Nunes Vicente.

Thanks to the excellent work of the Program Committee, the Organizing Committees, the help of a large number of student volunteer, and last but not least, the general chair Luis Nunes Vicente's exceptional leadership and organizational talent, ICCOPT 2013 was a sound success. The fourth edition of the ICCOPT series, has solidified the position of the ICCOPT conferences as one of the prime optimization conferences, providing a major international venue for the continuous optimization community to gather each year at the ISMP, ICCOPT and at the SIAM Optimization conferences.

### Looking forward – The next steps

In the Summer of 2013 the ICCOPT Steering Committee was rejuvenated. After ten years of service I am passing the torch to Jong-Shi Pang, the new Chair of the ICCOPT Steering Committee. The first action of the renewed committee was to select the site of ICCOPT 2016. As it was announced in Lisbon, ICCOPT-V, the next edition of the series, will take place in Tokyo, Japan. After a difficult decision in choosing among several well-written site proposals, the 2016 ICCOPT Steering Committee picked Tokyo, Japan as the host for this event. Under the enthusiastic leadership of Professor Shinji Mizuno and his local team, we look forward to another interesting conference in this series that promises to extend the impact of continuous optimization to the Asian countries and offer an opportunity for the local optimization experts to interact with their international colleagues.

Looking forward to meet all of you again at ICCOPT 2016 in Tokyo!

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## SIAM Conference on Optimization 2014

*May 19–22, 2014, Town and Country Resort & Convention Center, San Diego, California, USA.*

*Plenary Speakers*
Retsef Levi, Massachusetts Institute of Technology, USA / Joaquim R. R. A. Martins, University of Michigan, USA / Yurii Nesterov, Catholic University of Louvain, Belgium / Cynthia A. Phillips, Sandia National Laboratories, USA / Andy Philpott, The University of Auckland, New Zealand / Franz Rendl, Universität Klagenfurt, Austria / Francisco Santos, Universidad de Cantabria, Spain / Rekha R. Thomas, University of Washington, USA

*Invited Minitutorials*
*Mixed-integer Nonlinear Optimization*
Sven Leyffer, Argonne National Laboratory, USA Jeff Linderoth, University of Wisconsin Madison, USA Jim Luedtke, University of Wisconsin Madison, USA
*Polynomial Optimization*
Didier Henrion, University of Toulouse, France Monique Laurent, Centrum Wiskunde & Informatica, Amsterdam, The Netherlands, and Tilburg University, The Netherlands

*Minisymposia*
Over 100 minisymposia on various aspects of optimization theory, algorithms, and applications, including:

- Coordinate Descent Methods
- Derivative-Free Optimization
- Energy Applications
- Geometry of Linear Optimization
- Global Optimization
- Healthcare Applications
- Integer and Discrete Optimization
- Large-Scale Inverse Problems
- Multidisciplinary Design Optimization
- Nonlinear Optimization
- Numerical Linear Algebra
- PDE-Constrained Optimization
- Polynomial and Tensor Optimization
- Robust Optimization
- Semidefinite and Conic Optimization
- Stochastic Optimization

*Contributed Presentations (in Lecture or Poster Format)*

We look forward to seeing you in San Diego!

Miguel F. Anjos and Michael J. Todd
Organizing Committee Co-chairs

# IPCO 2014
# The 17th Conference on Integer Programming and Combinatorial Optimization

*Summer school: June 20–22, Conference: June 23–25, 2014, Bonn, Germany.* The IPCO conference is a forum for researchers and practitioners working on various aspects of integer programming and combinatorial optimization. The aim is to present recent developments in theory, computation, and applications. The scope of IPCO is viewed in a broad sense, to include algorithmic and structural results in integer programming and combinatorial optimization as well as revealing computational studies and novel applications of discrete optimization to practical problems.

In the preceding summer school, the following distinguished speakers have accepted our invitation to give lectures: Gérard Cornuéjols (Carnegie Mellon University, Pittsburgh), Friedrich Eisenbrand (École Polytechnique Fédérale Lausanne), András Frank (Eötvös University, Budapest), David Shmoys (Cornell University, Ithaca).

*Organizing Committee:*
Stephan Held (co-chair), Stefan Hougardy, Bernhard Korte, Jens Vygen (chair)

*Program Committee:*
Flavia Bonomo (Universidad de Buenos Aires), Sam Burer (University of Iowa), Gérard Cornuéjols (Carnegie Mellon University), Satoru Fujishige (Kyoto University), Michael Jünger (Universität zu Köln), Matthias Köppe (University of California, Davis), Jon Lee, chair (University of Michigan), Jeff Linderoth (University of Wisconsin), Jean-Philippe Richard (University of Florida), András Sebő (CNRS, Laboratoire G-SCOP, Grenoble), Maxim Sviridenko (University of Warwick), Chaitanya Swamy (University of Waterloo), Jens Vygen (Universität Bonn), David P. Williamson (Cornell University), Laurence Wolsey (Université catholique de Louvain)

The program committee is currently selecting approximately 33 papers among the record number of 143 submissions. The list of accepted papers will be published in early February 2014.

There will also be a poster session. Abstracts for posters can be submitted by March 31, 2014.

Registration is now open. The deadline for early registration is April 20, 2014.

http://www.or.uni-bonn.de/ipco/

*IPCO 2014 in Bonn (Michael Sondermann/Presseamt Bundesstadt Bonn)*

# Summer schools 2014 at the Centro Internazionale Matematico Estivo

*June – September, 2014, Firence, Italy.* In 2014, the following courses will be offered at the Centro Internazionale Matematico Estivo (International Mathematical Summer Center):

– Partial Differential Equations and Geometric Measure Theory June 2–7, 2014 – Cetraro (CS)
– Computational Electromagnetism June 9–14, – Cetraro (CS)
– Centralized and Distributed Multi-agent Optimization: Models and Algorithms. CIME-EMS Summer School in applied mathematics June 23–28, 2014 – Cetraro (CS)
– Mathematical Models and Methods for Living Systems CIME-CIRM Course September 1–6, 2013 – Levico Terme (TN)

Proposals for new courses are welcome and they can be submitted, preferably via e-mail, to cime@math.unifi.it.

CIME activity is carried out with the collaboration and financial support of: INdAM (Istituto Nazionale di Alta Matematica), MIUR (Ministero dell'Istruzione, dell'Università e della Ricerca), and Ente Cassa di Risparmio di Firenze.

http://web.math.unifi.it/users/cime/

# Mixed Integer Programming (MIP 2014)

*July 21–24, 2014, Ohio State University, Columbus, OH, USA.* This will be the eleventh in a series of annual workshops held in North America designed to bring the integer programming community together to discuss very recent developments in the field. The workshop series consists of a single track of invited talks and also features a poster session as an additional opportunity to share and discuss recent research. Registration details and a call for participation in the poster session will be announced later.

*Confirmed speakers:*
Karen Aardal (TU Delft), Warren Adams (Clemson), Alper Atamtürk (Berkeley), Timo Berthold (Xpress), Daniel Bienstock (Columbia), Pierre Bonami (IBM CPLEX), Sanjeeb Dash (IBM T. J. Watson), Juliane Dunkel (IBM Research – Zurich), Zonghao Gu (Gurobi), Raymond Hemmecke (TU Munich), Robert Hildebrand (ETH Zurich), Simge Küçükyavuz (Ohio State), Jeff Linderoth (UW-Madison), Andrea Lodi (University of Bologna), Marco Molinaro (Georgia Tech), Michele Monaci (University of Padova), Francois Margot (Carnegie Mellon), Giacomo Nannicini (Singapore University of Technology and Design), Laurent Poirrier (University of Padova), Sebastian Pokutta (Georgia Tech), Domenico Salvagnin (University of Padova), Stefan Vigerske (GAMS), Andreas Wächter (Northwestern), Minjiao Zhang (University of Alabama).

*Program Committee:*
Amitabh Basu (chair) (Johns Hopkins), Tobias Achterberg (IBM CPLEX, ZIB), Jon Lee (University of Michigan), Susan Margulies (US Naval Academy), Jim Ostrowski (University of Tennessee Knoxville).

*Local Committee:*
Simge Küçükyavuz (chair) (Ohio State), Ramteen Sioshansi (Ohio State).

Contact: mip2014@osu.edu
https://mip2014.engineering.osu.edu

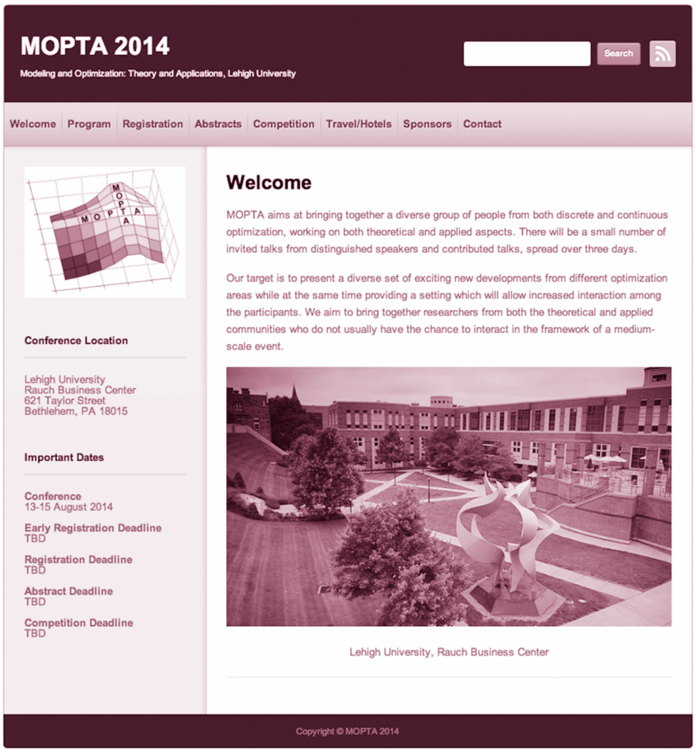
## MOPTA 2014

*August 13–15, 2014, Lehigh University Rauch Business Center Bethlehem, PA, USA.* MOPTA aims at bringing together a diverse group of people from both discrete and continuous optimization, working on both theoretical and applied aspects. There will be a small number of invited talks from distinguished speakers and contributed talks, spread over three days. Our target is to present a diverse set of exciting new developments from different optimization areas while at the same time providing a setting which will allow increased interaction among the participants. We aim to bring together researchers from both the theoretical and applied communities who do not usually have the chance to interact in the framework of a medium-scale event.

*Confirmed plenary speakers:*
Gerald Cornuejols (Carnegie Mellon University), Darinka Dentcheva (Stevens Institute of Technology), Warren Powell (Princeton University), Asu Ozdaglar (MIT), Andreas Wächter (Northwestern University).

*Organizing Committee:*
Boris Defourny (Chair), Tamás Terlaky, Frank E. Curtis, Katya Scheinberg, Ted K. Ralphs, Robert H. Storer, Aurélie C. Thiele, Larry V. Snyder, Eugene Perevalov, Luis F. Zuluaga.

We look forward to seeing you at MOPTA 2014!




http://coral.ie.lehigh.edu/~mopta/

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